DOES IT PAY TO GET A REVERSE MORTGAGE?

TECHNICA   APPENDIX

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MPEC with Dynamic Programming: Discrete and Continuous Choices

The panel data used in this study involve 3 years and 165 individuals. The available data are both continuous and discrete. The continuous data include consumption and nonhousing financial wealth. The discrete (or discretized) data are the individual's housing tenure (own versus rent), her moving decision, and her home value. I have additional data on the individuals' demographics, including age.

The MPEC with dynamic programming (DP) approach simultaneously solves the dynamic programming problem and the maximum likelihood estimation of the preference parameters.

Dynamic Programming with Approximation of the Value Function

ife-Cycle Model

One continuous state variable: financial wealth.

Two discrete state variables: previous-period housing tenure and previous-period house value.

One continuous choice variable: consumption.

Many discrete choices: Not move(N), Move to home with value h with housing tenure q (Mhq), where q ={own, rent}.

Backward Solution from Time   for True Value Functions

In each period, the household chooses whether to stay in the home or move out. If she moves out, she can either buy or rent a new home and can choose its value.   et the subscripts d^N, d^{Mhq} denote, respectively, the decision not to move and the decision to move to a home valued at h with housing tenure q. Housing tenure is a binary variable that takes a value of 1 if the household owns the home.

The last-period value function is known and equal to $V_T(W, H, Q)$, where W is the individual's nonhousing financial wealth, H her previous-period house value, and Q her previous-period housing tenure.

In periods $t = 1...(T-1)$ I define

$$\begin{aligned} V_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + y; H, Q) + \varepsilon_t^N \\ V_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q) + \varepsilon_t^{Mqh}, \end{aligned}$$

where M is the transaction cost:

$$M = Q[qh - H + \phi^{own}qh + \phi^{rent}(1-q)h] + (1-Q)(1-q)\phi^{rent}h$$

and ψ is the per period housing expense:

$$\psi = [Q\psi^{own} + (1-Q)\psi^{rent}]H + [q\psi^{own} + (1-q)\psi^{rent}]h.$$

c_{d^N} and $c_{d^{Mqh}}$ are the consumption levels, respectively, if the individual does not move and if she moves to home value h choosing housing tenure q. y is the household's per-period income. η_{t+1} is her survival probability. ε_t^N and ε_t^{Mqh} are extreme value type I errors.

Following Rust (1987), I assume that the additivity and the conditional independence assumptions hold.

To simplify the notation, I introduce the following expressions, which are evaluated at the optimal consumption level:

$$\begin{aligned} \widehat{V}_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + y; H, Q) \\ \widehat{V}_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q). \end{aligned}$$

The extreme value assumption on ε_t implies that I can reduce the dimensionality of the dynamic programming problem. The Bellman equation is given by the following closed-form solution:

$$
\begin{aligned}
V_t(W,H,Q) &= \Pr(N|W,H,Q)\ \ \widehat{V}_{d^N,t} + E(\varepsilon_t^N|N) \\
&\quad + \sum_h \sum_q \{\Pr(Mhq|W,H,Q)\ \ \widehat{V}_{d^{Mhq},t} + E(\varepsilon_t^{Mhq}|Mhq)\} \\
&= \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}
\end{aligned}
$$

Given V_{t+1}, the Bellman equation implies, for each wealth level W, three sets of equations that determine optimal consumption, $c^*_{d^N}$, $c^*_{d^{Mhq}}$, $V_t(W,H,Q)$, and $V_t'(W,H,Q)$:

Euler equations:

$$
\begin{aligned}
u'(c^*_{d^N},H) - \beta\eta_{t+1}V'_{t+1}(RW - c^*_{d^N} - \psi + y; H, Q) &= 0 \\
u'(c^*_{d^{Mhq}},h) - \beta\eta_{t+1}V'_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q) &= 0
\end{aligned}
$$

Envelope condition:

$$
V_t'(W,H,Q) = \Pr(N|W,H,Q)\ \ \widehat{V}'_{d^N,t} + \sum_h \sum_q \Pr(Mhq|W,H,Q)\ \ \widehat{V}'_{d^{Mhq},t}
$$

Bellman equation:

$$
V_t(W,H,Q) = \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}.
$$

The time $t = 1...(T-1)$ probabilities of not moving and moving to home value h with housing tenure q are:

$$
\Pr(N|W,H,Q) = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(V_t(W,H,Q))}
$$

$$
\Pr(Mhq|W,H,Q) = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(V_t(W,H,Q))}.
$$

Backward Solution from Time for Approximate Value Functions

et $\Phi(W,H,Q;a)$ and $\Phi_d(W,H,Q;b)$ be the functions that I use to approximate, respectively, the value functions $V(W,H,Q)$ and the policy functions $c^*_d(W,H,Q)$, with $d=\{d^N,d^{Mhq}\}$. If I assume that they are seventh-order polynomials centered at $\overline{W}$, then

$$\Phi(W,H,Q;a,\overline{W})=\sum_{k=0}^{7}a_{k,H,Q}(W-\overline{W})^k.$$

The time t value function is approximated by

$$V_t\,(W,H,Q)=\Phi(W,H,Q;a_t,\overline{W}_t)=\sum_{k=0}^{7}a_{k+1,H,Q,t}(W-\overline{W}_t)^k.$$

The time t policy functions are approximated by

$$c^*_{d,t}\,(W,H,Q)=\Phi(W,H,Q;b_{d,t},\overline{W}_t)=\sum_{k=0}^{7}b_{k+1,H,Q,d,t}(W-\overline{W}_t)^k,$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{W}$ on time, and the dependence of the policy functions on time is represented by the dependence of the b coefficients and the center $\overline{W}$.

I choose $\overline{W}_t=(W_t^{\max}+W_t^{\mathrm{mi}}\ )/2$, the period t average level of wealth. Note that $\overline{W}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, I drop it as an explicit argument of Φ. So, $\Phi(W,H,Q;a_t)$ will mean $\Phi(W,H,Q;a_t,\overline{W}_t)$.

I would like to find coefficients a_t and $b_{d,t}$ such that each time t Bellman equation, along with the Euler and envelope conditions, holds with the Φ approximation; that is, for each time $t<T-2$, I want to find coefficients a_t such that for all W

$$\Phi(W,H,Q;a_t)=\ln\left\{\exp(\widehat{V}_{d^N,t})+\sum_h\sum_q\exp(\widehat{V}_{d^{Mhq},t})\right\},$$

where

$$\begin{aligned}\widehat{V}_{d^N,t}&=u(c^*_{d^N},H)+\beta\eta_{t+1}\Phi_{t+1}(RW-c^*_{d^N}-\psi+y;H,Q;a_{t+1})\\ \widehat{V}_{d^{Mhq},t}&=u(c^*_{d^{Mhq}},h)+\beta\eta_{t+1}\Phi_{t+1}(RW-c^*_{d^{Mhq}}-\psi-M+y;h,q;a_{t+1}),\end{aligned}$$

and for time $t = T - 1$, I want to find coefficients a_t given that

$$\widehat{V}_{d^N,T \;\; 1} = u(c^*_{d^N}, H) + \beta\eta_T V_T(RW - c^*_{d^N} - \psi + y; H, Q)$$

$$\widehat{V}_{d^{Mhq},T \;\; 1} = u(c^*_{d^{Mhq}}, h) + \beta\eta_T V_T(RW - c^*_{d^{Mhq}} - \psi - M + y; h, q).$$

I need to solve the Bellman equation approximately. To this end, I define various errors.

First, I create a finite grid of asset levels I will use for approximating the value functions. et $W_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. Then I create a grid of home values. et $H_{j,t}$ be grid point j in the time t grid. Housing tenure is a binary variable. et $Q_{z,t}$ be grid point z in the time t grid.

Next I need to specify the various errors that may arise in the approximation. I will consider three errors and one side condition.

First, at each time t and for each $W_{i,t}$ and each previous-period home value $H_{j,t \;\; 1}$ and housing tenure $Q_{z,t \;\; 1}$, the absolute value of the Euler equations if consumption is, respectively, $c^*_{i,j,z,d^N,t}$ and $c^*_{i,d^{Mhq},t}$, which I denote as $\lambda^e_{i,j,z,t} \geq 0$, satisfies the inequality

$$-\lambda^e_{i,j,z,t} \leq u'(c^*_{i,j,z,d^N,t}, H_{j,t \;\; 1}) - \beta\eta_{t+1}\Phi'(RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t \;\; 1}, Q_{z,t \;\; 1}; a_{t+1}) \leq \lambda^e_{i,j,z,t}$$

$$-\lambda^e_{i,j,z,t} \leq u'(c^*_{i,d^{Mhq},t}, h_t)$$

$$-\beta\eta_{t+1}\Phi'(RW_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1}) \leq \lambda^e_{i,j,z,t},$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $W_{i,t}$ with consumption $c_{i,j,z,d^N,t}$ and $c_{i,d^{Mhq},t}$ is denoted by $\lambda^b_{j,z,t}$ and satisfies

$$-\lambda^b_{j,z,t} \leq \Phi(W_{i,t}, H_{j,t \;\; 1}, Q_{z,t \;\; 1}; a_t) - \ln\left\{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^b_{j,z,t},$$

where

$$\widehat{V}_{i,j,z,d^N,t} = u(c^*_{i,j,z,d^N,t}, H_{j,t\ \ 1}) + \beta\eta_{t+1}\Phi(RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t\ \ 1}, Q_{z,t\ \ 1}; a_{t+1})$$
$$\widehat{V}_{i,d^{Mhq},t} = u(c^*_{i,d^{Mhq},t}, h_t) + \beta\eta_{t+1}\Phi(RW_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1})$$

Third, the envelope condition errors, $\lambda^{env}_{j,z,t}$, satisfy

$$-\lambda^{env}_{j,z,t} \leq \Phi'(W_{i,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}; a_t) - \{f_{i,j,z,d^N,t}\ \Phi'(RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t\ \ 1}, Q_{z,t\ \ 1}; a_{t+1})$$

$$+\sum_h \sum_q [f_{i,d^{Mhq},t}\ \ \Phi'(RW_{i,t} - c_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1})]\} \leq \lambda^{env}_{j,z,t},$$

where

$$f_{i,j,z,d,t} = \Pr(d|W_{i,t}, H_{j,t}, Q_{z,t}) = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})}.$$

Fourth, I introduce the policy function errors:

$$-\lambda^{cons}_{i,j,z,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_{z,t}; b_t) - c^*_{i,j,z,d,t}(W_{i,t}, H_{j,t}, Q_{z,t}) \leq \lambda^{cons}_{i,j,z,d,t}.$$

Empirical Part

In the theoretical DP part I obtain the coefficients used in the approximation of the value function. In this part, for any individual data on financial wealth, previous-period home value and age, I calculate predicted consumption and the probability of moving. The individual makes the housing decision $d_{n,tp}$ and the consumption decision simultaneously.

et $c^{pred}_{n,tp}$ and $c^{data}_{n,tp}$ denote, respectively, the predicted and the true value of consumption for household n at time tp. For any given discrete choice on housing $d_{n,tp}$, using the real data on consumption, I calculate the measurement error:

$$\Pr(c_{n,tp}|d_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\ \frac{c^{data}_{,tp}\quad c^{pred}_{,tp})^2}{2\sigma^2}}.$$

The probability for the discrete choice on housing is given by

$$\Pr(d_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}) = \frac{e^{\widehat{V}_{d,\ \ ,tp}}}{\sum_m e^{\widehat{V}_{m,n,tp}}}.$$

Therefore the joint probability of making the discrete housing choice $d_{n,tp}$ and the continuous consumption choice $c_{n,tp}$ is given by

$$\Pr(d_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}) =$$

$$\Pr(d_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1})\ \ \Pr(c_{n,t}|d_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}).$$

The log-likelihood is given by:

$$(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}, \theta),$$

where N denotes the number of individuals in the sample and TP the number of time periods in the panel data.

MPEC

With these definitions, let

$$\Lambda = \sum_t \sum_i \sum_j \sum_z \lambda^e_{i,j,z,t} + \sum_t \sum_j \sum_z \lambda^b_{j,z,t} + \sum_t \sum_j \sum_z \lambda^{env}_{j,z,t} + \sum_t \sum_i \sum_j \sum_z {}_d\lambda^{cons}_{i,j,z,d,t},$$

and let P be a penalty parameter.

The MPEC approach to the estimation of the preference parameters is

$$\underset{\theta,a,c}{Max}\ (\theta) - P\ \ \Lambda$$

subject to

Bellman error

$$-\lambda^b_{j,z,t} \leq \Phi(W_{i,t}; a_t) - \ln\left\{\exp(\widehat{V}_{i,i,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})\right\} \leq \lambda^b_{j,z,t}$$

Euler errors

$$
\begin{aligned}
-\lambda^{e}_{i,j,z,t} &\leq u_{c;i,j,z,d^N,t} + \beta \Phi^{+}_{W;i,j,z,d^N,t} \leq \lambda^{e}_{i,j,z,t} \\
-\lambda^{e}_{i,j,z,t} &\leq u_{c;i,d^{Mhq},t} + \beta \Phi^{+}_{W;i,d^{Mhq},t} \leq \lambda^{e}_{i,j,z,t}
\end{aligned}
$$

envelope error

$$
-\lambda^{env}_{j,z,t} \leq \Phi_{W;i,z,t} - \{f_{i,j,z,d^N,t} \quad \Phi^{+}_{W;i,j,z,d^N,t} + \sum_{h} \sum_{q} [f_{i,d^{Mhq},t} \quad \Phi^{+}_{W;i,d^{Mhq},t}]\} \leq \lambda^{env}_{j,z,t}
$$

policy function error

$$
-\lambda^{cons}_{i,j,z,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_{z,t}; b_{d,t}) - c^{*}_{i,j,z,d,t}(W_{i,t}, H_{j,t}, Q_{z,t}) \leq \lambda^{cons}_{i,j,z,d,t}
$$

The probability of decision d is

$$
f_{i,j,z,d,t} = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})},
$$

where Φ^{+} denotes the approximation for the next-period value function, as described in the next subsection.

AMP

Backward Solution from Time for Approximate Value Functions in AMP

In order to formulate this problem in AMP , I need to list every quantity that is computed. The time-specific asset grids $W_{i,t}$ are fixed. The parameters are

$$W_{i,t}, \beta, \eta_{i,t}, R, \psi^{own}, \psi^{rent}, \phi^{own}, \phi^{rent}, \theta_B.$$

The basic variables of interest are

$$\begin{aligned}
&c_{i,j,z,d^N,t},\ c_{i,d^{Mhq},t} \\
&a_{k,j,z,t},\ b_{k,j,z,d,t} \\
&\lambda^{e}_{i,j,z,t},\ \lambda^{b}_{j,z,t},\ \lambda^{env}_{j,z,t},\ \lambda^{cons}_{i,j,z,d,t}.
\end{aligned}$$

AMP does not allow procedure programming; therefore, I need to define other variables to represent quantities defined in terms of other variables. I first need

$$\begin{aligned}
u_{i,j,z,d^N,t} &\equiv u\left(c^*_{i,j,z,d^N,t}, H_{j,t\ \ 1}\right) \\
u_{c;i,j,z,d^N,t} &\equiv u'\left(c^*_{i,j,z,d^N,t}, H_{j,t\ \ 1}\right) \\
W^+_{i,j,z,d^N,t} &\equiv RW_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y \\
f_{i,j,z,d^N,t} &= \Pr(N|W_{i,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}) \\
u_{i,d^{Mhq},t} &\equiv u\left(c^*_{i,d^{Mhq},t}, h_t\right) \\
u_{c;i,d^{Mhq},t} &\equiv u'\left(c^*_{i,d^{Mhq},t}, h_t\right) \\
W^+_{i,d^{Mhq},t} &\equiv RW_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{hq} - M_{j,z,d^{Mhq}} + y \\
f_{i,d^{Mhq},t} &= \Pr(Mhq|W_{i,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}).
\end{aligned}$$

I next use these variables to construct more variables:

$$
\begin{aligned}
\Phi_{i,j,z,t} &\equiv \Phi(W_{i,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}; a_t) \\
\Phi_{W;i,j,z,t} &\equiv \Phi'(W_{i,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}; a_t) \\
\Phi^{+}_{i,j,z,d^N,t} &\equiv \Phi(W^{+}_{i,j,z,d^N,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}; a_{t+1}) \\
\Phi^{+}_{W;i,j,z,d^N,t} &\equiv \Phi'(W^{+}_{i,j,z,d^N,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}; a_{t+1}) \\
\Phi^{+}_{i,d^{Mhq},t} &\equiv \Phi(W^{+}_{i,d^{Mhq},t}, h_t, q_t; a_{t+1}) \\
\Phi^{+}_{W;i,d^{Mhq},t} &\equiv \Phi'(W^{+}_{i,d^{Mhq},t}, h_t, q_t; a_{t+1}) \\
\Psi_{i,j,z,d,t} &\equiv \Phi(W_{i,t}, H_{j,t\ \ 1}, Q_{z,t\ \ 1}; b_{d,t})
\end{aligned}
$$

With these variables defined, the Bellman equation error inequality becomes

$$
-\lambda^b_{j,z,t} \leq \Phi_{i,j,z,t} - \ln\left\{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})\right\} \leq \lambda^b_{j,z,t},
$$

where

$$
\begin{aligned}
\widehat{V}_{i,j,z,d^N,t} &= u_{i,j,z,d^N,t} + \beta\eta_{t+1}\Phi^{+}_{i,j,z,d^N,t} \\
\widehat{V}_{i,d^{Mhq},t} &= u_{i,d^{Mhq},t} + \beta\eta_{t+1}\Phi^{+}_{i,d^{Mhq},t}
\end{aligned}
$$

the Euler equation error inequalities become

$$
\begin{aligned}
-\lambda^e_{i,j,z,t} &\leq u_{c;i,j,z,d^N,t} + \beta\Phi^{+}_{W;i,j,z,d^N,t} \leq \lambda^e_{i,j,z,t} \\
-\lambda^e_{i,j,z,t} &\leq u_{c;i,d^{Mhq},t} + \beta\Phi^{+}_{W;i,d^{Mhq},t} \leq \lambda^e_{i,j,z,,t},
\end{aligned}
$$

and the envelope error inequality becomes

$$
-\lambda^{env}_{j,z,t} \leq \Phi_{W;i,j,z,t} - \{f_{i,j,z,d^N,t}\ \ \Phi^{+}_{W;i,j,z,d^N,t} + \sum_h \sum_q [f_{i,d^{Mhq},t}\ \ \Phi^{+}_{W;i,d^{Mhq},t}]\} \leq \lambda^{env}_{j,z,t}.
$$

The probability of decision d is then

$$f_{i,j,d,t} = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})}.$$

The policy function errors are

$$-\lambda^{cons}_{i,j,z,d,t} \leq \Psi_{i,j,z,d,t} - c^*_{i,j,z,d,t} \leq \lambda^{cons}_{i,j,z,d,t}.$$

Empirical Part in AMP

I consider individuals in our sample such that $\quad ge^{data}_{n,tp} = 1...(T-2)$.

et $W^{data}_{n,tp}$, $\quad ge^{data}_{n,tp}$, $H^{data}_{n,tp\ \ 1}$ and $Q^{data}_{n,tp\ \ 1}$ denote the data on nonhousing financial wealth, age, previous-period home value and previous-period housing tenure for household n in year tp in the panel data. Given these data, the variables of interest are

$$c^{pred}_{d^N,n,tp} = \Psi_{d^N}(W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}; b_{\ \ ge^{data}_{,tp}})$$

$$c^{pred}_{d^{Mhq},n,tp} = \Psi_{d^{Mhq}}(W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}; b_{\ \ ge^{data}_{,tp}})$$

$$u^{pred}_{d^N,n,tp} \equiv u\left(c^{pred}_{d^N,n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}\right)$$

$$u_{c;d^N,n,tp} \equiv u' \ \ c_{d^N,n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1})$$

$$W^+_{d^N,n,tp} \equiv RW^{data}_{n,tp} - c^{pred}_{d^N,n,tp} - \psi(H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}) + y$$

$$f^{pred}_{d^N,n,tp} = \Pr(N|W^{data}_{n,t}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}, \quad ge^{data}_{n,tp})$$

$$u^{pred}_{d^{Mhq},n,tp} \equiv u\left(c^{pred}_{d^{Mhq},n,tp}, H^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}, Q^{data}_{n,tp\ \ 1}, Q^{choice}_{n,tp}\right)$$

$$u_{c;d^{Mhq},n,tp} \equiv u' \ \ c_{d^{Mhq},n,tp}, H^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}, Q^{data}_{n,tp\ \ 1}, Q^{choice}_{n,tp})$$

$$W^+_{d^{Mhq},n,tp} \equiv RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) - M(H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) + y$$

$$f^{pred}_{d^{Mhq},n,tp} = \Pr(Mhq|W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}, \quad ge^{data}_{n,tp}).$$

I next use these variables to construct more variables

$$\Phi^{data}_{n,tp} \equiv \Phi(W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}; a\ \ _{ge^{data}_{\ ,tp}})$$

$$\Phi^{data}_{W;n,tp} \equiv (\Phi^{data})'(W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}; a\ \ _{ge^{data}_{\ ,tp}})$$

$$\Phi^{+}_{d^N,n,tp} \equiv \Phi^{data}(W^{+}_{d^N,n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}; a\ \ _{ge^{data}_{\ ,tp}+1})$$

$$\Phi^{+}_{W,d^N,n,tp} \equiv \Phi'(W^{+}_{d^N,n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}; a\ \ _{ge^{data}_{\ ,tp}+1})$$

$$\Phi^{+}_{d^{Mhq},n,tp} \equiv \Phi(W^{+}_{d^{Mhq},n,tp}, H^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}, Q^{data}_{n,tp\ \ 1}, Q^{choice}_{n,tp}; a\ \ _{ge^{data}_{\ ,tp}+1})$$

$$\Phi^{+}_{W;d^{Mhq},n,tp} \equiv \Phi'(W^{+}_{d^{Mhq},n,tp}, H^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}, H^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}; a\ \ _{ge^{data}_{\ ,tp}+1})$$

$$\widehat{V}^{pred}_{d^N,n,tp} = u(c^{pred}_{d^{NM},n,tp}, H^{data}_{n,tp}) + \beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{NM},n,tp} - \psi(H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}) + y; H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}; a\ \ _{ge^{data}_{\ ,tp}+1})$$

$$\widehat{V}^{pred}_{d^{Mhq},n,tp} = u(c^{pred}_{d^{Mhq},n,tp}, H^{choice}_{n,tp}) + \beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp})$$
$$-M(H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) + y; H^{data}_{n,tp\ \ 1}, H^{choice}_{n,tp}, Q^{data}_{n,tp\ \ 1}, Q^{choice}_{n,tp}; a\ \ _{ge^{data}_{\ ,tp}+1}).$$

The probabilities of not moving and moving are

$$f^{pred}_{d^N,n,tp} = \Pr(H_{d^N,n,tp} | W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1},\ \ \ ge^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^N,\ \ ,tp})}{\exp(\widehat{V}^{pred}_{d^N,\ \ ,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},\ \ ,tp})}$$

$$f^{pred}_{d^{Mhq},n,tp} = \Pr(H_{d^{Mhq},n,tp} | W^{data}_{n,tp}, H^{data}_{n,tp\ \ 1}, Q^{data}_{n,tp\ \ 1},\ \ \ ge^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{Mhq},\ \ ,tp})}{\exp(\widehat{V}^{pred}_{d^N,\ \ ,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},\ \ ,tp})}.$$

The measurement error in consumption is normally distributed with mean 0 and variance σ^2:

$$\Pr(c_{n,tp} | d_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(c^{data}_{\ ,tp}\ \ \ c^{pred}_{d,\ \ ,tp})^2}{2\sigma^2}).$$