Perturbation Methods for a 2D Growth Models

Deterministic Model

In[7]:=
```
x = 0; Remove["Global`*"]
```

Setup

We define the Euler equation for a simple growth model.
fi[ki] is the production function; ci[k1,k2] is the (unknown) consumption policy function.
kiplus and ciplus are the next period's capital stock and consumption

In[8]:=
```
k1plus  =  f1[k1] - c1[k1, k2];
c1plus  =  c1[k1plus, k2plus];
k2plus  =  f2[k2] - c2[k1, k2];
c2plus  =  c2[k1plus, k2plus];
EulerEq  =
 {u1[c1[k1, k2], c2[k1, k2]] - β u1[c1plus, c2plus] f1'[k1plus],
  u2[c1[k1, k2], c2[k1, k2]] - β u2[c1plus, c2plus] f2'[k2plus]}
```

Out[12]=
```
{u1[c1[k1, k2], c2[k1, k2]] - β u1[c1[-c1[k1, k2] + f1[k1], -c2[k1, k2] + f2[k2]],
    c2[-c1[k1, k2] + f1[k1], -c2[k1, k2] + f2[k2]]] f1′[-c1[k1, k2] + f1[k1]],
  u2[c1[k1, k2], c2[k1, k2]] - β u2[c1[-c1[k1, k2] + f1[k1], -c2[k1, k2] + f2[k2]],
    c2[-c1[k1, k2] + f1[k1], -c2[k1, k2] + f2[k2]]] f2′[-c2[k1, k2] + f2[k2]]}
```

Choose utility and production functions. Put a free parameter, A, in f[k] so that we can later fix the steady state capital stock.

In[13]:=
```
u[x_, y_] = Log[x] + Sqrt[y];
u1[x_, y_] = D[u[x, y], x];
u2[x_, y_] = D[u[x, y], y];
f1[x_]  = x + A x^α;
f2[x_]  = x + A x^α;
α = 1. / 4;
β = 95 / 100;
```

We want the steady state capital stock to be k=1 since it makes it easier to understand the results.
Choose A so that βf'[1]=1.

In[20]:=
```
A = A //. Solve[f1'[1] == 1 / β, A][[1]]
```

Out[20]=
```
0.210526
```

Let's look at our Euler equation

In[21]:= **EulerEq**

Out[21]=

$$\left\{\frac{1}{c1[k1,\,k2]}-\frac{19\left(1+\frac{0.0526316}{\left(0.210526\,k1^{0.25}+k1-c1[k1,k2]\right)^{0.75}}\right)}{20\,c1\left[0.210526\,k1^{0.25}+k1-c1[k1,\,k2],\,0.210526\,k2^{0.25}+k2-c2[k1,\,k2]\right]},\right.$$

$$\left.\frac{1}{2\sqrt{c2[k1,\,k2]}}-\frac{19\left(1+\frac{0.0526316}{\left(0.210526\,k2^{0.25}+k2-c2[k1,k2]\right)^{0.75}}\right)}{40\sqrt{c2\left[0.210526\,k1^{0.25}+k1-c1[k1,\,k2],\,0.210526\,k2^{0.25}+k2-c2[k1,\,k2]\right]}}\right\}$$

ss is a list of substitutions that impose the steady state, k=1, and some substitutions that convert floating point versions of 1 and 0 to integer versions.

In[22]:= **ss = {k1 → 1, k2 → 1, 1. → 1, 0. → 0}**

Out[22]= {k1 → 1, k2 → 1, 1. → 1, 0. → 0}

The steady state consumption is defined next

In[23]:= **c1[1, 1] = c2[1, 1] = css = f1[1] - 1**

Out[23]= 0.210526

Now check the Euler equation at the steady state.

In[24]:= **EulerEq //. ss**

Out[24]= $\{8.88178\times10^{-16},\,2.22045\times10^{-16}\}$

sol will be the list of solutions for derivatives of c[k]. We begin the construction by setting sol to be the empty set.

In[25]:= **sol = {};**

▪ prep for pert

In[26]:=
```
EulerEqϵ = EulerEq /. k1 → 1 + ϵ ϰ1 /. k2 → 1 + ϵ ϰ2
```

Out[26]=

$$\Big\{\frac{1}{c1[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2]}-\left(19\left(1+\frac{0.0526316}{\left(1+\epsilon\varkappa1+0.210526\,(1+\epsilon\varkappa1)^{0.25}-c1[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2]\right)^{0.75}}\right)\right)\Big/$$
$$\Big(20\,c1\Big[1+\epsilon\varkappa1+0.210526\,(1+\epsilon\varkappa1)^{0.25}-c1[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2],$$
$$1+\epsilon\varkappa2+0.210526\,(1+\epsilon\varkappa2)^{0.25}-c2[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2]\Big]\Big),$$
$$\frac{1}{2\sqrt{c2[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2]}}-\left(19\left(1+\frac{0.0526316}{\left(1+\epsilon\varkappa2+0.210526\,(1+\epsilon\varkappa2)^{0.25}-c2[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2]\right)^{0.75}}\right)\right)\Big/$$
$$\Big(40\sqrt{c2\Big[1+\epsilon\varkappa1+0.210526\,(1+\epsilon\varkappa1)^{0.25}-c1[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2],}$$
$$\overline{1+\epsilon\varkappa2+0.210526\,(1+\epsilon\varkappa2)^{0.25}-c2[1+\epsilon\varkappa1,\,1+\epsilon\varkappa2]\Big]}\Big)\Big\}$$

In[27]:=
```
subs = {cc_^(i_,j_)[1, 1] → cc_{i,j}}
```

Out[27]=

$$\{cc\_^{(i\_,j\_)}[1,\,1]\to cc_{i,j}\}$$

▪ k pert

The Euler equation must hold at all k. Therefore, its derivative w.r.t. k must also be zero at all k. Compute the derivative of the Eulereq

In[28]:=
```
D[EulerEqϵ, ϵ];
% /. ϵ → 0 /. 1. → 1
```

Out[29]=

$$\{0.178125\,\left(1.05263\,\varkappa1-\varkappa2\,c1^{(0,1)}[1,1]-\varkappa1\,c1^{(1,0)}[1,1]\right)-$$
$$22.5625\,\left(\varkappa2\,c1^{(0,1)}[1,1]+\varkappa1\,c1^{(1,0)}[1,1]\right)+$$
$$22.5625\,\Big(c1^{(1,0)}[1,1]\,\left(1.05263\,\varkappa1-\varkappa2\,c1^{(0,1)}[1,1]-\varkappa1\,c1^{(1,0)}[1,1]\right)+$$
$$c1^{(0,1)}[1,1]\,\left(1.05263\,\varkappa2-\varkappa2\,c2^{(0,1)}[1,1]-\varkappa1\,c2^{(1,0)}[1,1]\right)\Big),$$
$$0.0408647\,\left(1.05263\,\varkappa2-\varkappa2\,c2^{(0,1)}[1,1]-\varkappa1\,c2^{(1,0)}[1,1]\right)-$$
$$2.5881\,\left(\varkappa2\,c2^{(0,1)}[1,1]+\varkappa1\,c2^{(1,0)}[1,1]\right)+$$
$$2.5881\,\Big(\left(1.05263\,\varkappa1-\varkappa2\,c1^{(0,1)}[1,1]-\varkappa1\,c1^{(1,0)}[1,1]\right)c2^{(1,0)}[1,1]+$$
$$c2^{(0,1)}[1,1]\,\left(1.05263\,\varkappa2-\varkappa2\,c2^{(0,1)}[1,1]-\varkappa1\,c2^{(1,0)}[1,1]\right)\Big)\}$$

In[30]:=
```
% /. subs // Simplify
```

Out[30]=

$$\{-22.5625\,\varkappa1\,(-0.116233+c1_{1,0})\,(0.0714964+c1_{1,0})+$$
$$c1_{0,1}\,(1.00937\,\varkappa2-22.5625\,\varkappa2\,c1_{1,0}-22.5625\,\varkappa2\,c2_{0,1}-22.5625\,\varkappa1\,c2_{1,0}),$$
$$0.0430155\,\varkappa2-2.5881\,\varkappa2\,c2_{0,1}^2+(0.0953509\,\varkappa1-2.5881\,\varkappa2\,c1_{0,1}-2.5881\,\varkappa1\,c1_{1,0})\,c2_{1,0}+$$
$$c2_{0,1}\,(0.0953509\,\varkappa2-2.5881\,\varkappa1\,c2_{1,0})\}$$

In[31]:=
```
SolveAlways[% == 0, {ϰ1, ϰ2}]
```

Out[31]=

$$\{\{c1_{0,1}\to 0.,\ c1_{1,0}\to -0.0714964,\ c2_{0,1}\to -0.111809,\ c2_{1,0}\to 0.\},$$
$$\{c1_{0,1}\to 0.,\ c1_{1,0}\to -0.0714964,\ c2_{0,1}\to 0.148651,\ c2_{1,0}\to 0.\},$$
$$\{c1_{0,1}\to 0.,\ c1_{1,0}\to 0.116233,\ c2_{0,1}\to -0.111809,\ c2_{1,0}\to 0.\},$$
$$\{c1_{0,1}\to 0.,\ c1_{1,0}\to 0.116233,\ c2_{0,1}\to 0.148651,\ c2_{1,0}\to 0.\}\}$$

There are two solutions, one corresponding to the stable manifold and the other one corresponding to the unstable manifold. We choose the second one because we know that c'[1]>0, and put it in the solution set.

In[32]:= **sol = Union[sol, % // Last]**

Out[32]= {$c1_{0,1}$ → 0., $c1_{1,0}$ → 0.116233, $c2_{0,1}$ → 0.148651, $c2_{1,0}$ → 0.}

In[33]:= **sol = sol /. 0. → 0**

Out[33]= {$c1_{0,1}$ → 0, $c1_{1,0}$ → 0.116233, $c2_{0,1}$ → 0.148651, $c2_{1,0}$ → 0}

■ k pert - degree 2

We now move on to the second derivative

In[34]:=
```
D[EulerEqϵ, {ϵ, 2}];
% /. ϵ → 0 /. 1. → 1 /. subs;
```

In[36]:=
```
% /. sol
```

Out[36]=
$$\{2.72335\,\kappa1^2 - 22.5625\,(\kappa2\,(\kappa2\,c1_{0,2} + \kappa1\,c1_{1,1}) + \kappa1\,(\kappa2\,c1_{1,1} + \kappa1\,c1_{2,0})) - 0.2375\,(1.15086\,\kappa1^2 - 0.75\,(-0.0394737\,\kappa1^2 - \kappa2\,(\kappa2\,c1_{0,2} + \kappa1\,c1_{1,1}) - \kappa1\,(\kappa2\,c1_{1,1} + \kappa1\,c1_{2,0}))) - 1.\,(2.53917\,\kappa1^2 - 22.5625\,(0.903981\,\kappa2\,(0.903981\,\kappa2\,c1_{0,2} + 0.936398\,\kappa1\,c1_{1,1}) + 0.936398\,\kappa1\,(0.903981\,\kappa2\,c1_{1,1} + 0.936398\,\kappa1\,c1_{2,0}) + 0.116233\,(-0.0394737\,\kappa1^2 - \kappa2\,(\kappa2\,c1_{0,2} + \kappa1\,c1_{1,1}) - \kappa1\,(\kappa2\,c1_{1,1} + \kappa1\,c1_{2,0})))),\ -0.0235791\,\kappa2^2 - 0.0544862\,(1.07255\,\kappa2^2 - 0.75\,(-0.0394737\,\kappa2^2 - \kappa2\,(\kappa2\,c2_{0,2} + \kappa1\,c2_{1,1}) - \kappa1\,(\kappa2\,c2_{1,1} + \kappa1\,c2_{2,0}))) + \frac{1}{2}\,(0.81495\,\kappa2^2 - 5.17619\,(\kappa2\,(\kappa2\,c2_{0,2} + \kappa1\,c2_{1,1}) + \kappa1\,(\kappa2\,c2_{1,1} + \kappa1\,c2_{2,0}))) - 0.5\,(0.665961\,\kappa2^2 - 5.17619\,(0.903981\,\kappa2\,(0.903981\,\kappa2\,c2_{0,2} + 0.936398\,\kappa1\,c2_{1,1}) + 0.936398\,\kappa1\,(0.903981\,\kappa2\,c2_{1,1} + 0.936398\,\kappa1\,c2_{2,0}) + 0.148651\,(-0.0394737\,\kappa2^2 - \kappa2\,(\kappa2\,c2_{0,2} + \kappa1\,c2_{1,1}) - \kappa1\,(\kappa2\,c2_{1,1} + \kappa1\,c2_{2,0}))))\}$$

In[37]:=
```
CoefficientList[%, {κ1, κ2}]
```

Out[37]=
$$\{\{\{0, 0, -6.92549\,c1_{0,2}\}, \{0, -12.5286\,c1_{1,1}, 0\}, \{-0.199701 - 5.57939\,c1_{2,0}, 0, 0\}\}, \{\{0, 0, -0.0243237 - 0.898741\,c2_{0,2}\}, \{0, -1.64579\,c2_{1,1}, 0\}, \{-0.744333\,c2_{2,0}, 0, 0\}\}\}$$

In[38]:=
```
% // Flatten // Union // Rest
```

Out[38]=
$$\{-6.92549\,c1_{0,2}, -12.5286\,c1_{1,1}, -0.199701 - 5.57939\,c1_{2,0}, -0.0243237 - 0.898741\,c2_{0,2}, -1.64579\,c2_{1,1}, -0.744333\,c2_{2,0}\}$$

In[39]:=
```
eqs = %
```

Out[39]=
$$\{-6.92549\,c1_{0,2}, -12.5286\,c1_{1,1}, -0.199701 - 5.57939\,c1_{2,0}, -0.0243237 - 0.898741\,c2_{0,2}, -1.64579\,c2_{1,1}, -0.744333\,c2_{2,0}\}$$

In[40]:=
```
vars = Variables[eqs]
```

Out[40]=
$$\{c1_{0,2}, c1_{1,1}, c1_{2,0}, c2_{0,2}, c2_{1,1}, c2_{2,0}\}$$

This is a linear expression in terms of the unknown c''[1]. Solve it

In[41]:=
```
Solve[eqs == 0, vars]
```

Out[41]=
$$\{\{c1_{0,2} \to 0., c1_{1,1} \to 0., c1_{2,0} \to -0.0357926, c2_{0,2} \to -0.0270642, c2_{1,1} \to 0., c2_{2,0} \to 0.\}\}$$

and add this solution to our solution set

In[42]:=
```
sol = Union[sol, %[[1]]] // Simplify
```

Out[42]=
$$\{c1_{0,1} \to 0, c1_{0,2} \to 0., c1_{1,0} \to 0.116233, c1_{1,1} \to 0., c1_{2,0} \to -0.0357926, c2_{0,1} \to 0.148651, c2_{0,2} \to -0.0270642, c2_{1,0} \to 0, c2_{1,1} \to 0., c2_{2,0} \to 0.\}$$

In[43]:=
```
sol = sol /. 0. → 0
```

Out[43]=
$$\{c1_{0,1} \to 0, c1_{0,2} \to 0, c1_{1,0} \to 0.116233, c1_{1,1} \to 0, c1_{2,0} \to -0.0357926, c2_{0,1} \to 0.148651, c2_{0,2} \to -0.0270642, c2_{1,0} \to 0, c2_{1,1} \to 0, c2_{2,0} \to 0\}$$

k pert - degree 3

We will now express the steps in a more compact manner

```
In[44]:= D[EulerEqϵ, {ϵ, 3}];
% /. ϵ → 0 /. 1. → 1 /. subs;
% /. sol;
CoefficientList[%, {κ1, κ2}];
% // Flatten // Union // Rest;
eqs = %
vars = Variables[eqs]
Solve[eqs == 0, vars]
sol = Union[sol, %[[1]]] // Simplify
```

```
Out[49]= {-8.69586 c1_{0,3}, -24.2945 c1_{1,2}, -22.437 c1_{2,1}, 0.370006 - 6.83767 c1_{3,0},
  0.0436706 - 1.10182 c2_{0,3}, -3.09976 c2_{1,2}, -2.8867 c2_{2,1}, -0.888667 c2_{3,0}}

Out[50]= {c1_{0,3}, c1_{1,2}, c1_{2,1}, c1_{3,0}, c2_{0,3}, c2_{1,2}, c2_{2,1}, c2_{3,0}}

Out[51]= {{c1_{0,3} → 0., c1_{1,2} → 0., c1_{2,1} → 0.,
   c1_{3,0} → 0.0541129, c2_{0,3} → 0.0396351, c2_{1,2} → 0., c2_{2,1} → 0., c2_{3,0} → 0.}}

Out[52]= {c1_{0,1} → 0, c1_{0,2} → 0, c1_{0,3} → 0., c1_{1,0} → 0.116233, c1_{1,1} → 0, c1_{1,2} → 0.,
  c1_{2,0} → -0.0357926, c1_{2,1} → 0., c1_{3,0} → 0.0541129, c2_{0,1} → 0.148651, c2_{0,2} → -0.0270642,
  c2_{0,3} → 0.0396351, c2_{1,0} → 0, c2_{1,1} → 0, c2_{1,2} → 0., c2_{2,0} → 0, c2_{2,1} → 0., c2_{3,0} → 0.}
```

```
In[53]:= sol = sol /. 0. → 0
```

```
Out[53]= {c1_{0,1} → 0, c1_{0,2} → 0, c1_{0,3} → 0, c1_{1,0} → 0.116233, c1_{1,1} → 0, c1_{1,2} → 0,
  c1_{2,0} → -0.0357926, c1_{2,1} → 0, c1_{3,0} → 0.0541129, c2_{0,1} → 0.148651, c2_{0,2} → -0.0270642,
  c2_{0,3} → 0.0396351, c2_{1,0} → 0, c2_{1,1} → 0, c2_{1,2} → 0, c2_{2,0} → 0, c2_{2,1} → 0, c2_{3,0} → 0}
```

■ k pert - degree 4

We will now express the steps in a more compact manner

```
In[54]:= D[EulerEqϵ, {ϵ, 4}];
        % /. ϵ → 0 /. 1. → 1 /. subs;
        % /. sol;
        CoefficientList[%, {κ1, κ2}];
        % // Flatten // Union // Rest;
        eqs = %
        vars = Variables[eqs];
        Solve[eqs == 0, vars];
        sol = Union[sol, %[[1]]] /. 0. → 0 // Simplify
```

Out[59]= $\{-10.2962\, c1_{0,4}, -39.0237\, c1_{1,3}, -55.1774\, c1_{2,2}, -34.4659\, c1_{3,1}, -1.08824 - 8.01592\, c1_{4,0}, -0.12473 - 1.28539\, c2_{0,4}, -4.89366\, c2_{1,3}, -6.95528\, c2_{2,2}, -4.37084\, c2_{3,1}, -1.02382\, c2_{4,0}\}$

Out[62]= $\{c1_{0,1} \to 0, c1_{0,2} \to 0, c1_{0,3} \to 0, c1_{0,4} \to 0, c1_{1,0} \to 0.116233, c1_{1,1} \to 0, c1_{1,2} \to 0, c1_{1,3} \to 0, c1_{2,0} \to -0.0357926, c1_{2,1} \to 0, c1_{2,2} \to 0, c1_{3,0} \to 0.0541129, c1_{3,1} \to 0, c1_{4,0} \to -0.135759, c2_{0,1} \to 0.148651, c2_{0,2} \to -0.0270642, c2_{0,3} \to 0.0396351, c2_{0,4} \to -0.0970362, c2_{1,0} \to 0, c2_{1,1} \to 0, c2_{1,2} \to 0, c2_{1,3} \to 0, c2_{2,0} \to 0, c2_{2,1} \to 0, c2_{2,2} \to 0, c2_{3,0} \to 0, c2_{3,1} \to 0, c2_{4,0} \to 0\}$

k pert - degree 5

We will now express the steps in a more compact manner

In[63]:=
```
D[EulerEqϵ, {ϵ, 5}];
% /. ϵ → 0 /. 1. → 1 /. subs;
% /. sol;
CoefficientList[%, {κ1, κ2}];
% // Flatten // Union // Rest;
eqs = %
vars = Variables[eqs];
Solve[eqs == 0, vars];
 sol = Union[sol, %[[1]]] /. 0. → 0 // Simplify
```

Out[68]= $\{-11.743\, c1_{0,5},\ -56.2726\, c1_{1,4},\ -107.486\, c1_{2,3},\ -102.245\, c1_{3,2},\ -48.4079\, c1_{4,1},\ 4.34358 - 9.11923\, c1_{5,0},\ 0.485097 - 1.45134\, c2_{0,5},\ -6.97658\, c2_{1,4},\ -13.3728\, c2_{2,3},\ -12.7716\, c2_{3,2},\ -6.07443\, c2_{4,1},\ -1.15038\, c2_{5,0}\}$

Out[71]= $\{c1_{0,1} \to 0,\ c1_{0,2} \to 0,\ c1_{0,3} \to 0,\ c1_{0,4} \to 0,\ c1_{0,5} \to 0,\ c1_{1,0} \to 0.116233,\ c1_{1,1} \to 0,\ c1_{1,2} \to 0,\ c1_{1,3} \to 0,\ c1_{1,4} \to 0,\ c1_{2,0} \to -0.0357926,\ c1_{2,1} \to 0,\ c1_{2,2} \to 0,\ c1_{2,3} \to 0,\ c1_{3,0} \to 0.0541129,\ c1_{3,1} \to 0,\ c1_{3,2} \to 0,\ c1_{4,0} \to -0.135759,\ c1_{4,1} \to 0,\ c1_{5,0} \to 0.47631,\ c2_{0,1} \to 0.148651,\ c2_{0,2} \to -0.0270642,\ c2_{0,3} \to 0.0396351,\ c2_{0,4} \to -0.0970362,\ c2_{0,5} \to 0.33424,\ c2_{1,0} \to 0,\ c2_{1,1} \to 0,\ c2_{1,2} \to 0,\ c2_{1,3} \to 0,\ c2_{1,4} \to 0,\ c2_{2,0} \to 0,\ c2_{2,1} \to 0,\ c2_{2,2} \to 0,\ c2_{2,3} \to 0,\ c2_{3,0} \to 0,\ c2_{3,1} \to 0,\ c2_{3,2} \to 0,\ c2_{4,0} \to 0,\ c2_{4,1} \to 0,\ c2_{5,0} \to 0\}$

■ k pert - degree more

We will now express the steps in a more compact manner

In[72]:=
```
sol5 = sol;
```

In[73]:=
```
sol = sol5
```

Out[73]= $\{c1_{0,1} \to 0, c1_{0,2} \to 0, c1_{0,3} \to 0, c1_{0,4} \to 0, c1_{0,5} \to 0, c1_{1,0} \to 0.116233, c1_{1,1} \to 0, c1_{1,2} \to 0, c1_{1,3} \to 0, c1_{1,4} \to 0, c1_{2,0} \to -0.0357926, c1_{2,1} \to 0, c1_{2,2} \to 0, c1_{2,3} \to 0, c1_{3,0} \to 0.0541129, c1_{3,1} \to 0, c1_{3,2} \to 0, c1_{4,0} \to -0.135759, c1_{4,1} \to 0, c1_{5,0} \to 0.47631, c2_{0,1} \to 0.148651, c2_{0,2} \to -0.0270642, c2_{0,3} \to 0.0396351, c2_{0,4} \to -0.0970362, c2_{0,5} \to 0.33424, c2_{1,0} \to 0, c2_{1,1} \to 0, c2_{1,2} \to 0, c2_{1,3} \to 0, c2_{1,4} \to 0, c2_{2,0} \to 0, c2_{2,1} \to 0, c2_{2,2} \to 0, c2_{2,3} \to 0, c2_{3,0} \to 0, c2_{3,1} \to 0, c2_{3,2} \to 0, c2_{4,0} \to 0, c2_{4,1} \to 0, c2_{5,0} \to 0\}$

In[74]:=
```
Do[
 d0 = D[EulerEqϵ, {ϵ, jjj}];
 d1 = d0 /. ϵ → 0 /. 1. → 1 /. subs;
 d2 = d1 /. sol;
 d3 = CoefficientList[d2, {κ1, κ2}];
 eqs = d3 // Flatten // Union // Rest;
 vars = Variables[eqs];
 solnew = Solve[eqs == 0, vars];
 sol = Union[sol, solnew[[1]]] /. 0. → 0 // Simplify,
 {jjj, 6, 8}]
```

In[75]:=
```
Length[sol]
```

Out[75]= 88

In[76]:=
```
sol
```

Out[76]= $\{c1_{0,1} \to 0, c1_{0,2} \to 0, c1_{0,3} \to 0, c1_{0,4} \to 0, c1_{0,5} \to 0, c1_{0,6} \to 0, c1_{0,7} \to 0, c1_{0,8} \to 0, c1_{1,0} \to 0.116233, c1_{1,1} \to 0, c1_{1,2} \to 0, c1_{1,3} \to 0, c1_{1,4} \to 0, c1_{1,5} \to 0, c1_{1,6} \to 0, c1_{1,7} \to 0, c1_{2,0} \to -0.0357926, c1_{2,1} \to 0, c1_{2,2} \to 0, c1_{2,3} \to 0, c1_{2,4} \to 0, c1_{2,5} \to 0, c1_{2,6} \to 0, c1_{3,0} \to 0.0541129, c1_{3,1} \to 0, c1_{3,2} \to 0, c1_{3,3} \to 0, c1_{3,4} \to 0, c1_{3,5} \to 0, c1_{4,0} \to -0.135759, c1_{4,1} \to 0, c1_{4,2} \to 0, c1_{4,3} \to 0, c1_{4,4} \to 0, c1_{5,0} \to 0.47631, c1_{5,1} \to 0, c1_{5,2} \to 0, c1_{5,3} \to 0, c1_{6,0} \to -2.14781, c1_{6,1} \to 0, c1_{6,2} \to 0, c1_{7,0} \to 11.8364, c1_{7,1} \to 0, c1_{8,0} \to -77.0954, c2_{0,1} \to 0.148651, c2_{0,2} \to -0.0270642, c2_{0,3} \to 0.0396351, c2_{0,4} \to -0.0970362, c2_{0,5} \to 0.33424, c2_{0,6} \to -1.48609, c2_{0,7} \to 8.10074, c2_{0,8} \to -52.3138, c2_{1,0} \to 0, c2_{1,1} \to 0, c2_{1,2} \to 0, c2_{1,3} \to 0, c2_{1,4} \to 0, c2_{1,5} \to 0, c2_{1,6} \to 0, c2_{1,7} \to 0, c2_{2,0} \to 0, c2_{2,1} \to 0, c2_{2,2} \to 0, c2_{2,3} \to 0, c2_{2,4} \to 0, c2_{2,5} \to 0, c2_{2,6} \to 0, c2_{3,0} \to 0, c2_{3,1} \to 0, c2_{3,2} \to 0, c2_{3,3} \to 0, c2_{3,4} \to 0, c2_{3,5} \to 0, c2_{4,0} \to 0, c2_{4,1} \to 0, c2_{4,2} \to 0, c2_{4,3} \to 0, c2_{4,4} \to 0, c2_{5,0} \to 0, c2_{5,1} \to 0, c2_{5,2} \to 0, c2_{5,3} \to 0, c2_{6,0} \to 0, c2_{6,1} \to 0, c2_{6,2} \to 0, c2_{7,0} \to 0, c2_{7,1} \to 0, c2_{8,0} \to 0\}$