Solving Asset Pricing Models: A Statistical Approach

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Agenda

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- Introduce a new general-purpose estimation method for structural models, the *empirical projection method*.

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- Introduce a new general-purpose estimation method for structural models, the *empirical projection method*.
- Demonstrate the method on several asset-pricing models.

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$$E_t(P_t - m_{t+1}(P_{t+1} + D_{t+1})) = 0,$$

where m_{t+1} is a function of some other variables, such as aggregate consumption, C_t.

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I propose a third.

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Parameters are then chosen to either match specific moments, or maximize a likelihood.

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The answer depends on the choice of distributions.

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so

$$0 = E(E_t(F)I_t^i) = E(FI_t^i).$$

GMM, cont'd

In GMM, parameters are chosen to minimize

$$\sum_i E(FI_t^i)^2$$

or something like it.

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The method is an empirically-based version of projection methods.

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We can't find this function exactly, but we can approximate it,

$$P(C_t, D_t) = \sum_{i=1}^{N} a_i P^i(C_t, D_t)$$

Projection Methods, cont'd

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This gives N equations in N unknowns.

Galerkin method

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The choice of W is arbitrary. I exploit this.

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Then

$$\begin{aligned} L_i E_t(F) &= E(E_t(F)P^i) \\ &= E(FP^i). \end{aligned}$$

But I can compute this without knowing the distribution of x!

Empirical Galerkin method, cont'd

Just compute

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from the data, for each i.

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Then choose the a_i so that each of the above equations are zero.

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You can use this to compute a time series of P predicted by the model. Then you can compute moments, test goodness-of-fit, etc.

Application to Asset Pricing

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- CRRA – people are impatient, and hate risk.
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I take several estimates from the literature, and see how they do in replicating the time series of risk-free rates and S&P 500 price-dividend ratios.

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- For external habit: log consumption growth rate and dividend/consumption ratio, and log S_t – the surplus consumption ratio of Campbell-Cochrane.

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All data is Shiller annual dataset.

Sources of Estimates

I take model parameter estimates from the literature:

- Hansen-Singleton (1982) – CRRA
- Ferson-Constanides (1991) – internal habit
- Campbell-Cochrane (1999) – external habit (calibrated)
- Tallarini-Zhang (2005) – external habit (estimated)

Table: CRRA Estimates

	Data	Hansen and Singleton (1982)
β	NA	0.9751
γ	NA	0.9001
$E(R)$	0.0752	0.0472
$\sigma(R)$	0.1660	0.0542
$E(\log(P/D))$	3.2105	3.5572
$\sigma(\log(P/D))$	0.3573	0.1496
$E(R^f)$	0.0450	0.0439
$\sigma(R^f)$	0.0243	0.0114

Figure: CRRA Predictions: Risky Asset

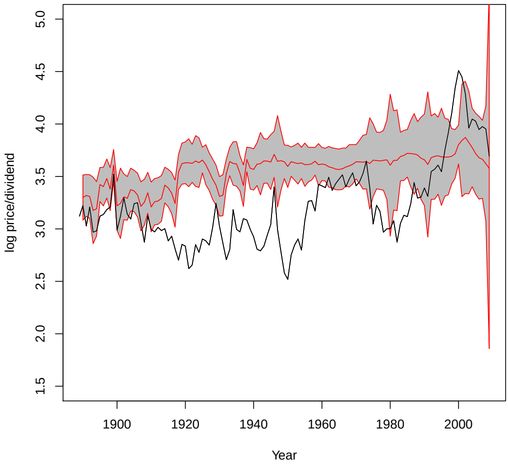


Table: One-lag Habit Estimates

	Data	Ferson-Constanides (1991)
β	NA	1.0250
γ	NA	2.5800
h	NA	0.1200
$E(R)$	0.0752	0.0308
$\sigma(R)$	0.1660	0.0821
$E(\log(P/D))$	3.2105	4.6178
$\sigma(\log(P/D))$	0.3573	0.2072
$E(R^f)$	0.0450	0.0296
$\sigma(R^f)$	0.0243	0.0515

Figure: Comparison of one-lag habit estimates

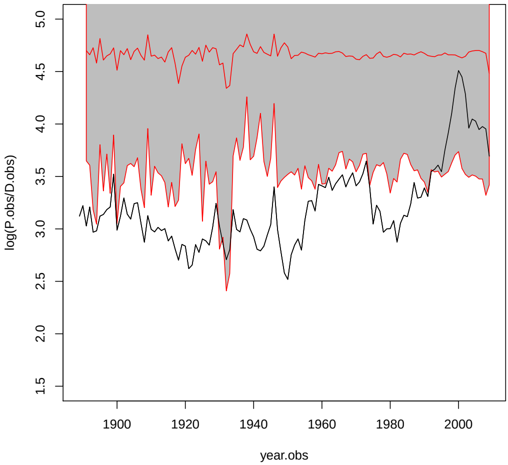
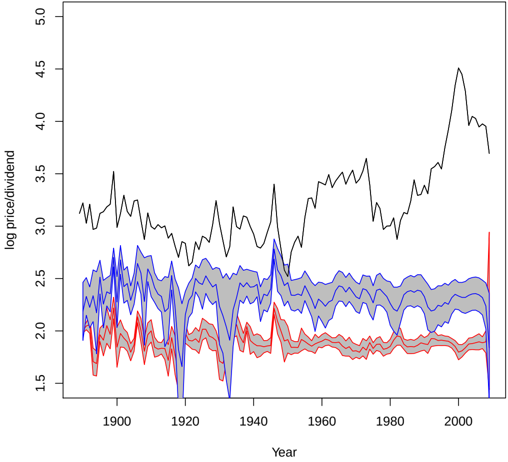


Table: External Habit Estimates

	Data	Campbell-Cochrane (1999)	Tallarini-Zhang (2005)
β	NA	0.8900	1.0100
γ	NA	2.0000	6.0000
ϕ	NA	0.8700	0.9000
$E(R)$	0.0752	0.1719	0.1223
$\sigma(R)$	0.1660	0.0994	0.1725
$E(\log(P/D))$	3.2105	1.8948	2.3441
$\sigma(\log(P/D))$	0.3573	0.0748	0.1066
$E(R^f)$	0.0450	0.1660	0.1017
$\sigma(R^f)$	0.0243	0.0403	0.0998

Figure: Time Series of External Habit Price Predictions



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- It is a GMM estimator, but turned to a different purpose. Ordinary GMM takes data and gives you parameter estimates. Here we take parameter estimates, and produce predictions of the prices.
- This allows us to evaluate the failures of models along different dimensions, rather than simply accepting or rejecting.