How Far Can We Trust Computation?

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Abstract: Computation is playing a growing role in economics. Some applications, such as empirical estimation, are standard but computation is also being used in novel and controversial ways. Sometimes computation is used to solve problems for which there is no existence theorem. Computation is increasingly being used as a substitute for theory where a list of examples is presented instead of theorems, even though there are cases where computation has produced nonsense results. These observations raise questions about the reliability of numerical methods and the extent to which we can trust them. We take up these issues in an effort to consider what computation can and cannot do in economics. We first argue that economists should adopt the research paradigm of the physical sciences instead of pure mathematics. This is desirable since computationally intensive analysis of complex economic models is more appropriate for studying the real world than analyses of simpler tractable models. We then admit that simple, ad hoc, "intuitive" computational schemes are often unreliable, but that the numerical analysis literature does offer us reliable methods. We then present general strategies for using these methods to address the legitimate concerns of critics, and examine the tradeoffs between computational and theoretical approaches to economic analysis.

Introduction

- Computational power creates new opportunities
  - Traditional roles
    - Empirical analysis
    - Applied general equilibrium
  - Nontraditional roles
    - Substitute for theory
    - Complement of theoretical analyses
- Problems and Questions
  - What can computational methods *reliably* do?
  - Where does computation fit into economic methodology?
  - How do we address criticisms?
- Overview
  - Computational power and potential economic applications are increasing exponentially
  - What can we learn from computational applications in science?
  - Can computation replace theorems in economic theory?
  - What structural changes are necessary in economics to achieve potential?
- Key points
  - The objective of economics is to understand economic phenomenon
  - Computational tools can be trusted *if used carefully*

Hardware Progress - "You ain't seen nothing yet"

- Semiconductors
  - Moore's law - "density doubles every 18 months"
  - 3D chips, Asynchronous chips (no clock), Parallelism
  - Current trend – speed doubling every 2 years – continues for another 10 years
- Future technologies
  - Spintronics
  - Optical computing
  - Quantum computing

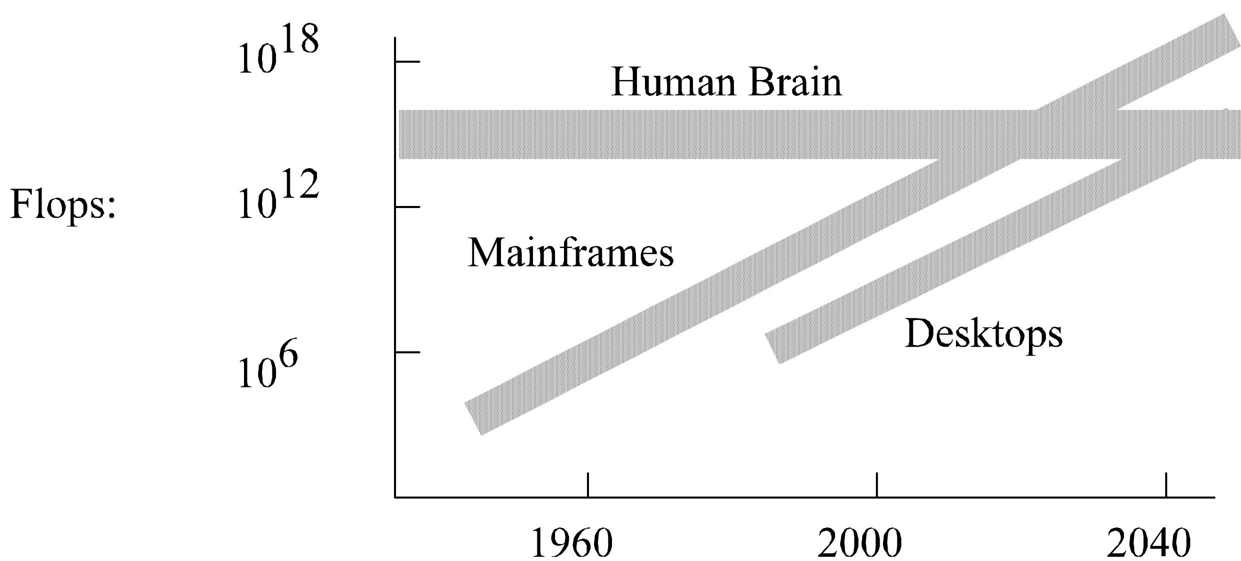


Figure 1: Trends in computation speed: flops vs. year

Software Progress

- Numerical Analysis
  - Recent Examples
    * Linear programming - Interior point methods
    * Nonlinear equations, complementarity problems
  - Numerical analysis progress has been as great as hardware progress!
  - Only small amount of numerical analysis is used in economics
- Systems and Languages
  - Parallelism: Combine many cheap processors
  - Net computing
  - Powerful languages - Fortran 90, C, C++
  - Symbolic languages, e.g. Mathematics, make it possible to do previously unthinkable algebraic computations, e.g., Judd-Guu (*ET*, 2001)
  - Sophisticated programming environments -GAMS, Mathcode C++
  - User friendly environments for less serious problems - Matlab, Gauss

What can economists compute now?

- Optimization
  - Dynamic programming
  - Mechanism design
- General equilibrium
  - Arrow-Debreu general equilibrium - Scarf
  - Perfect foresight models, Fair-Taylor, L-B-J
  - Dynamic, stochastic recursive models
- Asset markets
  - Asymmetric information - Bernardo-Judd
  - Incomplete asset markets - Brown et al., Schmedders, Judd-Guu
- Games
  - Finite games- Lemke-Howson, Wilson, McKelvey
  - Supergames - Cronshaw-Luenberger, Judd-Yeltekin-Conklin
- Dynamic games
  - Closed-loop (a.k.a., Markov perfect) - Wright-Williams, Miranda, Ha-Sibert, Pakes
  - Supergames with states - Judd-Yeltekin
  - Policy games - R. Chang, Sleet-Yeltekin

How Science uses Computation

- Methods used in science
  - Experimentation
  - Theories, models, and deductive methods
    - Produce theorems, closed-form solutions
    - Not common (except for freshman physics courses)
  - Computational mode - standard procedure
    - Write down equations
    - Compute solutions
- Examples of computational successes in astronomy
  - The Red Spot of Jupiter
  - Origin of the moon
  - Shape of Galaxies
- Economics as a science
  - Astronomy and economics – observational sciences
  - Red Spot $\sim$ Kydland-Prescott RBC analysis
  - Differences: Precise theories of science versus qualitative theories in economics

Economics vs. Science Approach: Equity Premium Puzzle Example

- Question: Why is return on stocks so much higher than bonds?
- Many possible explanations, each one found inadequate
  - Risk aversion, recursive utility, habit formation
  - “Keeping up with the Joneses”
  - Incomplete risk markets, Trading costs
  - “Junior can’t borrow”
  - Uncertainty about premium
- Consider: Would a meteorologist ask if weather is caused by
  - gravity alone?
  - heating and cooling alone?
  - the earth’s rotation alone?
- As with the weather, it is unreasonable to ask for a single dominant cause of economic phenomena

- Radical idea: examine a model with *all* of these elements
  - A general equilibrium perspective
  - Can x alone explain equity premium puzzle? – Wrong question
  - How do x, y, etc. together affect asset prices? - Right question
  - Need to examine interactions – Subadditive? Superadditive?
- Problem: Analysis would require substantial computational effort and produce only approximately accurate results
- Tukey: "Far better an approximate answer to the right question ... than an exact answer to the wrong question..."

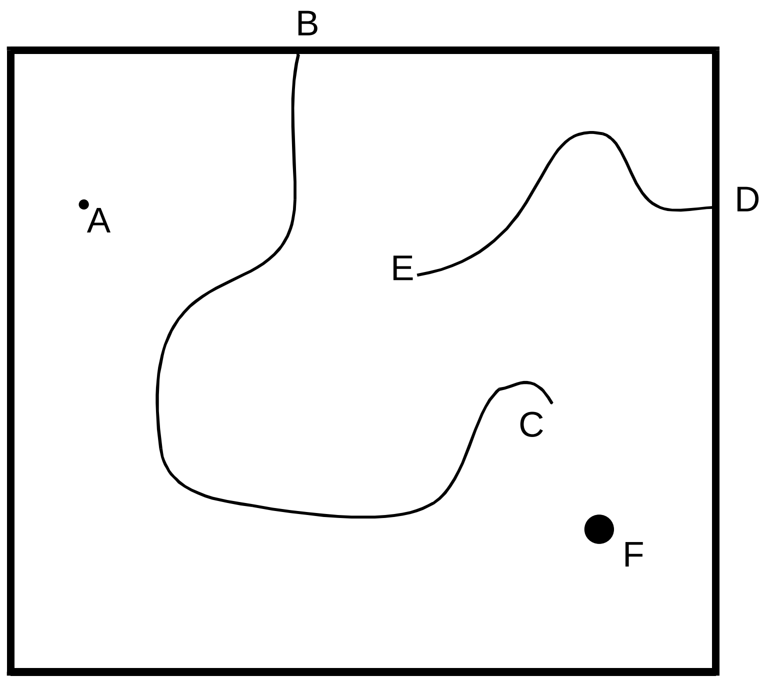
Frisch's Definition of "Econometrics"

- Computation is a natural part of econometrics as defined by Frisch (1933)
  - ".. as long as we confine ourselves to statements in general terms about one economic factor having an effect on some other factor, almost any sort of relationship may be selected.."
  - Economic analysis requires a comparison of magnitudes of complex interactions operating in all directions.
  - "Mathematics is indispensable ... necessary for discussing issues safely and consistently"
- Computational methods are necessary to examine the complex relationships in modern dynamic stochastic economies.

A Computational Approach to Theory

- Theory: A Definition
  - Define terms, concepts
  - State assumptions
  - Determine the implications of the theory
- Two ways to ascertain implications of a theory
  - Deductive Theory - traditional favorite in economics
    - Prove general theorems about general case
    - Add auxiliary assumptions to make tractable
    - Prove more precise theorems about tractable cases
  - Computational Theory
    - Specify parameterized versions of the theory
    - Compute specific models, i.e., fix parameter values
    - Summarize results of computations

- Theory as Exploration: The Typical Scenario
  - A theory defines a general class of models
  - Deductive theory often focuses on a thread of instances of the theory
  - Computation can examine a finite collection of general instances
  - Properties of model are piecewise continuous in the parameters



Typical graph of tractable cases

- Complaints by theorists
  - "You produce only examples, we produce theorems"
  - "If your results are true then you should be able to prove it"
  - "Where is your existence theorem?"
  - "Numerical results have errors"

"You produce only examples, we produce theorems"

- *Reply 1*: Theorem is just a plural of example
  - Deductive theory examines a continuum, but often a measure zero set, of examples
  - Computations often can examine only a finite collection of examples, but you can choose them
- *Reply 2*: Many of these examples are more relevant than tractable models
  - Example: Executive incentives
    - Question: How much should executive compensation be tied to corporate performance?
    - Model: principal-agent model
    - Jensen-Murphy argument
      - Find that total executives' incentives are small, about $3 per thousand
      - Argue that this is too little to create proper incentives
      - Conjecture that sensible risk aversion cannot explain this
      - Argue that political constraints are explanation
    - Haubrich computations: compute optimal contract for reasonable tastes and technology
    - Results: optimal share is small, often *about $3 per thousand*
  - *Relevance* of examples is more important than *cardinality*

- *Reply 2*: Perturbation methods can produce theorems
  - $F(x, \delta, \epsilon) = 0$ expresses a theory, with general solution $x(\delta, \epsilon)$
  - Solve $F(x, \delta, 0) = 0$ to get $x(\delta, 0)$ – thread of special tractable cases.
  - Perturbation methods compute $x(\delta, \epsilon)$ for small ϵ – fattens the thread.
  - Computers can do the algebra (sometimes enormous) required by perturbation methods
  - Example: Asset equilibrium (Judd-Guu, 2001)
    * Asset returns and consumption known when risk, σ^2, is zero
    * Use perturbation methods (Lyaponuv-Schmidt) to compute how asset returns change as σ^2 increases.
    * Example application: How does the introduction of an option affect price of underlying? (Answer: increase)

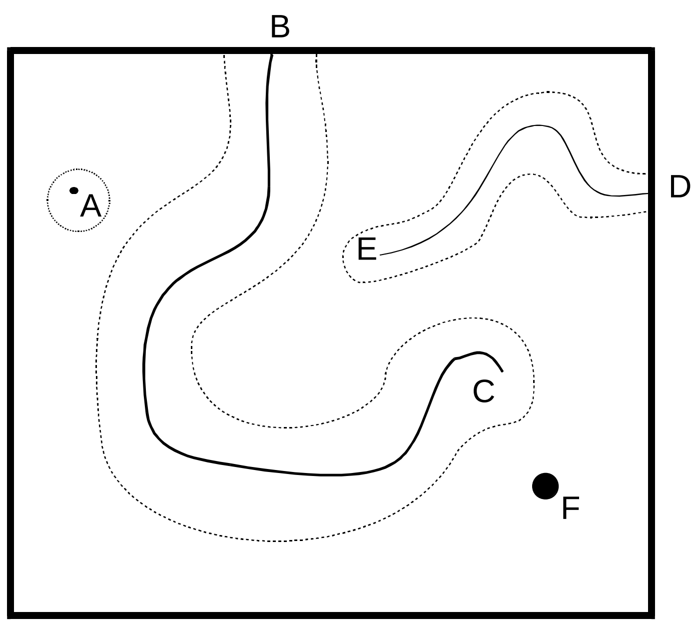


Figure 2: Perturbation methods fatten up tractable cases

- *Reply 3*: Computation can give results when there are no theorems
  - Simple general statements are not likely to be globally true
  - There may still be useful patterns
  - Example: Quirmbach
    * Question: What ex post market structure best encourages ex ante innovation among competitors?
    * Model: A two-period model of innovation then production
    * Computations: Search across demand functions, costs, R&D success rates, game forms
    * Results: Bertrand and Cournot were roughly the same, better than ex post collusion.
    * There are no reasonable theorems.

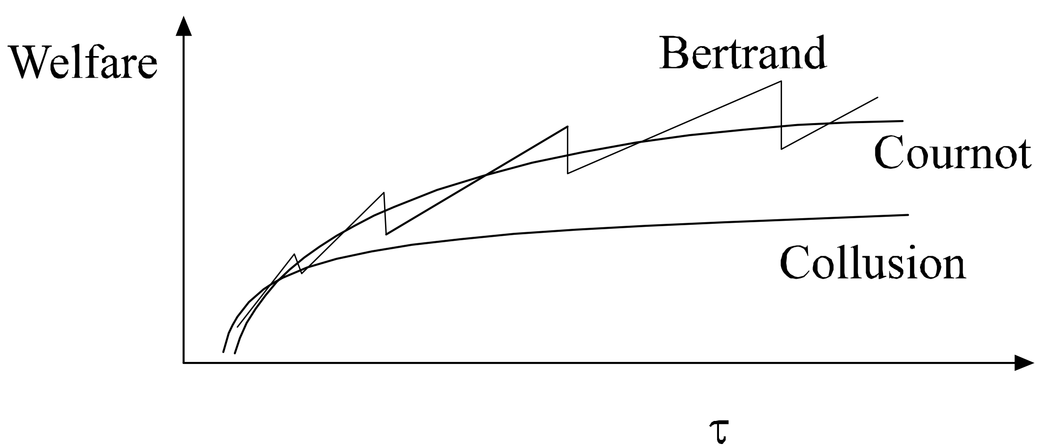


Figure 3: Welfare vs. probability of innovation success

- *Reply 4*: Computation can explore complex, intractable models
  - Marimon-McGrattan-Sargent exploration of genetic algorithm learning
  - Scott Page's model of local public goods provision
  - Agent-based Computational Economics is general area with this perspective

"If your results are true then you should be able to prove them"

- *Reply 1*: Leave the 19th century
  - Hilbert problem #2. Can it be proven that the axioms of logic are consistent?
  - Godel first incompleteness theorem: There exists true but unprovable statements.
- *Reply 2*: Efficiency
  - Objective is to gain economic insight about economic problems
  - If computing many examples is less costly, it is a more efficient way to achieve the goal.

“Where is your existence theorem?”

- *Reply 1*: Economists should aim to be scientists, not mathematicians.
  - General relativity theory
    - Proposed by Einstein
    - First nontrivial example came later
    - Still no general existence theorem
  - Navier-Stokes equation
    - Equation:
      - $u(x,t) \in R^n$ velocity of fluid, $p(x,t) \in R$ pressure
      - $x \in R^n$, $t \geq 0$
      - $f(x,t) \in R^n$ force, ν viscosity

$$\begin{aligned} \frac{\partial}{\partial t} u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} &= \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t) \\ \text{div } u &= \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \\ u(x,0) &= u^{\circ}(x) \end{aligned}$$

    - Applications: Fluid flow, such as boats and airplanes.
    - Numerical methods are used to “solve” these problems in practice.
    - Question: Do there exist physically reasonable solutions (e.g., smooth) to this problem? Are they unique? Smooth?
    - Prize: Solve problem, earn $1,000,000
    - Did lack of existence theorem keep you from flying here?

- *Reply 2*: Computation can produce a relevant existence theorem
  - Computation has errors that are interpretable
    - errors in optimization
    - lack of market clearing
  - If errors are uniformly bound by ε then you have an ε-equilibrium
  - ε-equilibria are all we can reasonably ask for from humans (God runs nature and does not allow errors in his world but people are imperfect.)
  - Multiplicity
    - We can check for size of ε-equilibria set
    - If it is large then theory is not precise even if there is a unique equilibrium

“Numerical results have errors”

Reply 1: Yes, but they can be controlled

- In econometrics, we
  - Take data, use statistics to find point estimate
  - Report point estimate, *and*
  - Report accuracy diagnostics
    - standard errors, p-value
    - Bayesian posterior variance
- In computational work, we
  - Takes specifications, and compute equilibrium
  - Report results, *and*
  - Should report accuracy diagnostics
    - den Haan-Marcet (1994): statistical evaluation of equilibrium
    - Judd (1992), backward error analysis of numerical analysis:
      - Formulate problem in dimensionless fashion
      - report L^p norms of errors of Euler equation errors, market non-clearing, etc.

Reply 2: Yes, but are they worse then errors made by pure theory?

- Trade-off: specification error of focus on tractable models versus numerical errors of numerical solutions to more general models.
- “All models are wrong but some are useful” – G. E. P. Box

Systematic Approaches to Computational Theory

- Perturbation Methods: Fattening the Thread
  - $F(x, \delta, \epsilon) = 0$ expresses a theory, with general solution $x(\delta, \epsilon)$
  - Deductive theory solves $F(x, \delta, 0) = 0$ to get $x(\delta, 0)$ – the thread of special cases.
  - Perturbation methods compute $x(\delta, \epsilon)$ for small ϵ – fattens the thread.
  - Example: consumption function in growth models

$$C(k, \sigma^2) \doteq C(k^*, 0) + C_k(k^*, 0) + C_{\sigma^2}(k^*, 0)\sigma^2 + \cdots$$

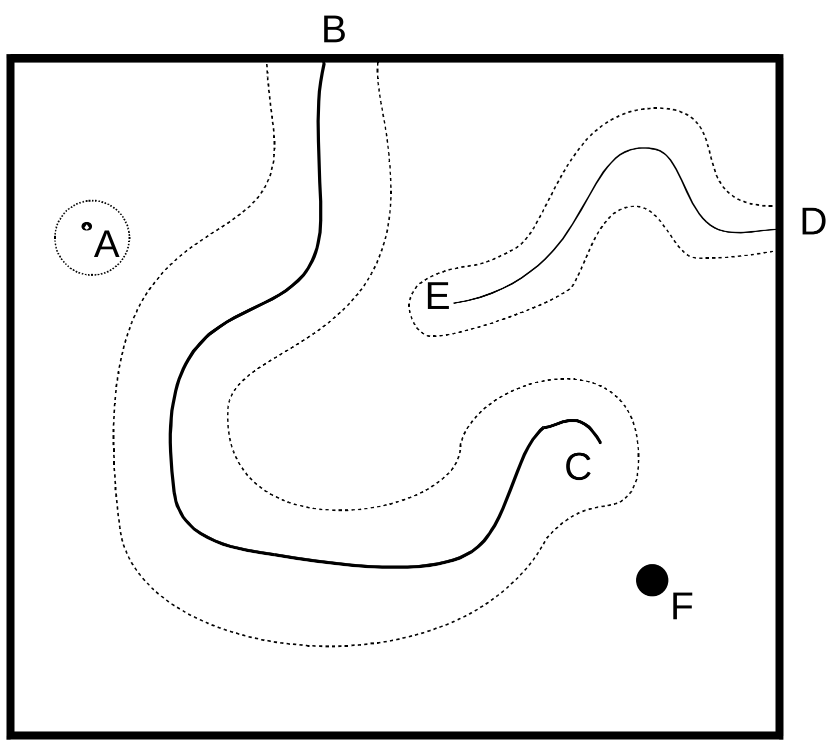


Figure 4: Perturbation methods fatten up tractable cases

- Monte Carlo Sampling
  - Draw N points independently from the model space according to a probability measure μ.
  - Test proposition at each $x \in X$
  - If proposition is true at $x \in X$, then state "We conclude with $1 - (1-\epsilon)^N$ confidence that the set of counterexamples has μ-measure less than ϵ."
- Quasi-Monte Carlo Sampling
  - Construct a N-point set X with low discrepancy (i.e., "uniformly spread") in a metric d. Let mesh be δ.
  - Test proposition at each $x \in X$
  - If proposition is true at each $x \in X$, then "No set of counterexamples contains a ball of diameter δ."
  - If proposition is true at each $x \in X$, interval arithmetic is used, and Lipschitz bounds apply and can be computed a priori, then Proposition is *proven*!
- Sample and Search
  - Formulate proposition as $f(x) > 0$
  - Construct a N-point set X
  - Check $f(x) > 0$ at $x \in X$
  - Use each point as the initial point in $\min_x f(x)$

- Regression Methods of Summarizing Results
  - Construct a N-point set X of instances.
  - Compute quantities of interest (price, quantity, welfare, etd.) at each $x \in X$
  - "Regress" the quantities of interest on the model parameters, compute "covariances" among model parameters and equilibrium outcomes.
- Presentation of Computational Results
  - Tables, Graphs – limited
  - "Confidence probabilities"
  - "Regression" results
  - Need ways to describe robust patterns

Deductive versus Computational Theory

	Deductive methods	Computational methods
Approach:	Prove theorems	compute examples
Validity:	absolute	limited by numerical error
Range:	continua	finite number of examples
Generality:	simplifications made for tractability	limited only by computational methods
Existence:	proven	present examples of ϵ-equilibrium
Comp. statics:	usually need special functional forms or strong qualitative assumptions	impose empirically motivated restrictions
Errors:	specification errors	numerical errors
Inputs:	mathematical theory skills	computer time, computational skills

The Current Problems of Computational Economics

- Poor scholarship
  - Reinventing the wheel (e.g., rational expectations, dynamic games)
  - Vocabulary inconsistent with mathematical practice (e.g., "linear approximation")
  - Nonsense results appear in published papers, making theorists' criticisms appear valid!
  - Incomplete reporting and checking of results (e.g., stochastic simulators seldom report standard errors)
  - Lack of knowledge about numerical analysis - "Monte Carlo is only way to do high (>6) dimensional integration"

- Poor graduate training in numerical methods
  - Most training is narrow, field-specific.
  - Few academic programs commit resources to a computational course (exceptions: U. Chicago, NU (Kellogg), Ohio State Ag. Econ.)
  - Example: Linear approximation of rational expectations models
    - Standard “method”
      - Compute steady state
      - Take linear approx. of constraints and quadratic approx. of objective
      - Solve L-Q dynamic programming problem
    - Conclusion (c. 1990): “Higher-order approximations are not possible since we cannot exactly solve quadratic-cubic DP problems”
    - Fact: Mathematicians have been taking high-order approximations of RBC-like models since 1970’s.
    - June, 2002: Three papers presented at SCE conference on efficient code to take order k approximations of arbitrary nonlinear rational expectations models.

- Typical journal policies hinder dissemination of computational work
  - Theorists (economic and econometric) are required to show proofs
  - Empirical papers must describe methods, report standard errors, etc.
  - Experimentalists have strict protocols of reporting - See "Guidelines for submission of manuscripts on experimental economics" by Palfrey and Porter in *Econometrica* (1991)
  - *But*, journal editors say "get rid of computational details"
- Little effort at disseminating, sharing software
- Challenges for Computational Economics
  - Improve graduate training
  - Get journals to treat computational work like other work
  - Create high but reasonable standards

The Future of Computation in Economics

- Technology – Hardware and Software
  - Computing costs will continue to decrease
  - New computing environments and technologies can be exploited
  - Economists will catch up to numerical analysis frontier
  - Numerical analysis will find better ways to exploit new technologies
  - Economists will develop problem-specific methods (as in CGE)
- Economists can address criticisms if they
  - Use accurate, efficient techniques
  - Apply accuracy tests to their results
  - Are transparent in their use of numerical methods
- An Economic Theory of Future Economic Research
  - Inputs: Human time and computers
  - Output: Understanding of economic systems
  - Trend: Falling price of computation
  - Prediction: Comparative advantage principles imply
    - Substitute computer power for human effort to analyze specific models
    - Human activity focusses on formulating concepts and models