Does it Pay to Get a Reverse Mortgage?

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Motivation

Empirical Evidences:

- The Baby Boomer retirement started in 2001.
- By 2030, one out of five people is projected to be 65 years or older.
- More than 80 percent of older households own their homes, which are worth approximately $4 trillion.
- In the 1990s, reverse mortgages became available and provided a new way to convert home equity into cash.

Reverse Mortage

- Reverse mortgages let homeowners
  a) remain in their homes for as long as possible
  b) borrow against their home equity at terms that include large up-front costs and high interest rates.
- There are no monthly or other payments to be made during the term of the loan.
- Repayment of principal plus cumulated interests is triggered by moving, is repaid out of house sale proceeds, and is capped by the value of those proceeds (nonrecourse loan).
- To be eligible:
  a) a borrower must be 62 or older
  b) own the home outright or have a low loan balance
  c) have no other liens against the home
  d) no credit or income requirements

Example of Loan Calculation

Your home is in San Francisco County. Your attained age is 62.

Value of your home	$100,000
Loan principal limit	$49,200
Less loan fees to lender	$2,000
Less Mortgage Insurance	$2,000
Less other closing costs	$1,914
Less service fee set-aside	$4,631
Cash available to you	$38,655
Current interest rate index	2.8%
Lender's margin	3.10%
Current loan interest rate	5.18%
HUD Mortgage Insurance	0.50%
Current effective loan rate	5.68%

Source reversemortgage.com (October 2008)

The Reverse Mortgage Market

- The reverse mortgage market was created in 1987 with the HUD (Department of Housing and Urban Development) program called Home Equity Conversion Mortgage (HECM)
- Reverse mortgages are specifically designed for house-rich but cash-poor homeowners. However, these homeowners have not bought them.
- Potential Market: 13.2 million older households (Stucki 2005)
- Actual Market: 265,234 federally insured reverse mortgages in 2007, about 1% of the 30.8 million households with at least one member age 62 and older in 2006

My Study

- Questions:
  1- Does it pay to get a reverse mortgage?
  2- Why do house-rich but cash-poor homeowners choose not to cash in the home equity through a reverse mortgage but prefer to mantain low level of consumption?
- This study estimates a dynamic structural life-cycle model of retiree consumption, housing and moving decisions. These decisions are made in light of lifespan and mobility uncertainty.
- Data: Subsample of single retirees from the Health and Retirement Study that could qualify for a reverse mortgage.

My Contribution

1. I obtain reasonable estimates for the structural preference parameters.
2. I provide a plausible explanation for the relative weakness in reverse mortgage demand.
   Reverse mortgages:
   - A bad option for house-rich but cash-poor households
   - PROS: Provide liquidity and a form of longevity insurance
   - CONS: Large up-front costs and the moving risk
3. I use a set of tools from numerical analysis to estimate and solve the empirical model.

Empirical Evidence about Reverse Morgagees

- Between 1993 and 2004, the median annual income of reverse mortgage borrowers increased from $12,289 to $18,240.
- The median net worth among the general population of older households, excluding home equity, was $23,369 in 2000.
- Close to half of reverse mortgage borrowers (46 percent) have homes worth $100,000 to $200,000, compared with only about one-third of general homeowners (34 percent)
- Davidoff et al. (2007) shows that, empirically, reverse mortgagees have exited homes unusually rapidly.
- About 60% of loan terminations are attributed to death and about 40% to moving out.

Literature Review

- Life-cycle and Precautionary Savings:
  Gourinchas and Parker (2002), Cagetti (2003),
  French (2005), Hubbard et al. (1994),
  Palumbo (1999), Hurd (1989)
- Housing and Portfolio Choice:
  Cocco (2005), Yao and Zhang (2005)
- Discrete Choice:
  Rust (1987), Hotz and Miller (1993),
  Aguirragabiria and Mira (2002), Keane and Wolpin (1997)
- Numerical Analysis:
  Judd and Su (2007)

Model

- Preferences

$$U_{it}(C_{it}, H_{it}) = \frac{(C_{it}^{1-\omega}(\psi^{rent} H_{it})^{\omega})^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it})$$

where $\varepsilon_{it}(d_{it})$ is a vector of unobserved utility components associated to the discrete housing choice and it is Extreme Value Type I distributed

- Budget Constraint

$$A_{it+1} = (1+r)A_{it} + y - C_{it} - \psi_{it} - M_{it}$$

- Bequest Function:

$$b(TW_{it}) = \theta_B \frac{TW_{it}^{1-\gamma}}{1-\gamma}$$

Choice Set

- Discrete Choice: Housing

$$d_{it}^{1} = \begin{cases} D_{it}^{M} = 1 & \text{if household } i \text{ moves out of her house in period } t \\ D_{it}^{M} = 0 & \text{otherwise} \end{cases}$$

$$d_{it}^{2} | d_{it}^{1} = \begin{cases} D_{it}^{O} = 1 & \text{if household } i \text{ owns her house in period } t \\ D_{it}^{O} = 0 & \text{if household } i \text{ rents her house in period } t \end{cases}$$

$$d_{it}^{3} | d_{it}^{1}, d_{it}^{2} = H_{it}$$

The discrete choice set is

$$d_{it} = \{d_{it}^{1}, d_{it}^{2}, d_{it}^{3}\}$$

- Continuous Choice: Consumption C_{it}

Housing Expenses

- Per-Period Cost

$$\psi_{it} = [D_i^O \psi^{own} + (1 - D_i^O)\psi^{rent}]H_{it}^*$$

where

$$H_{it}^* = D_{it}^M H_{it} + (1 - D_{it}^M)H_{it-1}$$

- Moving Cost

$$M_{it} = D_{it}^M D_{it-1}^O [D_{it}^O H_{it} - H_{it-1} + H_{it}\phi(D_{it}^O)] + D_{it}^M (1 - D_{it-1}^O)(1 - D_{it}^O) H_{it} \phi^{rent}$$

where the transaction costs are:

$$\phi(D_{it}^O) = [D_{it}^O \phi^{own} + (1 - D_{it}^O)\phi^{rent}]$$

Value Function

$$V_t(X_{it}, \varepsilon_{it}) = \max_{d_{it}, C_{it}} U(d_{it}, C_{it}) + \varepsilon_{it}(d_{it})$$

$$+\beta\eta_{it+1} E[V_{it+1}(W_{it+1}, H^*_{it}, D^O_{it}, \varepsilon_{it+1}|X_{it}, C_{it})] + b(TW_{it+1})$$

subject to

$$W_{it+1} = (1+r)W_{it} + y - C_{it} - \psi_{it} - M_{it}$$

$$H^*_{it} = D^M_{it} H_{it} + (1 - D^M_{it}) H_{it-1}$$

$$C_{it} \geq C_{MIN}$$

State Space: $X_{it} = \{W_{it}, H_{it-1}, D^O_{it-1}, Age_{it}\}$

Preference parameters to estimate: $\theta = \{\gamma, \omega, \sigma, \theta_B\}$

- Bellman Equation:

$$V_t(X_{it}, \varepsilon_{it}) = \max_{d_{it}, C_{it}} [U(d_{it}, C_{it}) + \varepsilon_{it}(d_{it}) + \beta\eta_{it+1} EV(X_{it+1})]$$

$$= \max_{d_{it}} \left\{ \left[\max_{C_{it}} \{[U(d_{it}, C_{it}) + \beta\eta_{it+1} EV(X_{it+1})] | d_{it}\} \right] + \varepsilon_{it}(d_{it}) \right\}$$

- Inner Maximization (consumption conditional on housing)

$$r(X_{it}, d_{it}) = \max_{C_{it}} [U(d_{it}, C_{it}) + \beta\eta_{it+1} EV_{t+1}(X_{it+1}) | d_{it}]$$

- Outer Maximization (housing)
Conditional Choice Probabilities

$$P(j|X_{it}, \theta) = \frac{\exp\{r(X_{it}, j)\}}{\sum_{k \in d_{it}(X_{it})} \exp\{r(X_{it}, k)\}}$$

where

$$EV_{t+1}(X_{it+1}) = \ln \left[\sum_{k \in d_t(X_t)} \exp\{r(X_{it}, k)\} \right]$$

Loglikelihood

- Measurement Error in Consumption

$$\Pr(c_{n,t}|d^H_{n,tp}, A^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c^{data}_{n,tp} - c^{pred}_{n,tp})^2}{2\sigma^2}}$$

- Discrete Choice Probability

$$\Pr(d^H_{n,tp}|A^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{e^{V_{d,n,tp}}}{\sum_m e^{Vm,n,tp}}$$

- Joint Probability of Housing and Consumption Choice

$$\Pr(d^H_{n,tp}, c_{n,tp}|A^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \Pr(d^H_{n,tp}|A^{data}_{n,tp}, H_{n,tp}, Q^{data}_{n,tp}) \cdot \Pr(c_{n,t}|d^H_{n,tp}, A^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp})$$

- Log-Likelihood

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d^H_{n,tp}, c_{n,tp}|A^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}, \theta)$$

Four Tools from Applied Mathematics

Basic Idea:
Use methods and software developed by computational scientists and mathematicians to reduce the computational burdens of structural estimation and to significantly increase the precision in the economic results.

- Mathematical Programming with Equilibrium Constraints
- Flexible Polynomial Approximation
- Shape Preservation
- Envelope Theorem

Simple Life-Cycle Model: One continuous state variable

- Backward Solution for the True Value Function

The last period value function is known and equal to $V_T(W)$
In periods $t = 1...(T-1)$ the Bellman equation is:

$$V_t(W) = \max_c (u(c) + \beta E V_{t+1}(RW - c))$$

Given V_{t+1}, the Bellman equation implies, for each wealth level W, three equations that determine optimal consumption, c^*, $V_t(W)$, and $V_t'(W)$:

- Bellman equation: $V_t(A) = u(c^*) + \beta V_{t+1}(RW - c^*)$
- Euler equation: $u'(c^*) - \beta V_{t+1}'(RW - c^*) = 0$
- Envelope condition: $V'(W) = \beta R V'(RW - c^*)$

Backward Solution for the Approximate Value Function

- Choose a functional form and a finite grid of wealth levels

Time t value function is approximated by

$$V_t(W) = \Phi(W; a_t) = \sum_{k=0}^{7} a_{k+1,t}(W - \overline{W}_t)^k$$

- We would like to find coefficients a_t such that each time t Bellman equation, along with the Euler equation and envelope condition, holds with the Φ approximation

$$\Phi(W; a_t) = \max_c (u(c) + \beta\Phi_{t+1}(RW - c; a_{t+1}))$$

- Define three set of errors, $\lambda_t^b \geq 0, \lambda_{i,t}^e \geq 0, \lambda_t^{env} \geq 0$, that satisfy the following inequalities

- Bellman error

$$-\lambda_t^b \leq \Phi(W_{i,t}; a_t) - [u(c_{i,t}^*) + \beta\Phi_{t+1}(RW_{i,t} - c_{i,t}^*; a_{t+1})] \leq \lambda_t^b$$

- Euler error

$$-\lambda_{i,t}^e \leq u'(c_{i,t}^*) - \beta\Phi'(RW_{i,t} - c_{i,t}^*; a_{t+1}) \leq \lambda_{i,t}^e$$

- Envelope error

$$-\lambda_t^{env} \leq \Phi'(W_{i,t}; a_t) - R\beta\Phi'_{t+1}(RW_{i,t} - c_{i,t}^*; a_{t+1}) \leq \lambda_t^{env}$$

Dynamic Programming with Approximation of the Value Function

- Minimize the sum of the errors:

$$\min_{a,c,\lambda} \sum_t \sum_i \lambda^e_{i,t} + \sum_t \lambda^b_t + \sum_t \lambda^{env}_t$$

subject to:

- Bellman error
- Euler error
- Envelope error
- Transversality condition

where the Transversality condition:

$$\Phi(W_{i,t}; a_{i,t}) \geq \Phi(W_{i-1,t}; a_t) + \frac{(\Phi(W_{i+1,t}; a_t) - \Phi(W_{i-1,t}; a_t))}{(W_{i+1,t} - W_{i-1,t})}(W_{i,t} - W_{i-1,t})$$

Empirical Part

- We have continuous data on assets and consumption.
- We assume that the measurement error in consumption is normally distributed with mean 0 and unknown variance σ^2.
- We can use the Euler equation to recover the predicted value of consumption.
- The probability that household n chooses consumption $c_{n,tp}$ in period tp is:

$$\Pr(c_{n,tp}|W_{n,tp}^{data}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c_{n,tp}^{data} - c_{n,tp}^{pred})^2}{2\sigma^2}}$$

Therefore the log-likelihood is given by:

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(c_{n,tp}|A_{n,tp}^{data}, \theta)$$

Structural Estimation with Dynamic Programming

- Conventional Approach
  1. Take a guess of the structural parameters
  2. Solve the dynamic programming problem
  3. Calculate the log-likelihood
  4. Repeat 1,2,3 until the log-likelihood is maximized
- Constrained Optimization (MPEC) Approach

$$\max_{a,c,\lambda,\theta} \mathcal{L}(\theta) - Penalty \cdot \Lambda$$

- Bellman error
- Euler error
- Envelope error
- Transversality condition

where $\Lambda = \sum_t \sum_i \lambda^e_{i,t} + \sum_t \lambda^b_t + \sum_t \lambda^{env}_t$

Go to the Appendix

Dynamic Programming

- Minimize the Sum of Errors

$$\Lambda = \sum_t \sum_i \sum_j \sum_z \lambda^{e}_{i,j,z,t} + \sum_t \sum_j \sum_z \lambda^{b}_{j,z,t} + \sum_t \sum_j \sum_z \lambda^{env}_{j,z,t} + \sum_t \sum_i \sum_j \sum_z \sum_d \lambda^{cons}_{i,j,d,z,t}$$

subject to:

$$-\lambda^{e}_{i,j,z,t} \leq EulerEquation_{i,j,z,t} \leq \lambda^{e}_{i,j,z,t}$$

$$-\lambda^{b}_{j,z,t} \leq BellmanEquation_{j,z,t} \leq \lambda^{b}_{j,z,t}$$

$$-\lambda^{env}_{j,z,t} \leq EnvelopeCondition_{j,z,t} \leq \lambda^{env}_{j,z,t}$$

$$-\lambda^{cons}_{i,j,d,z,t} \leq PolicyFunction_{i,j,d,z,t} \leq \lambda^{cons}_{i,j,d,z,t}$$

Solving DP and Estimation with the MPEC

$$\underset{\theta,a,c}{Max} \mathcal{L}(\theta) - Penalty \cdot \Lambda$$

subject to:
Euler error
Bellman error
Envelope error
Policy function error
Transversality condition

where

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d^{H}_{n,tp}, c_{n,tp} | A^{data}_{n,tp}, H^{data}_{n,tp}, \theta)$$

Data

The Health and Retirement Study (HRS) and The Consumption and Activities Mail Survey (CAMS). US data (2000-2005).

- We select a group of 165 households that qualify for a reverse mortgage.
- Characteristics: 62 years old or older, single, retiree, homeowner, complete information about consumption and financial situation.

	Percentiles			Min	Max
	25%	50%	75%		
H	$40,000	$70,000	$90,000	$ 3,000	$170,000
W	$6,000	$25,000	$69,500	$0	$276,548
C	$6,347	$9,774	$15,409	$650	$84,380
ss	$7,200	$9,600	$11,748	$0	$ 18,907
Age	69	75	80	66	86

Financial Portfolio Composition

	Percentiles			Min	Max
	25%	50%	75%		
Stocks	$0	$0	$0	$ 0	$125,000
Chck	$750	$3,600	$10,000	$ 0	$100,000
Cds	$0	$0	$5,300	$ 0	$273,548
Tran	$1,000	$4,000	$8,000	$ 0	$30,000
Bonds	$0	$0	$0	$ 0	$80,000
IRA	$0	$0	$2,5000	$ 0	$137,000
Debt	$0	$0	$0	$ 0	$12,000

- For almost all the retirees in the sample, the financial portfolio does not contain risky assets.

Moving Decision

- In each period, about 8% of the households in our sample moves out of their homes.
- Among those who moved, about 20% decide to rent a new house, while about 80% buy a new house.
- The moving decision does not appear to be strictly related with age.
- About 50% of the retirees move near or with children or other relatives or friends. About 25% move for financial reasons, and the remaining 25% move because of health problems, weather or climate reasons, to have a better location, or for other reasons.

Results

Parameter	Variable	Estimate
γ	Coefficient of relative risk aversion	1.4196 (0.013)
ω	Preference parameter over housing	0.5325 (0.032)
σ	s.d. of measurement error in consumption	1.206 (0.640)
θ_B	Degree of altruism	0.000 (0.001)

- The standard errors are computed using a bootstrap procedure.

Simulation of Welfare Gain from Reverse Mortgage

The welfare gain from a reverse mortgage is calculated as a percentage increase in the initial non-housing financial wealth that makes the household without reverse mortgage as well off in expected utility terms as with the reverse mortgage.

Simulation of Welfare Gain from Reverse Mortgage

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	\$8,500	\$ 10,600
Cash-Rich	\$107,800	\$ 90,200

where:

Financial Wealth	House Value
Cash-Poor: < \$40,000	House-Poor: <\$60,000
Cash-Rich: > \$40,000	House-Rich: >\$60,000

Simulation of Welfare Gain from Reverse Mortgage

Median Welfare Gain, Baseline Case

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	-9%(- $ 767)	-14%(- $ 1,525)
Cash-Rich	50%($ 53,302)	41% ($ 36,863)

- PROS of RM: liquidity and a form of longevity insurance.
- CONS of RM: high up-front cost and moving risk.

No Moving Risk

Median Welfare Gain

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	207%($ 17,600)	430%($ 45,622)
Cash-Rich	18%($ 19,054)	54%($ 48,883)

No Up-Front Costs

Median Welfare Gain

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	2%($ 146)	-12%($ 1,374)
Cash-Rich	56%($ 60,021)	49%($ 43,784)

10% Cut in Current Income

Median Welfare Gain

	HOUSE	
	House-Poor	**House-Rich**
FINANCIAL WEALTH		
Cash-Poor	-20% ($ 1,657)	-16% ($ 1,654)
Cash-Rich	49% ($ 53,298)	46% ($ 41,728)

Conclusions

- Innovative structural dynamic life-cycle model of consumption, housing and mobility choice to calculate the welfare benefits of allowing retirees to cash in their home equity through a reverse mortgage.
- First application of a set of four mathematical tools to estimate and solve an empirical model.
- Reverse mortgages provide liquidity and a form of longevity insurance, but introduce a new risk, the moving risk. These financial instruments are risky especially for house-rich but cash-poor homeowners.
- Gambling can make someone who is initially poor relatively rich. However, luck plays an important role in this gamble.

Thank you

Continuous and Discrete State Variables

Let W be a continuous state variable and J be a discrete state variable.
Time t value function is approximated by

$$V_t\left(W,J\right) = \Phi(W,J;a_t) = \sum_{k=0}^{7} a_{k+1,t}(W-\overline{W}_t)^k$$

The constrained optimization approach to a life-cycle model with continuous and discrete state variables is:

$$Minimize \sum_i \sum_j \sum_t \lambda^e_{i,j,t} + \sum_j \sum_t \lambda^b_{j,t} + \sum_j \sum_t \lambda^{env}_{j,t}$$

subject to

- Bellman Error:

$$-\lambda^b_{j,t} \leq \Phi(W,J;a_t) - [u(c^*,J) + \beta\Phi(RW-c^*,J;a_{t+1})] \leq \lambda^b_{j,t}$$

- Euler Error

$$-\lambda^e_{i,j,t} \leq \; u'(c^*,J) - \beta\Phi'(RW-c^*,J;a_{t+1}) \leq \lambda^{env}_{i,j,t}$$

- Envelope Error:

$$-\lambda^{env}_{j,t} \leq \Phi'(W,J;a_t) - R\beta\Phi'(RW-c^*,J;a_{t+1}) \leq \lambda^{env}_{j,t}$$

Appendix: DP with Approximation of the Value Function

- Euler Equations:

$$u'(c^*_{dN}, H) - \beta\eta_{t+1}RV'_{t+1}(RW - c^*_{dN} - \psi + y; H, Q) = 0$$

$$u'(c^*_{dMhq}, h) - \beta\eta_{t+1}RV'_{t+1}(RW - c^*_{dMhq} - \psi - M + y; h, q) = 0$$

- Bellman Equation:

$$V_t(W, H, Q) = \ln\left\{\exp(\widehat{V}_{dN,t}) + \sum_q \sum_h \exp(\widehat{V}_{dMhq,t})\right\}$$

- Envelope Condition:

$$V'_t(W, H, Q) = \Pr(NM|W, H, Q) \cdot \widehat{V}'_{dN,t} + \sum_q \sum_h \Pr(Mhq|W, H, Q) \cdot \widehat{V}'_{dMhq,t}$$

Go back

- Value Function Approximation

$$V_t\,(W, H, Q) = \Phi(W, H, Q; a_t, \overline{W}_t) = \sum_{k=0}^{7} a_{k+1,H,Q,t}(W - \overline{W}_t)^k$$

- Policy Function Approximation

$$c^*_{d,t}\,(W, H, Q) = \Phi(W, H, Q; b_{d,t}, \overline{W}_t) = \sum_{k=0}^{7} b_{k+1,H,Q,d,t}(W - \overline{W}_t)^k$$

We would like to find coefficients a_t and $b_{d,t}$ such that each time t Bellman equation, along with the Euler and Envelope conditions, holds with the Φ approximation

Go back

- Euler Errors

$$-\lambda^{e}_{i,j,z,t} \leq u'(c^{*}_{i,j,d^N,t}, H_{j,t}) - \beta R \Phi'(RW_{i,t} - c^{*}_{i,j,d^N,t} - \psi + y; H_{j,t}, Q_t; a_{t+1}) \leq \lambda^{e}_{i,j,z,t}$$

$$-\lambda^{e}_{i,j,z,t} \leq u'(c^{*}_{i,j,d^{Mhq},t}, H_{t+1}) - \beta R \Phi'(RW_{i,t} - c^{*}_{i,j,d^{Mhq},t} - \psi - M + y; H_{t+1}, Q_{t+1}; a_{t+1}) \leq \lambda^{e}_{i,j,z,t}$$

- Bellman Error

$$-\lambda^{b}_{j,z,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_t; a_t) - \ln\left\{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_{q}\sum_{h}\exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^{b}_{j,z,t}$$

where

$$\widehat{V}_{i,j,d^N,t} = u(c^{*}_{i,j,d^N,t}, H_{j,t}) + \beta\eta_{t+1}\Phi(RW_{i,t} - c^{*}_{i,j,d^N,t} - \psi + y; H_{j,t}, Q_t; a_{t+1})$$

$$\widehat{V}_{i,j,d^{Mhq},t} = u(c^{*}_{i,j,d^{Mhq},t}, H_{t+1}) + \beta\eta_{t+1}\Phi(RW - c^{*}_{i,j,d^{Mhq},t} - \psi - M + y; H_{t+1}, Q_{t+1}; a_{t+1})$$

Go back

- Envelope Error

$$-\lambda^{env}_{j,z,t} \leq \Phi'(W_{i,t}, H_{j,t}, Q_t; a_t) - \{f_{i,j,d^N,t} \cdot \Phi'(RW_{i,t} - c^*_{i,j,d^N,t} - \psi + y; H_{j,t}, Q_t; a_{t+1})$$

$$+ \sum_q \sum_h [f_{i,j,d^{Mhq},t} \cdot \Phi'(RW_{i,t} - c_{i,j,d^{Mhq},t} - \psi - M; H_{t+1}, Q_{t+1}; a_{t+1})]\} \leq \lambda^{env}_{j,z,t}$$

where

$$f_{i,j,d,t} = \Pr(d|W_{i,t}, H_{j,t}, Q_t) = \frac{\exp(\widehat{V}_{i,j,d,t})}{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_q \sum_h \exp(\widehat{V}_{i,j,d^{Mhq},t})}$$

- Policy Function Error

$$-\lambda^{cons}_{i,j,z,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_t; b_t) - c^*_{i,j,d,t}(W_{i,t}, H_{j,t}, Q_t) \leq \lambda^{cons}_{i,j,z,d,t}$$

Go back

Dynamic Programming

- Minimize the Sum of Errors

$$\Lambda = \sum_t \sum_i \sum_j \sum_z \lambda^{e}_{i,j,z,t} + \sum_t \sum_j \sum_z \lambda^{b}_{j,z,t} + \sum_t \sum_j \sum_z \lambda^{env}_{j,z,t} + \sum_t \sum_i \sum_j \sum_z \sum_d \lambda^{cons}_{i,j,d,z,t}$$

subject to:

$$-\lambda^{e}_{i,j,z,t} \leq EulerEquation_{i,j,z,t} \leq \lambda^{e}_{i,j,z,t}$$

$$-\lambda^{b}_{j,z,t} \leq BellmanEquation_{j,z,t} \leq \lambda^{b}_{j,z,t}$$

$$-\lambda^{env}_{j,z,t} \leq EnvelopeCondition_{j,z,t} \leq \lambda^{env}_{j,z,t}$$

$$-\lambda^{cons}_{i,j,d,z,t} \leq PolicyFunction_{i,j,d,z,t} \leq \lambda^{cons}_{i,j,d,z,t}$$

Go back

Loglikelihood

- Measurement Error in Consumption

$$\Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c^{data}_{n,tp} - c^{pred}_{n,tp})^2}{2\sigma^2}}$$

- Discrete Choice Probability

$$\Pr(d^H_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{e^{V_{d,n,tp}}}{\sum_m e^{V_{m,n,tp}}}$$

- Joint Probability of Housing and Consumption Choice

$$\Pr(d^H_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \Pr(d^H_{n,tp}|W^{data}_{n,tp}, H_{n,tp}, Q^{data}_{n,tp}) \cdot \Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp})$$

- Log-Likelihood

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d^H_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}, \theta)$$

Go back

Solving DP and Estimation with the MPEC

$$\underset{\theta,a,c}{Max}\mathcal{L}(\theta) - Penalty \cdot \Lambda$$

subject to:
Euler error
Bellman error
Envelope error
Policy function error

where

$$\mathcal{L}(\theta) = \sum_{n=1}^{N}\sum_{tp=1}^{TP} \log \Pr(d^H_{n,tp}, c_{n,tp} | W^{data}_{n,tp}, H^{data}_{n,tp}, \theta)$$

We assume that there is a measurement error in consumption $\sim N(0, \sigma^2)$

Go back

Empirical Evidence about Reverse Morgagees

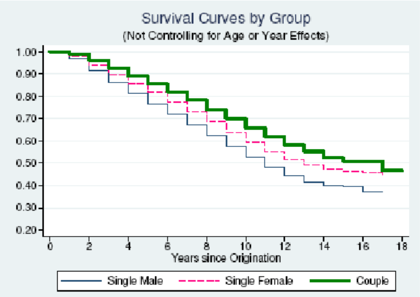

Figure: Survival Curves of HECM Loans for Single Males, Single Females, and Couples (Bowen et al.,2008)