Optimal Income Taxation with Multidimensional Types

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PRELIMINARY AND INCOMPLETE

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Introduction

- Optimal income taxation: Mirrlees
  - Heterogeneous productivity
  - Utilitarian (or redistributive) objective
  - Standard cases: clear pattern of binding IC constraints; tax rates in [0,1].
- Criticism of Mirrlees - not enough heterogeneity
- Multidimensional heterogeneity
  - Little theory; special cases only
  - No clear pattern of binding IC constraints
  - Revelation principle still holds, producing a nonlinear optimization problem with IC constraints.
  - Clearly more realistic than 1-D models.

Two-D Types - Productivity and Elasticity of Labor Supply

- $u^j(c,l) = \log c - l^{1/\eta_j+1}/(1/\eta_j + 1)$
- w_i is productivity type i.
- (c_{ij}, y_{ij}) is allocation for (i,j)-type taxpayer.
- Zero tax solution for type (i,j) is $(l^*_{ij}, c^*_{ij}, y^*_{ij}) = (1, w_i, w_i)$.
- Problem:

$$
\begin{aligned}
\max_{(y,c)} \quad & \sum_{i=1}^N \sum_{j=1}^N \lambda_{ij}\, u^j(c_{ij}, y_{ij}/w_i) \\
& u^j(c_{ij}, y_{ij}/w_i) - u^j(c_{i'j'}, y_{i'j'}/w_i) \geq 0 \quad \forall (i,j), (i',j') \\
& \sum_{i=1}^N \sum_{j=1}^N c_{ij} \leq \sum_{i=1}^N y_{ij} \\
& \sum_{i=1}^N \sum_{j=1}^N c_{ij} \geq 0,
\end{aligned}
$$

- We choose the following parameters:
  - $N = 5$, $w_i = i$
  - $\lambda_i = 1$
  - $\eta = (1, 1/2, 1/3, 1/5, 1/8)$.
  - We use the zero tax solution (c^*, y^*) as a starting point for the NLP solver.

Table 6. $\eta = (1, 1/2, 1/3, 1/5, 1/8)$, $w = (1, 2, 3, 4, 5)$

(i,j)	c_{ij}	y_{ij}	$MTR_{i,j}$	$ATR_{i,j}$	l_{ij}/l^*_{ij}	c_{ij}/c^*_{ij}	Utility	
							Judd-Su	Mirrlees
$(1,1)$	1.68	0.42	0.28	-2.92	0.42	1.68	0.4294	.3641
$(1,2)$	1.77	0.62	0.32	-1.86	0.62	1.77	0.4952	.3138
$(1,3)$	1.79	0.65	0.51	-1.75	0.65	1.79	0.5378	.6601
$(1,4)$	1.83	0.77	0.50	-1.37	0.77	1.83	0.5700	.7830
$(1,5)$	1.86	0.86	0.43	-1.16	0.86	1.86	0.5940	.8760
$(2,1)$	1.86	0.86	0.60	-1.16	0.43	0.93	0.5308	.3751
$(2,2)$	2.03	1.39	0.50	-0.45	0.69	1.01	0.5973	.6180
$(2,3)$	2.07	1.50	0.56	-0.38	0.75	1.03	0.6512	.7189
$(2,4)$	2.16	1.74	0.46	-0.24	0.87	1.08	0.7006	.8181
$(2,5)$	2.20	1.83	0.46	-0.20	0.91	1.10	0.7413	.9085
$(3,1)$	2.20	1.83	0.55	-0.20	0.61	0.73	0.6053	.5496
$(3,2)$	2.47	2.49	0.43	0.00	0.83	0.82	0.7157	.7269
$(3,3)$	2.47	2.49	0.53	0.00	0.83	0.82	0.7878	.8158
$(3,4)$	2.55	2.68	0.52	0.04	0.89	0.85	0.8520	.9057
$(3,5)$	2.62	2.85	0.42	0.07	0.95	0.87	0.8965	.9672
$(4,1)$	3.36	4.00	0.16	0.15	1.00	0.84	0.7127	.7090
$(4,2)$	3.36	4.00	0.16	0.15	1.00	0.84	0.8794	.8664
$(4,3)$	3.36	4.00	0.15	0.15	1.00	0.84	0.9627	.9402
$(4,4)$	3.36	4.00	0.15	0.15	1.00	0.84	1.0461	1.0080
$(4,5)$	3.36	4.00	0.15	0.15	1.00	0.84	1.1017	1.0476
$(5,5)$	4.00	5.14	0	0.22	1.02	0.80	1.2439	1.1487
$(5,4)$	4.11	5.24	-0.05	0.21	1.04	0.82	1.1928	1.1331
$(5,3)$	4.34	5.43	-0.12	0.20	1.08	0.86	1.1188	1.0877
$(5,2)$	4.49	5.56	-0.11	0.19	1.11	0.89	1.0428	1.0286
$(5,1)$	4.87	5.87	-0.15	0.17	1.17	0.97	0.8933	.8901

Table 7. Binding IC$[(i,j),\ (i',j')]$

(i,j)	$(i'j')$	(i,j)	$(i'j')$
		$(4,1)$	$(3,2)$, $(3,3)$, $(3,5)$, $(4,2)$, $(4,3)$, $(4,4)$, $(4,5)$
$(1,2)$	$(1,1)$	$(4,2)$	$(4,1)$, $(4,3)$, $(4,4)$, $(4,5)$
$(1,3)$	$(1,2)$	$(4,3)$	$(4,1)$, $(4,2)$, $(4,4)$, $(4,5)$
$(1,4)$	$(1,3)$	$(4,4)$	$(4,1)$, $(4,2)$, $(4,3)$, $(4,5)$
$(1,5)$	$(1,4)$, $(2,1)$	$(4,5)$	$(4,1)$, $(4,2)$, $(4,3)$, $(4,4)$
$(2,1)$	$(1,4)$, $(1,5)$	$(5,1)$	$(4,1)$, $(4,2)$, $(4,3)$, $(4,4)$, $(4,5)$
$(2,2)$	$(1,5)$, $(2,1)$	$(5,2)$	$(4,1)$, $(4,2)$, $(4,3)$, $(4,4)$, $(4,5)$, $(5,1)$
$(2,3)$	$(2,2)$	$(5,3)$	$(5,2)$
$(2,4)$	$(2,3)$	$(5,4)$	$(5,3)$
$(2,5)$	$(2,4)$, $(3,1)$	$(5,5)$	$(5,4)$
$(3,1)$	$(2,3)$, $(2,5)$		
$(3,2)$	$(2,5)$, $(3,1)$, $(3,3)$		
$(3,3)$	$(3,2)$		
$(3,4)$	$(3,2)$, $(3,3)$		
$(3,5)$	$(3,4)$		

Numerical Issues

- LICQ (linear independence constraint qualification)
  - "The gradients of the binding constraints are linearly independent."
  - A sufficient condition in convergence theorems for most algorithms
  - Essentially a necessary condition for good convergence rate
  - Will fail when there are more binding constraints than variables
  - Mangasarian-Fromowitz and Robinson are not sufficient for convergence of current algorithms
- Software and Hardware
  - AMPL - modelling language commonly used in OR
  - Desktop computers, primarily through NEOS