GSSA Steps

Guess a value for the b coefficients, and define the transition law for k to be

$K(k, a) = \Psi(k, a; b)$

Run simulation

(a_t is generated once in initialization process, used in all simulations)

$$k_{t+1} = K(k_t,\ a_t),\ \ t = 1,\ \ldots,\ T$$

$$c_t = (1 - \delta)\, k_t + a_t\, f(k_t) - k_{t+1}$$

Euler Equation

The Euler equation is

$$1 = E\left\{\frac{u'(c_{t+1})}{u'(c_t)}(1-\delta-a_{t+1}\,f'(k_{t+1}))\,\middle|\,k_t,\,a_t\right\}$$

Construct regression data

Using the simulated data, define

$$y_t = E\left\{\frac{u'(c_{t+1})}{u'(c_t)}(1-\delta-a_{t+1}\,f'(k_{t+1}))\,k_{t+1}\,\middle|\,k_t,\,a_t\right\}$$

Relate simulation data to Euler equation

If we have the equilibrium K(.,.) function, then

$$\begin{aligned} y_t &= E\left\{\frac{u'(c_{t+1})}{u'(c_t)}(1-\delta - a_{t+1} f'(k_{t+1}))\, k_{t+1} \,\middle|\, k_t\right\} \\ &= 1 * k_{t+1} \\ &= K(k_t,\, a_t) \\ &= \Psi(k_t,\, a_t;\, b) \end{aligned}$$

Design regression

To see if that is true, we choose b' so as to minimize

$$\sum_t (y_t - \Psi(k_t, a_t; b'))^2$$

which in our case is nothing more than a linear regression

Approximate y_t

We need to approximate y_t. There are several possibilities

Rational expectations, Monte Carlo:

$y_t = \left(\frac{u'(c_{t+1})}{u'(c_t)} (1 - \delta - a_{t+1} f'(k_{t+1})) k_{t+1} \right)$ + noise

where noise has mean zero; so we define

$\text{yhat}_t = \left(\frac{u'(c_{t+1})}{u'(c_t)} (1 - \delta - a_{t+1} f'(k_{t+1})) k_{t+1} \right)$

and regress yhat on Ψ(k,a;b)

(This is PEA procedure)

Rational expectations, quadrature:

yhat_t = a quadrature formula for

$$E\left\{\frac{u'(c_{t+1})}{u'(c_t)}\left(1-\delta-a_{t+1}\,f'(k_{t+1})\right)k_{t+1}\;\middle|\;k_t\right\}$$

The quadrature nodes are points in the distribution of $a_{t+1} \mid a_t$.
For each quadrature node, we compute the c_{t+1} under K(k,a;b)
The quadrature weights are related to the distribution of $a_{t+1} \mid a_t$

Rational beliefs, quadrature:

yhat^j = a quadrature formula for

$$E^j \left\{ \frac{u'\left(c^j{}_{t+1}\right)}{u'\left(c^j{}_t\right)} R_{t+1}\, k^j{}_{t+1} \,\middle|\, \text{state}_t \right\}$$

where the quadrature weights and nodes are determined by agent j's distribution of $a_{t+1} \mid a_t$

Bond prices:

Let $m_{i,t}$ be the expected MRS of tomorrow's consumption with respect to today's consumption:

$$m_{i,t} = \beta\, E\left\{ \frac{u_i'(c_{i,t+1})}{u_i'(c_{i,t})} \,\middle|\, \text{state}_t \right\}$$

Let q be the price of a one-period bond that delivers one unit of consumption tomorrow. q depends on the current state, so we let $q = \Psi(\text{state}; b)$ approximate the equilibrium q function

The regression chooses b' so as to minimize

$$\sum_i \sum_t (m_{i,t} - \Psi(\text{state}_t; b'))^2$$

which in our case is nothing more than a linear regression where we have multiple observations at each time t