Supporting Information

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SI Text

DSICE Model

The stochastic IAM that we use in the present analysis is an adapted version of the DSICE framework used in ref. 1. The latter builds on the DICE-CJL model (2), which itself is a numerically stable version of DICE 2007 (3) with a flexible time period length. The DICE model has been applied in numerous studies. Besides those associated with risk and uncertainty, the model parameters used for our analysis are calibrated to the same levels as those used for DICE. Generally, a multidimensional stochastic IAM (e.g., the one in the present study) requires heavy computation, but DSICE makes it tractable by using an efficient and accurate dynamic programming algorithm (4).

In this section, we present the major equations of the model used in this study. For details on some specific functional forms and parameter levels, the reader is referred to ref. 2.

Like DICE, DSICE computes the time paths of the optimal carbon emission control for the world. A social planner is set to weigh the costs and benefits of emission control. Uncertainty (stochasticity) of climate change effects is included in such a way that the social planner makes emission control decisions by projecting future developments of climate and the economy that are not precisely known at the times of decisions. The model finds the levels of consumption and emission control that maximize the expected present value of global social welfare

$$\max_{C_t>0, 0\le\mu_t\le 1} \mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t u(C_t, S_t)\right\}$$

where $\mathbb{E}$ represents the expectation operator, β signifies the discount factor, and C and μ are consumption and the emission control rate. Furthermore, S represents the nonmarket benefits from ecosystem services. The exact functional forms for C and S are given by Eqs. **1** and **2** and repeated here for convenience:

$$u(C_t, S_t) = \left[(1-\gamma)\left(\frac{C_t}{L_t}\right)^{1-1/\sigma} + \gamma\left(\frac{S_t}{L_t}\right)^{1-1/\sigma}\right]^{(1-\alpha)\sigma/(\sigma-1)} \frac{L_t}{1-\alpha}$$

$$S_t = \frac{S_0\left(1 - I_t J_S^*\right)}{1+\lambda\left(T_t^{AT} - T_0^{AT}\right)^2}.$$

Here, σ represents substitutability between the goods, whereas α is the risk aversion parameter and γ is the share parameter of ecosystem benefits in the welfare. (When $\sigma = 1$, the utility function becomes $u(C_t, S_t) = [(C_t/L_t)^{1-\gamma}(S_t/L_t)^{\gamma}]^{1-\alpha}[L_t/(1-\alpha)]$.) The total world labor supply is denoted by L. Furthermore, λ is a scaling parameter to enforce consistency with the DICE model calibration, and J_S^* represents the tipping point damage level for the environmental goods. We let I_t be the indicator function showing whether the tipping event has happened at time t or not. That is, $I_t = 0$ means that it has not happened and $I_t = 1$ means that it has happened at time t. It is stochastic, and it's dynamic evolution is determined by

$$I_{t+1} = g(I_t, \mathbf{T}_t, \omega_t)$$

with ω being an independent and identically distributed random process. The process therefore can be represented as a discrete Markov chain with the Markov chain probability transition matrix at time t being

$$\begin{bmatrix} 1-p_t & p_t \\ 0 & 1 \end{bmatrix}$$

where its (i, j) element is the transition probability from state i to j for I_t. State 1 represents the pretipping state with $I_t = 0$, and state 2 is the absorbing state in the transition matrix, representing the irreversible nature of the tipping point with $I_t = 1$. Furthermore, the tipping probability p_t is given by

$$p_t = 1 - \exp\left\{-\nu \max\left\{0,\ T_t^{AT} - 1\right\}\right\},$$

where ν is a calibrated hazard rate factor so that $p_t = 5\%$ at $T_t^{AT} = 4$ °C. The likelihood of tipping is therefore endogenous on global warming.

Economic output at year t is produced from capital k_t and labor supply L_t according to

$$f_t(k_t, L_t) = A_t k_t^{\alpha} L_t^{1-\alpha},$$

where α is the capital share and A_t is a total productivity factor. Both evolve exogenously (see ref. 2 for the exact functional forms).

Output is affected by global average atmospheric temperature T_t^{AT} (temperature increase relative to the 1900 level). Climate factors reduce output by $1 - \Omega(T_t^{AT})$ where

$$\Omega\left(T_t^{AT}, I_t\right) = \frac{1 - I_t J_Y^*}{1+\pi_2\left(T_t^{AT}\right)^2},$$

where π_2 is a coefficient of the damage factor and J_Y^* is the tipping damage level for the output. Abatement effort μ will reduce total net CO_2 emissions at some cost as a share of output. Therefore, the net output function at year t is

$$\mathcal{Y}_t\left(k_t, T_t^{AT}, I_t, \mu_t\right) = \left(1 - \theta_{1,t}\mu_t^{\theta_2}\right)\Omega\left(T_t^{AT}, I_t\right) f_t(k_t, L_t),$$

where $\mu_t \in [0, 1]$ is the emission control rate, θ_1 is an exogenously given variable describing the effectiveness of the abatement technology, and θ_2 is a parameter. In total, the dynamics of the economic sector are driven by the dynamics of its stock of physical capital. The latter is given by

$$k_{t+1} = (1-\delta)k_t + \mathcal{Y}_t\left(k_t, T_t^{AT}, I_t, \mu_t\right) - C_t,$$

where δ is the annual rate of depreciation of capital.

The structure of the carbon cycle in this study is adapted from the DICE 2007 model (3). The CO_2 concentrations for the carbon cycle are modeled by a three-box module with

$$\mathbf{M}_t = \left(M_t^{AT}, M_t^{UP}, M_t^{LO}\right)^{\top},$$

representing carbon concentrations in the atmosphere (M_t^{AT}), the upper oceans (M_t^{UP}), and the lower oceans (M_t^{LO}). The transition system of the CO_2 concentration from year t to year $t+1$ is

$$\mathbf{M}_{t+1} = \mathbf{\Phi}^{M}\mathbf{M}_t + \left(\mathcal{E}_t(k_t, \mu_t), 0, 0\right)^{\top},$$

with

$$\mathcal{E}_t(k, \mu) = \sigma_t(1-\mu) f_t(k, L_t) + E_t^{Land},$$

where the first term denotes the endogenous emission from industrial production (σ_t is an exogenously evolving abatement technology path) and the second term denotes an exogenous projection of carbon emissions from biological processes. Furthermore, the carbon cycle transition matrix is given by

$$\mathbf{\Phi}^{\mathrm{M}} = \begin{bmatrix} 1-\phi_{12} & \phi_{12}\varphi_1 & 0 \\ \phi_{12} & 1-\phi_{12}\varphi_1-\phi_{23} & \phi_{23}\varphi_2 \\ 0 & \phi_{23} & 1-\phi_{23}\varphi_2 \end{bmatrix},$$

where ϕ_{12} and ϕ_{23} are parameters denoting fluxes in the carbon cycle and φ_1 and φ_2 denote preindustrial carbon ratios of the three boxes (see ref. 1 for a detailed calibration).

The CO_2 concentrations in the atmosphere affect the global average surface temperature via radiative forcing:

$$\mathcal{F}_t\left(M_t^{\mathrm{AT}}\right) = \eta \log_2\left(M_t^{\mathrm{AT}}/M_*^{\mathrm{AT}}\right) + F_t^{\mathrm{EX}},$$

where η is an exogenous forcing parameter, M_*^{AT} is the preindustrial carbon concentration in the atmosphere, and F_t^{EX} denotes exogenous radiative forcing (see ref. 2 for the exact functional forms).

Here, we also make use of the DICE 2007 two-box model for the climate. The global mean temperature is represented by a two-layer model,

$$\mathbf{T}_t = \left(T_t^{\mathrm{AT}}, T_t^{\mathrm{LO}}\right)^\top,$$

representing the average temperature in the atmosphere (T_t^{AT}) and the lower oceans (T_t^{LO}). The transition system of the global average temperature from year t to year $t+1$ is

$$\mathbf{T}_{t+1} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{T}_t + \left(\xi_1 \mathcal{F}_t\left(M_t^{\mathrm{AT}}\right), 0\right)^\top,$$

where

$$\mathbf{\Phi}^{\mathrm{T}} = \begin{bmatrix} 1-\xi_1\eta/\xi_2-\xi_1\xi_3 & \xi_1\xi_3 \\ \xi_4 & 1-\xi_4 \end{bmatrix},$$

where ξ_1, ξ_3, and ξ_4 are parameters calibrated in ref. 2 and ξ_2 is the climate sensitivity parameter.

The stochastic optimization problem of the social planner becomes

$$\begin{aligned} \max_{C_t,\mu_t} \quad & \mathbb{E}\left\{ \sum_{t=0}^{\infty} e^{-\rho t} u(C_t, S_t) \right\} \\ \text{s.t.} \quad & k_{t+1} = (1-\delta)k_t + \mathcal{Y}_t\left(k_t, T_t^{\mathrm{AT}}, I_t, \mu_t\right) - C_t, \\ & \mathbf{M}_{t+1} = \mathbf{\Phi}^{\mathrm{M}}\mathbf{M}_t + \left(\mathcal{E}_t(k_t, \mu_t), 0, 0\right)^\top, \\ & \mathbf{T}_{t+1} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{T}_t + \left(\xi_1\mathcal{F}_t\left(M_t^{\mathrm{AT}}\right), 0\right)^\top, \\ & I_{t+1} = g(I_t, \mathbf{T}_t, \omega_t). \end{aligned}$$

Its solution method is described in ref. 1.

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4. Cai Y, Judd K (2010) Stable and efficient computational methods for dynamic programming. *J Eur Econ Assoc* 8(2-3):626–634.