LogLog Example - FIXG

Mathematica details

Input general equations

Print out equations

Specifications

In[4762]:= $\sigma_1 = 1;\ \sigma_2 = 1;$

In[4763]:=
```
ucc[c_] = If[σ1 == 1, Log[c + cb], (c + cb)^(1-σ1) / (1 - σ1)];
ull[leis_] = If[σ2 == 1, Log[leis + lb], (leis + lb)^(1-σ2) / (1 - σ2)];
```

$$ucc[c\_] = If\left[\sigma_1 == 1, Log[c + cb], (c + cb)^{1-\sigma_1} / (1 - \sigma_1)\right];$$
$$ull[leis\_] = If\left[\sigma_2 == 1, Log[leis + lb], (leis + lb)^{1-\sigma_2} / (1 - \sigma_2)\right];$$

In[4765]:= $\beta = 96 / 100;$

In[4766]:=
```
uc[z_] = ucc[z]
ul[z_] = ull[z]
```

Out[4766]= Log[cb + z]

Out[4767]= Log[lb + z]

In[4768]:=
```
λm = uc'[1];
λM = Min[uc'[cb + 1 / 10 000 000], 100];
```

Sequential solution of optimization problem

Goal: for a given (b, λ) state, determine the solutions for ($\mathbb{C}$, $\mathbb{L}$, $\mathbb{G}$, p, TR).

In[4770]:=
```
CompSys // NTable
```

Out[4770]//TableForm=

	1
1	$\lambda == \frac{1}{cb+\mathbb{C}}$
2	$\lambda == \frac{1}{1+lb-\mathbb{L}} + \lambda\,\tau$
3	$p\,\lambda == \frac{24\,\lambda_+}{25}$
4	$b + TR + \mathbb{L} == b_+\,p + \mathbb{C} + \mathbb{L}\,\tau$
5	$g_M + \mathbb{C} == \mathbb{L}$
6	$p\,\mu_{TR} + \frac{24}{25}\,Vb[b_+, \lambda_+] == 0$
7	$\frac{24}{25}\left(\frac{b_+\,\mu_{TR}}{\lambda} + V\lambda[b_+, \lambda_+]\right) == 0$
8	$\mu_{TR} + Vb[b, \lambda] == 0$
9	$V\lambda[b, \lambda] == -\frac{(cb+\mathbb{C})^2\left(\frac{\lambda}{cb+\mathbb{C}} - \frac{\lambda}{1+lb-\mathbb{L}} + \mu_{TR}\left(-\frac{\mathbb{L}}{(1+lb-\mathbb{L})^2} + \lambda\,\tau - \frac{-b_+\,p+\mathbb{L}-\mathbb{L}\,\tau}{(cb+\mathbb{C})^2}\right)\right)}{\lambda}$
10	$\mu_{TR} \geq 0$ && $TR\,\mu_{TR} == 0$ && $TR \geq 0$

Because we have a fixed choice of λplus and b_+, only equations 1-5 are relevant for finding policy choices.

In[4771]:=
```
tmp = polsys = CompSys[[1 ;; 5]]
```

Out[4771]=

$$\left\{\lambda == \frac{1}{cb+\mathbb{C}},\ \lambda == \frac{1}{1+lb-\mathbb{L}} + \lambda\,\tau,\ p\,\lambda == \frac{24\,\lambda_+}{25},\ b + TR + \mathbb{L} == b_+\,p + \mathbb{C} + \mathbb{L}\,\tau,\ g_M + \mathbb{C} == \mathbb{L}\right\}$$

Problem reduction

Government expenditure, λplus and b_+ are all fixed.
The solution strategy is simple:
gov is pinned down exogenously
consumption is pinned down by λ
labor supply must equal consumption plus gov
Labor supply and equation 2 (labor supply foc) pins down τ
If τ>1, then state is INFEASIBLE.
The budget constraint then pins down TR.
If TR>=0, we are okay. Otherwise INFEASIBILITY.

We first solve out for gov, consumption, labor and price

In[4772]:=
```
gsol = {𝔾 → g_M}
```
Out[4772]= $\{g_M \to g_M\}$

In[4773]:=
```
csol = Solve[tmp[[1]], ℂ][[1]]
```
Out[4773]= $\left\{\mathbb{C} \to \frac{1 - cb\,\lambda}{\lambda}\right\}$

In[4774]:=
```
lbsol = Solve[tmp[[5]], 𝕃][[1]]
```
Out[4774]= $\{\mathbb{L} \to g_M + \mathbb{C}\}$

In[4775]:=
```
psol = Solve[tmp[[3]], p][[1]]
```
Out[4775]= $\left\{p \to \frac{24\,\lambda_+}{25\,\lambda}\right\}$

We collect the solutions. polsols give us the static decisions as functions of λ (a state) and τ (the other static choice)

In[4776]:=
```
polsols = {csol, lbsol, gsol, psol} // Flatten;
```
and apply them to the system.

In[4777]:=
```
tmp = tmp //. polsols // Simplify;
tmp // NTable
```

Out[4778]//TableForm=

	1
1	True
2	$\lambda == \lambda \left(\frac{1}{-1+(1+cb-g_M+lb)\,\lambda}+\tau\right)$
3	True
4	$b+g_M+TR+cb\,\tau == \frac{24\,b_+\,\lambda_+}{25\,\lambda}+\left(g_M+\frac{1}{\lambda}\right)\tau$
5	True

Equation 2 fixes τ:

In[4779]:=
```
tausol = Solve[tmp[[2]], τ][[1]]
```

Out[4779]= $\left\{\tau \to \frac{-2+\lambda+cb\,\lambda-g_M\,\lambda+lb\,\lambda}{-1+\lambda+cb\,\lambda-g_M\,\lambda+lb\,\lambda}\right\}$

Equation 4 is the budget constraint

In[4780]:= **budget = tmp[[4]]**

Out[4780]= $b + g_M + TR + cb\ \tau == \frac{24\ b_+\ \lambda_+}{25\ \lambda} + \left(g_M + \frac{1}{\lambda}\right)\ \tau$

Solution for TR is

In[4781]:= **TRsolτ = Solve[budget, TR][[1]]**

Out[4781]= $\left\{TR \to \frac{-25\ b\ \lambda - 25\ g_M\ \lambda + 24\ b_+\ \lambda_+ + 25\ \tau - 25\ cb\ \lambda\ \tau + 25\ g_M\ \lambda\ \tau}{25\ \lambda}\right\}$

If TRsolτ implies a negative TR, then the state is infeasible.

We use the solutions to the solutions for $\mathbb{C}$, $\mathbb{L}$, $\mathbb{G}$ and τ to compute utility as a function of the state (b, λ)

In[4782]:= **GovObj //. polsols /. tausol // Simplify**

Out[4782]= $\text{Log}\left[1 + cb - g_M + lb - \frac{1}{\lambda}\right] + \text{Log}\left[\frac{1}{\lambda}\right] + \frac{24}{25} V[b^+, \lambda_+]$

Code

Given g_M, b, λ, bplus and λplus, we:

First compute τ

In[4783]:=
```
tausol /. subT0eqs
```

Out[4783]=
$$\left\{\tau == \frac{-2+\lambda+\text{cb}\,\lambda-g_M\,\lambda+\text{lb}\,\lambda}{-1+\lambda+\text{cb}\,\lambda-g_M\,\lambda+\text{lb}\,\lambda}\right\}$$

If $\tau > 1$, then the state is infeasible.

Then, using the τ computed above, compute TRcheck

In[4784]:=
```
TRsolτ /. subT0eqs /. TR → "TRcheck"
```

Out[4784]=
$$\left\{\text{TRcheck} == \frac{-25\,b\,\lambda-25\,g_M\,\lambda+24\,b_+\,\lambda_++25\,\tau-25\,\text{cb}\,\lambda\,\tau+25\,g_M\,\lambda\,\tau}{25\,\lambda}\right\}$$

If TRcheck<0 then we have an infeasible transition

If TRcheck>=0, then we have τ and TR and we finish by computing the following in sequence:

In[4785]:=
```
csol /. subTOeqs
```

Out[4785]= $\left\{\mathbb{C} == \frac{1 - cb\,\lambda}{\lambda}\right\}$

In[4786]:=
```
lbsol /. gsol /. subTOeqs
```

Out[4786]= $\{\mathbb{L} == g_M + \mathbb{C}\}$

In[4787]:=
```
psol /. subTOeqs
```

Out[4787]= $\left\{p == \frac{24\,\lambda_+}{25\,\lambda}\right\}$

and the objective function

In[4788]:=
```
GovObj //. polsols /. tausol // Simplify
```

Out[4788]= $\text{Log}\left[1 + cb - g_M + lb - \frac{1}{\lambda}\right] + \text{Log}\left[\frac{1}{\lambda}\right] + \frac{24}{25} V[b^+, \lambda_+]$