Beijing

May, 2009

Problem Set 1

Due May 20, 2009

Use Fortran, Matlab, C, Gauss, Maple, or Mathematica or similar language. If you don't know a language, learn Matlab. Your written solutions to all exercises should explain what you did and what you found (be concise!), followed by a listing of your computer code and output. *Do not just give me computer code and output.* For example, tables are often useful. Send answers by e-mail to Xing or Mindy in the form of either pdf documents or Mathematica notebooks.

These exercises aim to get you on the computer, familiar with your software, and aware of numerical errors.

1. Compute

$$(1682\,xy^4 + 3x^3 + 29\,xy^2 - 2x^5 + 832)/107751$$

for $x = 192119201$ and $y = 35675640$. Does your computer get the right answer? By the way, the only way to surely get the right answer is to use infinite precision arithmetic, using either the computer or doing it by hand.

2. Solve the linear equations

$$\begin{aligned} 64919121\,x - 159018721\,y &= 1 \\ 41869520.5\,x - 102558961\,y &= 0 \end{aligned}$$

Compute the solution using double precision arithmetic on your computer and then compute the solution analytically. Do they agree?

3. Write programs to determine the relative and absolute speeds (expressed as operations per second) of addition, multiplication, division, exponentiation (e^x), logarithm (base 10), the sine function, and the tangent function on your computer with your software.

4. Write a program which determines your machine ϵ and your machine zero.

5. Apply the stopping rule I described in class (the one where you estimate the rate of convergence) to the following sequences. In each case telling me the final appoximate answer and how many iterations it took to get to an approximation within ε of the truth. (By the way, a sequence may not converge, so do not do more than 100,000 iterations in any case):

(a) $x_k = \sum_{n=0}^{k} \frac{4^n}{n!}$;
(b) $x_k = \sum_{n=1}^{k} n^{-3}$;
(c) $x_k = \sum_{n=1}^{k} n^{-1.7}$;
(d) $x_k = \sum_{n=1}^{k} n^{-0.92}$,
(e) $x_0 = 5$, $x_{k+1} = (2/3)x_k + x_k^{-2}$
(f) $x_0 = 5$, $x_k = (1 + x_k - 4x_k^2)/(4 - 6x_k)$

Use $\varepsilon = .01$, .001, and .0001. For each ε determine the number of terms computed before the stopping rule makes a choice and compute the error of the answer. Hint: use tables such as

Approximate result

	ϵ : .01	.001	.0001
(a):	?	?	?
(b):	?	?	?
(c):	?	?	?
(d):	?	?	?

Number of terms

	ϵ : .01	.001	.0001
(a):	?	?	?
(b):	?	?	?
(c):	?	?	?
(d):	?	?	?

Repeat this using the following stopping rule: stop when the difference between successive iterates is ε. Compare the performance of the two rules by comparing which one is a more reliable rule for estimating the error and which is better at detecting the true nature of convergence.