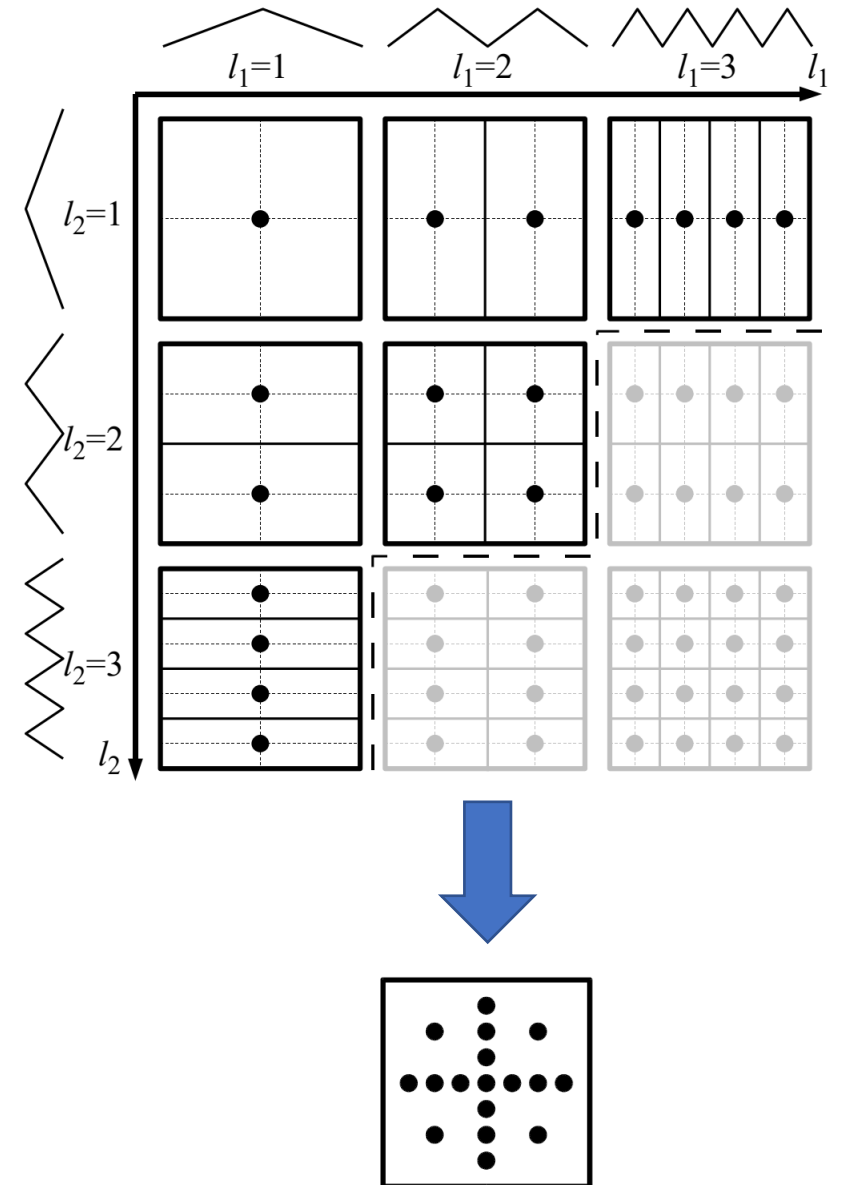
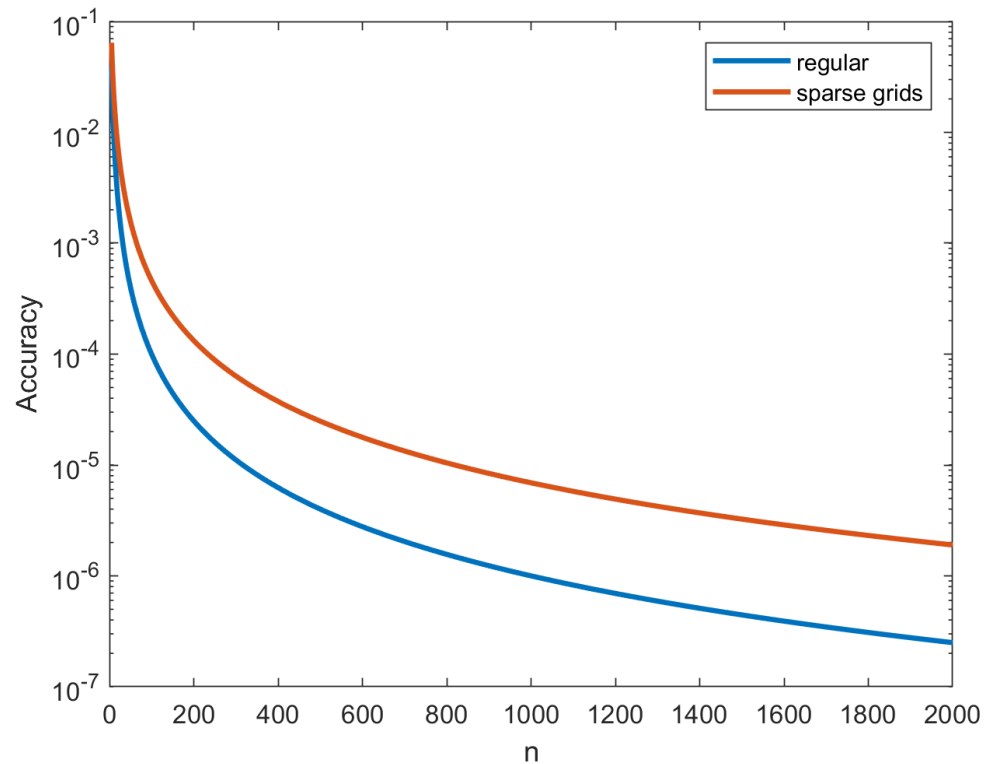


Hierarchical Sparse Grid

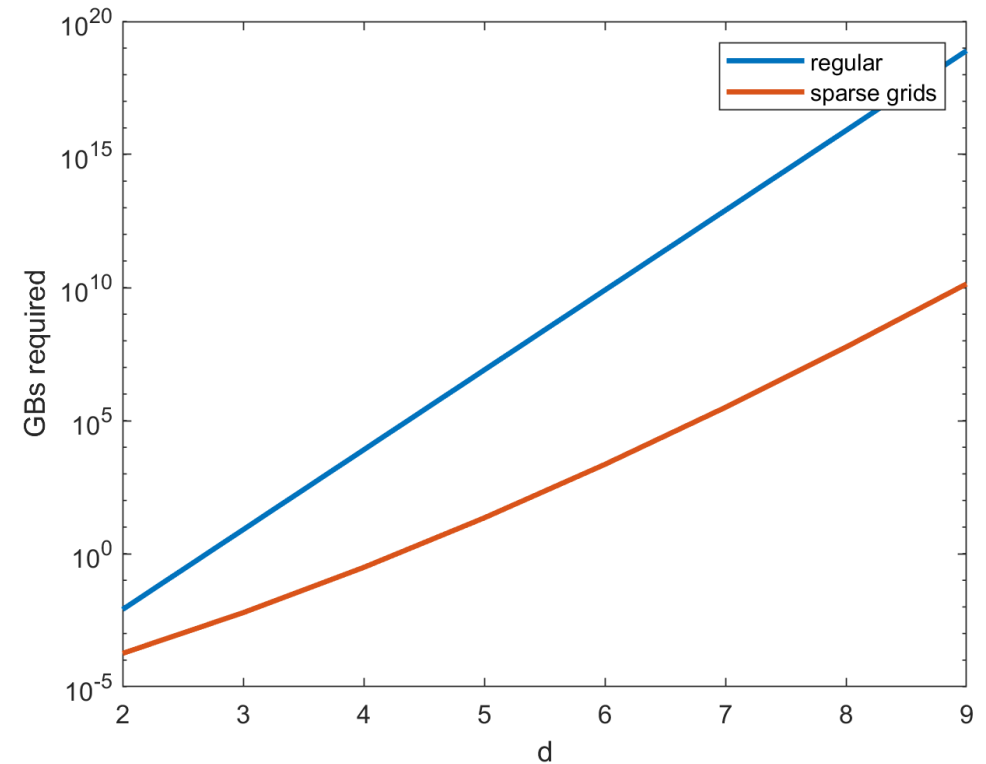
- Approximate $f: \mathbb{R}^d \rightarrow \mathbb{R}$ by combining univariate basis functions $\phi_{level,index}: [0, 1] \rightarrow \mathbb{R}$
- Desiderable properties of ϕ
 - Local support
 - Easy to construct and evaluate
- Basis functions can be
 - Piece-wise linear
 - Global polynomials
 - B-splines



	Regular grid	Sparse grid
Complexity	$O(n^d)$	$O(n \log(n)^{d-1})$
Accuracy	$O(n^{-2})$	$O\left(n^{-2} \log(n^{-1})^{d-1}\right)$



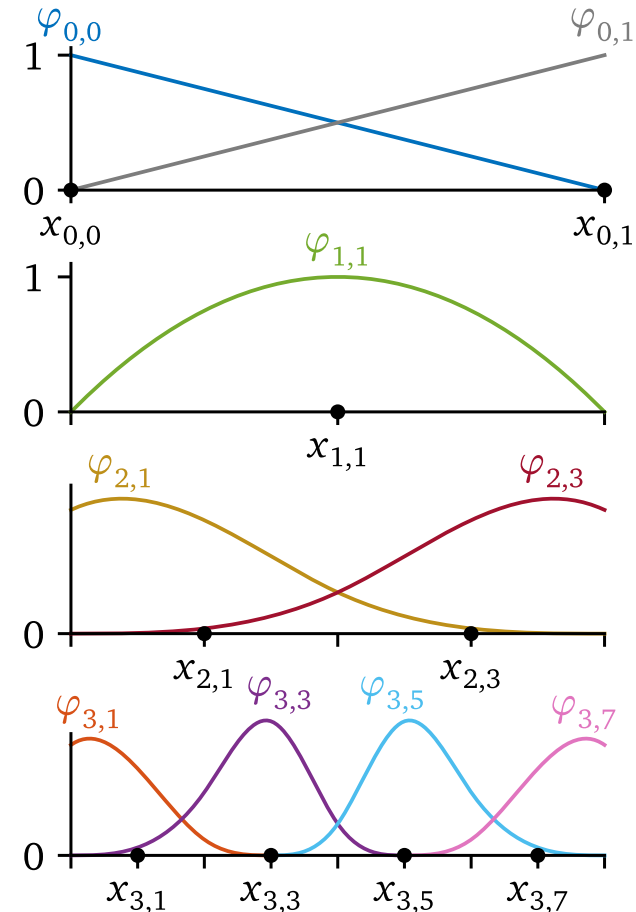
Accuracy as a function of the number of grid points for $d = 2$.



GB required to achieve an accuracy of 10^{-6} .

Hierarchical B-Splines of degree p

- Standard basis functions are piece-wise linear
 - Thus, only C^0 which can be insufficient
- B-splines
 - $C^{p-1} \rightarrow$ Gradient-based optimization
 - Still locally supported
 - Important as otherwise cost of evaluating sparse grid approximation explodes



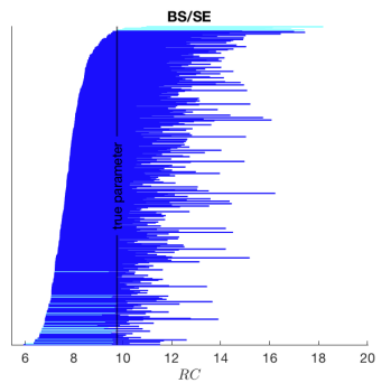
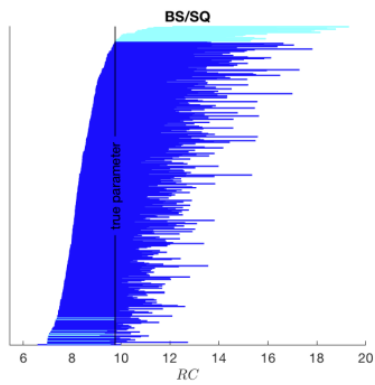
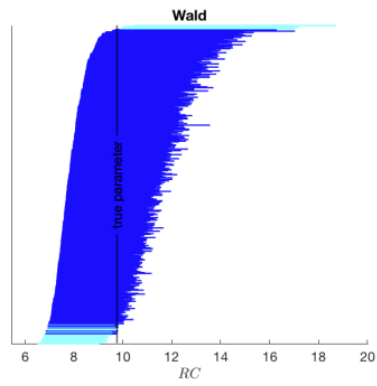
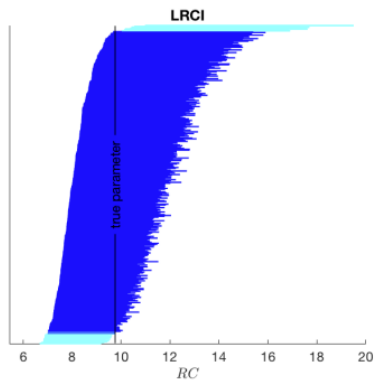
Bus Engine Replacement Model: Rust (1987)

Recall the implicit likelihood optimization problem

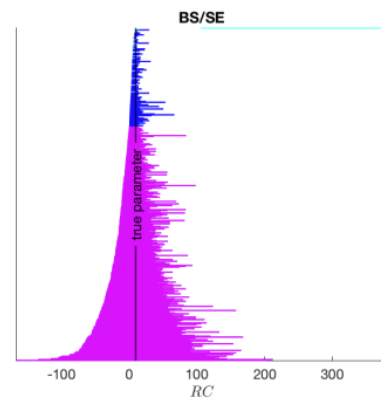
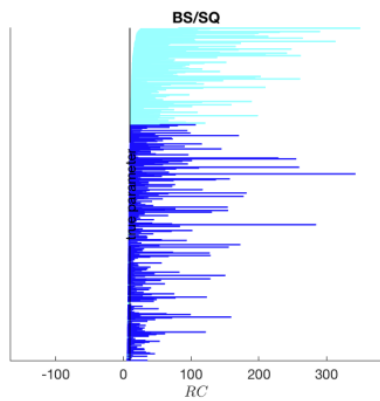
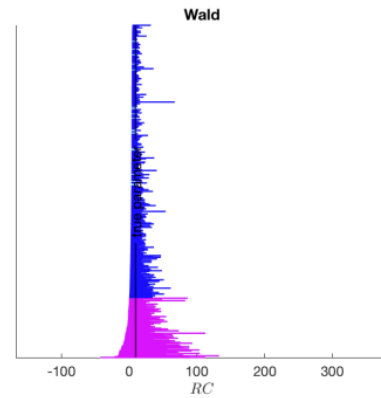
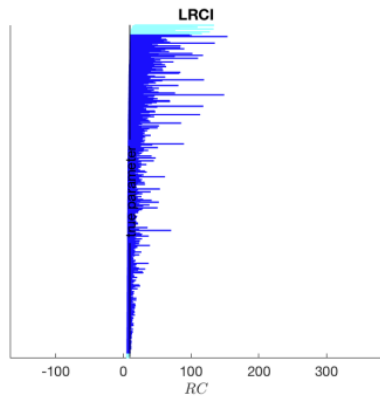
$$\begin{aligned} \max_{RC, \theta_{11}} L(EV, RC, \theta_{11}; data) \\ EV = T(EV) \end{aligned}$$

EV is solved with every evaluation of $L(EV, RC, \theta_{11}; data)$ by using a non-linear solver

Goal: find $\gamma\%$ - confidence set $\{\theta_j: \max_{\theta_{j-1}} L(\theta) + 0.5 \chi_1^2(\gamma) \geq L(\hat{\theta})\}$

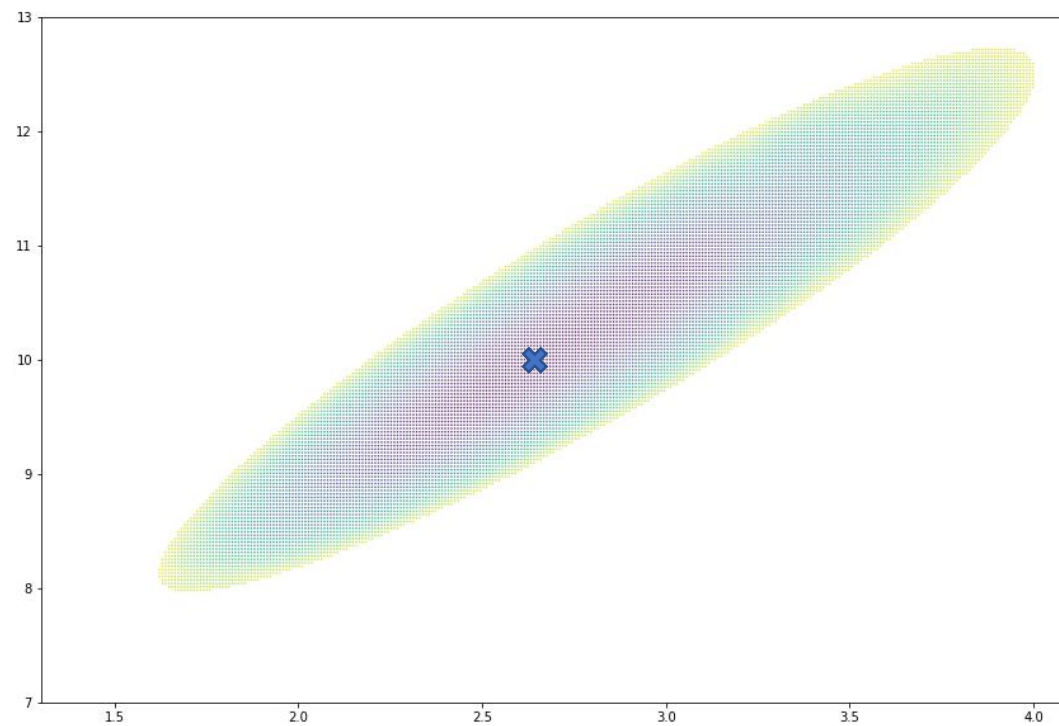


- CIs from simulated data sets of size equal to Rust (1987)
- **Blue** = True parameter in CI
- **Cyan** = True parameter not in CI

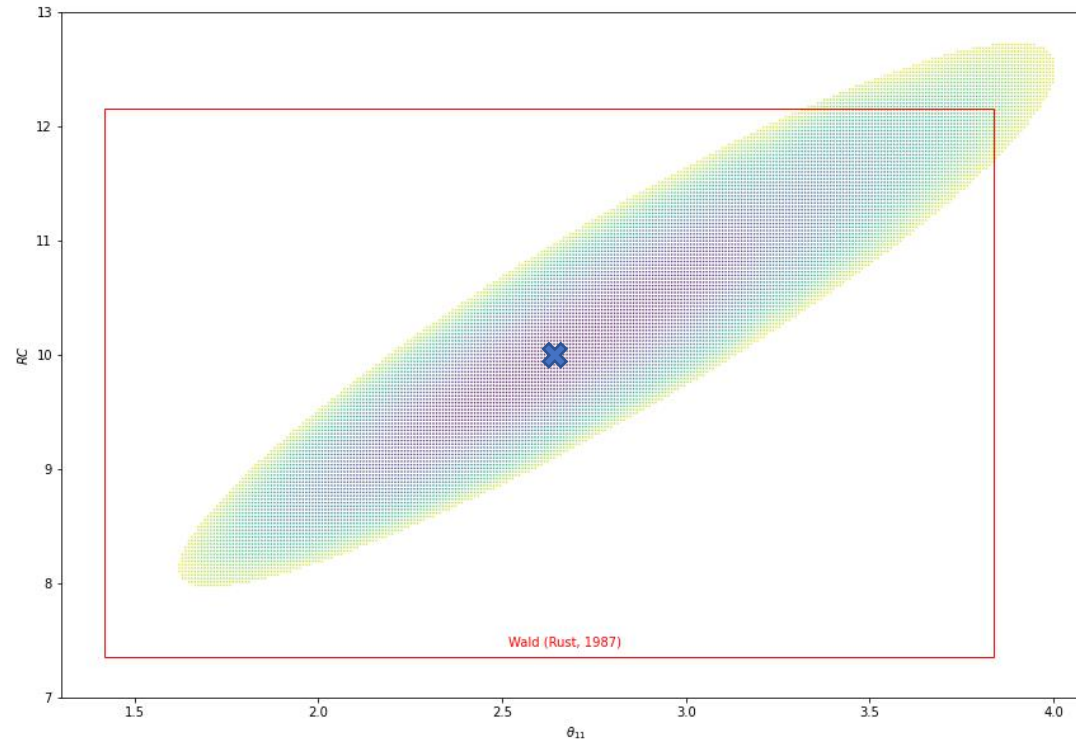


- CIs from simulated data sets of size = 1/10th of Rust (1987)
- **Blue** = True parameter in CI
- **Cyan** = True parameter not in CI
- **Violet** = CI includes nonsense values

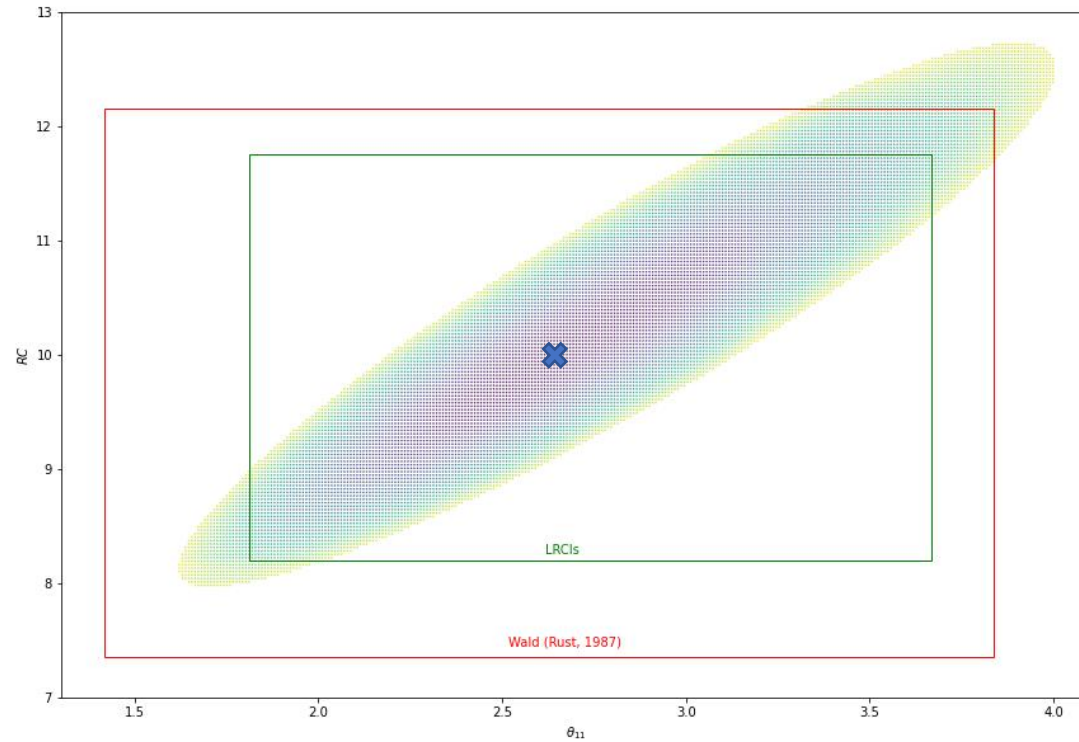
Bus Engine Replacement Model: Rust (1987)



Bus Engine Replacement Model: Rust (1987)



Bus Engine Replacement Model: Rust (1987)



References

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 - <https://github.com/SGpp/SGpp>
 - <https://github.com/SGpp/SGpp/wiki/Quick-Start>