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ECONOMICS OF THE LIFE-CYCLE: REVERSE MORTGAGE, MORTGAGE AND MARRIAGE

by

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I dedicate this dissertation with love to my family, my mother Loredana, my father Giorgio, and my brother Emanuele, who have always supported me despite the distance.

ECONOMICS OF THE LIFE-CYCLE:

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ABSTRACT

My dissertation spans the area of the economics of the life-cycle with an emphasis on applications of numerical methods to this area.

My first chapter entitled "Does it Pay to Get a Reverse Mortgage?" estimates a structural dynamic life-cycle model of consumption, housing, and mobility choice to calculate the welfare benefits of allowing retirees to cash in their home equity through a reverse mortgage. My main contributions are twofold. First, I provide a plausible explanation for the relative weakness of the demand for reverse mortgages, namely reverse mortgages introduce a new risk to their purchasers, the moving risk. Second, I use a recently developed set of tools from numerical analysis to estimate an empirical model. Specifically, these tools include the Mathematical Programming with Equilibrium Constraints (MPEC) approach, a flexible polynomial approximation, shape preservation, and the imposition of the Envelope Theorem in calculating value functions.

My second chapter "Marrying for Money" examines the financial gains from marriage. Specifically, the paper deals with marriage as an implicit insurance contract against the risk of earning loss, of disability, and of running out of money because of greater than average longevity. The main finding is that, even though economies of shared living are the dominant factor in the

financial gain from marriage, the risk-sharing opportunities provided by the family can play an important role.

My third chapter "Does it Pay to Pay Off Your Mortgage?" examines whether retirees, who have enough financial assets to pay off their mortgage, should pay it off or keep it and invest. I find that those with more initial wealth are better off from paying off their mortgage, whereas those with less initial wealth are worse off. The welfare gains to the wealthy can run as high as 4 percent of their initial assets. These gains reflect the fact that the nominal mortgage interest rate exceeds the nominal return one can earn on safe bonds. This holds for those with low initial wealth, but such households are typically liquidity constrained, so paying off their mortgage comes at cost of less consumption smoothing.

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List of Abbreviations

AARP	American Association of Retired People
AMPL	A Mathematical Programming Language
AR(1)	First-order Autoregressive Process.
ARM	Adjustable-rate Mortgage
CAMS	Consumption and Activities Mail Survey
CPS	Current Population Survey
DP	Dynamic Programming
FHA	Federal Housing Administration
FRM	Fixed-rate Mortgage
HECM	Home Equity Conversion Mortgage
HRS	Health and Retirement Study
HUD	Department of Housing and Urban Development
IRA	Individual Retirement Account
MPEC	Mathematical Programming with Equilibrium Constraints
OECD	Tax-Deferred Account
TDA	Tax-Deferred Account
U.S.	United States of America

Chapter 1

Does it Pay to Get a Reverse Mortgage?

1.1 Introduction

This paper estimates a structural dynamic life-cycle model of consumption, housing and mobility choice to calculate the welfare benefits of allowing retirees to cash in their home equity through a reverse mortgage. Our main contribution is twofold. First, we provide a plausible explanation for the relative weakness of the reverse mortgage demand. The reverse mortgage market was created in 1987 by the Department of Housing and Urban Development (HUD) and, after 20 years, it is still at 1 percent of its potentiality. Reverse mortgages are federally-insured private loans specifically designed for house-rich but cash-poor homeowners, as safe instruments able to relieve their financial pressure. However, these homeowners have not bought them. Our model explains this fact by showing that reverse mortgages are risky financial instruments; namely, moving becomes a risky proposition. Second, we use a recently developed set of tools from Applied Mathematics to estimate an empirical model. Specifically, this set includes the Mathematical Programming with Equilibrium Constraints (MPEC) approach, a flexible polynomial approximation, shape preservation, and the imposition of the Envelope Theorem for calculating the value functions. This is the first paper to use the Envelope Theorem in this way, and the first to use the four tools in combination.

Empirical evidence shows that the house is the major asset of most retirees. More than 80 percent of retirees own their homes (Munnel et al., 2007), for a total value of approximately \$4 trillion. Hence, many economists and policymakers have acknowledged homes as a potential source of savings to finance retirement consumption. According to the traditional life-cycle model, developed first by Modigliani and Brumberg (1954), Ando and Modigliani (1963), and Friedman (1957), individuals make their saving choices to smooth consumption over their lifetime. Theoretically, households build savings during their working period and divest those savings to meet their consumption needs at older ages. However, empirically, this pattern is not followed. This is specifically true of home equity. On average, seniors citizens tend not to cash in the savings locked in their home equity. Instead, homeownership rates remain stable until later in life. Before the advent of the reverse mortgage, selling and moving out represented the best way to liquidate home equity.

A HUD reverse mortgage is a loan available to homeowners, 62 years of age or older, and allows for the release of home equity while living in the same home. Reverse mortgages differ from conventional home loans in several respects. These loans are federally-insured and regulated by the Federal Housing Administration (FHA). In addition, there are no income or other credit requirements, and there is no risk of default or foreclosure. Nevertheless, reverse mortgages are characterized by large up-front costs and high interest rates. The main reason for the high up-front costs is the mortgage insurance premium paid to the FHA. By charging this premium, the FHA insures the borrower against the risk of lender's default. A reverse mortgage accrues interest beginning with the first payment to the borrower. While there are no interest payments for the length of the loan, moving triggers the repayment of the borrowing plus accumulated interest. It is repaid out of house sale proceeds, and is capped by the value of those proceeds. Stucki (2005) estimates the potential market for reverse mortgages at 13 million households. Although 86% of seniors know what a reverse mortgage is, in 2007 only 1 percent of the 30.8 million seniors in the United States closed a reverse mortgage contract. Several economists advocated strong public policy support for reverse mortgages. The relative weakness of the demand for these financial instruments reveals that these federally-insured loans are unable to meet retirees' needs and wants. Therefore our focus is on the study of this government failure or, in other words, of the systemic reasons that prevent the HUD reverse mortgages from becoming a common tool to finance consumption in retirement. In particular, why do house-rich but cash-poor homeowners choose not to cash in the savings locked in their house through a HUD reverse mortgage but prefer to maintain low levels of consumption? Should the government promote reverse mortgages as a way for older Americans to financially support themselves in retirement?

There are many psychological reasons why older homeowners may be reluctant to tap their home equity, such as aversion to debt and desire to keep the house debt free. In this paper we provide a rational explanation for their behavior, namely the moving risk. Since moving triggers the repayment of the borrowing, exogenous and unpredictable events that force the retiree to move out can not be disregarded. Therefore, assessing the potential for reverse mortgages requires jointly analyzing consumption, housing and moving decisions. The degree of risk aversion and the preference of housing over consumption are not observable; hence, we use a structural model to estimate these preference parameters. Our estimated structural model is sufficiently rich that it can be used to perform policy experiments and to evaluate the welfare gain from reverse mortgages under different conditions.

Financial, demographic and consumption data on reverse mortgagees are not publicly available. Consequently, we select a subsample of single retirees from the Health and Retirement Study (HRS) that could represent a potential target segment for reverse mortgages, according to estimates from the public policy perspective. Empirical evidence shows that these retirees support their consumption mainly with Social Security income and tend not to divest their home equity at older ages. Typically, their non-housing financial assets are a fraction of their home value. Our model features consumption, liquid saving and illiquid houses. Houses can be owned or rented. Moving is costly. Households are subject to life span uncertainty and to housing preference shocks that could force them to move. Within this framework, we compute the benefit of allowing people to cash in housing wealth through a reverse mortgage. We first estimate the model and solve for the optimal consumption and housing choice without a reverse mortgage. Then, we use the estimates of the structural preference parameters to calculate how much better or worse off the HRS retirees would be with a reverse mortgage. Our subsample includes both discrete and continuous data. Therefore, we extend the literature on discrete choice by also including continuous choices.

We address the estimation difficulties in solving the model by using four mathematical tools: Mathematical Programming with Equilibrium Constraints (MPEC), flexible polynomial approximation, shape preservation and Envelope Theorem. In the past decade, there has been a significant increase in computer speed and technological progress in algorithms and software used to solve large-scale problems. While many economic applications involve nonlinear large-scale and optimization problems, very few economic problems have been examined using mathematical programming approaches. This paper solves an economic policy question using cutting-edge methods in computational science and state-of-the-art software. It represents an example of interaction between economics and computational science.

This paper yields two main findings. First, we obtain reasonable estimates of the structural preference parameters. Specifically, retirees are risk averse and greatly value their house compared to consumption. This parameter configuration suggests that older homeowners prefer to make safe investments and maintain large savings to buffer themselves against unexpected shocks. While the house is a safe asset, it prevents quick access to the resources accumulated in the working period. Thus, home equity is the most important component of precautionary savings in retirement. Second, our model explains why house-rich but cash-poor homeowners have not bought reverse mortgages with issues related to the moving risk. Reverse mortgages provide liquidity and a form of longevity insurance; however, moving becomes a risky proposition. If homeowners move out, they have to repay the minimum between the house value and the outstanding debt, and their up-front costs are lost. Both consumption and housing profiles are affected in the periods following the move. We find that reverse mortgages are a very bad option for house-rich but cash-poor homeowners. For such homeowners, taking out the standard reverse mortgage and borrowing the maximum permitted amount reduces expected utility, on average, to the same degree as a 120 percent loss in financial assets. On the other hand, cash-rich homeowners benefit from the contract. Our findings might be examined in light of the empirical evidence that retirees tend to not divest their home equity, finance their consumption mostly with Social Security income and tilt their financial portfolio towards safe assets. This might explain why, after almost 20 years, the reverse mortgage is still a niche product.

The structure of the paper is as follows. Section 2 contains the literature review. Section 3 explains the features of a reverse mortgage contract, evaluates the lender's expected gain and provides some empirical evidence about reverse mortgagees. Section 4 presents the household's life-cycle model. Section 5 describes the solution method. Section 6 illustrates the HRS data. Section 7 contains the results and the welfare analysis. Section 8 presents some policy experiments. Section 9 provides conclusions.

1.2 Literature Review

This paper draws on three main sources of economic literature: life-cycle and precautionary savings, housing and portfolio choice and discrete choice.

We build on the studies of life-cycle behavior in Kotlikoff and Summers (1981), Carroll and Summers (1991), Kotlikoff et al. (2001), and Attanasio et al. (1997). Hubbard, Skinner, and Zeldes (1994) and Carroll (1997) parameterize and simulate life-cycle consumption models with uncertainty. Gourinchas and Parker (2002), Cagetti (2003), and French (2005) structurally estimate life-cycle models of consumption, of wealth accumulation and of labor supply, retirement, and savings behavior. Hubbard, Skinner, and Zeldes (1994), Palumbo (1999) and Hurd (1989) represent good attempts at modeling consumer behavior after retirement. However, in these papers housing is not taken into account. Given the empirical evidence that for most retirees the house is their major asset, we extend this literature examining the optimal consumption and housing choice for older homeowners.

Specifically, we follow Cocco (2005) and Yao and Zhang (2005 a,b) by explicitly modeling the housing decision and allowing households to derive utility from both housing and other consumption goods. Meyer and Speare (1985) studies types and determinants of senior mobility.

Additionally, we build on the literature of discrete choice models. The framework was introduced by Rust (1987,1988), and extended in Hotz and Miller (1993) and in Aguirregabiria and Mira (2002). However, most of the theoretical papers and the empirical applications focus only on discrete choice. Given that our sample involves both discrete and continuous data, we extend this literature by including continuous choices.

Finally, we follow Judd and Su (2008) who applied the MPEC approach to estimate the Zucher bus model (Rust, 1987). In this paper we present the first application of the MPEC approach to an empirical structural model with finite horizon dynamic programming.

1.3 Reverse Mortgage

The reverse mortgage market was created in 1987 with the HUD program called Home Equity Conversion Mortgage (HECM). The United States Congress passed the FHA Reverse Mortgage Legislation, the Housing and Community Development Act of 1987, (S. 825) on December 22, 1987. President Ronald W. Reagan signed FHA Reverse Mortgage Legislation (S. 825) on February 5, 1988. In 1996 the Federal National Mortgage Association (Fannie Mae) created the Home Keeper reverse mortgage to address needs unsatisfied by the HECM program, such as individuals with higher property values, condominium owners, and seniors wishing to use a reverse mortgage to purchase a new home.[1] These two reverse mortgages allow nearly every senior citizen to access the equity in her home without moving out or taking a conventional mortgage.

[1] Few lending institutions offers Non-HECM reverse mortgages. These loans could be larger than the HECM limit, are not federally insured, have private insurance and the interest rate is usually higher.

We briefly present the main features and requirements of the HECM senior reverse mortgage program.

A reverse mortgage is a particular kind of home equity loan that allows the owner to cash in some of the equity in her home. The loan does not have to be repaid so long as the borrower lives in the house. To be eligible for a reverse mortgage, a borrower must be 62 years of age or older, own the home outright (or have a low loan balance) and have no other liens against the home. The borrower does not have to satisfy any credit or income requirements. She can receive the proceeds in one of the following ways: a lump sum at the beginning, monthly payments until a fixed term or a life-long annuity, by establishing a credit-line with or without accrual of interest on the credit balance, or a combination of the above. There are no monthly or other payments to be made during the term of the loan. However, a reverse mortgage accrues interest charges, beginning when the first payment is made to the borrower. When she dies or relocates, the repayment is capped at the house value only (nonrecourse loan). The amount of the loan is a function of the age of the borrower and any co-applicant, the current value of the property and expected property appreciation rate, the current interest rate and interest rate volatility.

A reverse mortgage is just one of several financial instruments that allow a homeowner to secure liquid funds against the equity in her house. In general, Home Equity Conversion Products could be useful to all those who are house-rich but cash-poor. Conventional home equity loans are different from reverse mortgages in four respects. First, they require the payment of interests and some principal before moving. Second, the maximum amount of money that can be borrowed is determined by several variables including credit history and income. Third, the failure to repay the loan or meet loan requirements may result in foreclosure. Fourth, the up-front costs are generally lower.

In the early 1990s, projections of the potential demand for reverse mortgages varied between 800,000 older households (Merrill et al., 1993) and more than 11 million older households (Rasmussen et al., 1995). A more recent study (Stucki, 2005) estimated the potential market at 13.2 million older households. Moving from the potential market to the actual market, only 265,234 federally insured reverse mortgages were issued at the end of 2007 (HUD, 2007b). This represents about 1% of the 30.8 million households with at least one member aged 62 and older in 2006 (U.S. Census Bureau, 2006) and about 2% of the potential market as estimated by Stucki.

1.3.1 Lender's Perspective

We assume that the reverse mortgage borrower i chooses to receive the proceeds as a lump sum at the closure of the contract in time j.

The maximum amount that can be initially borrowed $V_{i,j}$ is assumed to be a fraction of the house value and a function of the borrower's age. In general, the higher the age of the borrower, the larger the amount that can be borrowed.

$$V_{i,j} = \kappa_i H_{i,j} \tag{1.1}$$

At the closure of the contract, the retiree has to pay some up-front costs, which we denote as $F_{i,j}$. They are assumed to be a fraction λ of the house value plus some additional cash for closing costs f. Specifically, they include an origination fee that covers the lender's operating expenses (2% of the house value), an up-front mortgage insurance premium MIP (2% of the house value), an appraisal fee and certain other standard closing costs (about $2000-4000).

$$F_{i,j} = \lambda H_{i,j} + f \tag{1.2}$$

Reverse mortgage up-front costs have been significantly larger than those for conventional home loans. This fact has often been cited as one of the main motives for the relative weakness in demand. The main reason for the high up-front costs is the MIP charged by the FHA. In addition to the initial MIP, the FHA charges an ongoing 0.5% annual premium on the loan balance. By charging MIPs, the FHA insures the borrower against the risk of lender's default. Additionally, it insures the lender against the risk that the outstanding debt exceeds the house value at loan termination. Thus, in this contract the FHA bears the risk of default and this explains the higher insurance premium compared to the insurance payments made on conventional loans. Until now, due to house price growth and borrowers' rapid mobility, the FHA has experienced small losses and retained substantial reserves.

Let $\overline{B}_{i,j}$ denote the cash available to borrower i at time j, after the payment of the up-front costs. $\overline{B}_{i,j}$ is the lender's initial cost. A reverse mortgage accrues interest charges, beginning when the first payment is made. Thereafter, the interest is compounded annually. Let $G_{i,t}$ be the outstanding debt at time t:

$$G_{i,t} = \overline{B}_{i,j} \sum_{j=1..t} (1 + i_D)^{t-j} \tag{1.3}$$

where i_D is the nominal interest rate on a reverse mortgage. In present value, the repayment in period t for household i is:

$$D_{i,t} = \frac{\min(H_{i,t}, G_{i,t})}{R^{t-j}} \tag{1.4}$$

If the borrower moves out of the house or dies at time t, she would be required to repay the minimum between the house value and the outstanding debt. Let $\eta_{i,t}$ be household i's probability of being alive at time t and let $m_{i,t}$ be her probability of moving at time t. The expected gain for the lender is:

$$EGain_{i,j} = F_{i,j} + \sum_{t=j+1..T} \eta_{i,t-1}\{(1-\eta_{i,t})(1-m_{i,t}) + \eta_{i,t} m_{i,t}\} D_{i,t} \tag{1.5}$$

A simple calculation, without taking into account the interest rate risk, the house price risk and the possibility of adverse selection, shows that a 62 year old homeowner with a \$100,000 house value could borrow about \$47,000, \$31,000, or \$10,000 respectively if she closes the Monthly Adjusting HECM, the Annually Adjusting HECM, or the Fannie Mae Home Keeper contract. This represents the actual cost for the lender. Given female survival probabilities and the US mobility rate, the expected gain for the lender is about \$74,000, \$64,000, or \$30,000.

1.3.2 Empirical Evidence on Reverse Mortgage Borrowers

Financial, demographic and consumption data for reverse mortgagees are not currently available. However, in December 2006, the AARP conducted the first national survey of homeowners who had considered these loans. We briefly summarize their findings.

Between 1993 and 2004, the median annual income of reverse mortgage borrowers increased from \$12,289 to \$18,240 (HUD, 2007b). For a third of borrowers (33 percent) the self-reported income was less than \$20,000, and for nearly two-thirds (62 percent) it was less than \$30,000. According to census data, the elderly median net worth, excluding home equity, was \$23,369 in 2000. More than half of reverse mortgage borrowers in the AARP Survey (54 percent) reported having less than \$25,000 in financial savings, but their average net worth is not available. Reverse mortgagees tend to be house-richer than typical older homeowners. Nearly half of reverse mortgage borrowers (46 percent) have homes worth \$100,000 to \$200,000, compared with only about one-third of general homeowners (34 percent). Average property values of borrowers were \$142,000 in 2000, while the median house value was \$65,624 for households without this loan. More than half (57 percent) of reverse mortgagees in 2000 were single women. Bowen Bishop and Shan (2008), using all 18 years of HECM loan data, presents the first systematic evidence on loan characteristics over time. Figure 1.1 and 1.2 are from Bishop and Shan (2008). Figure 1.1 presents the loan survival curves for single male, single female and couples. Figure 1.2 shows the termination hazard rates corresponding to the survival curves plotted in Figure 1.1. These hazard rates have an inverse-U shape. This implies that the termination hazard is low immediately after the closure of the contract and then increases with time. However, if the reverse mortgage contract has not been terminated within 10 years, the borrower is expected to remain in the home for a very long time. Davidoff and Welke (2007) shows that reverse mortgage borrowers have exited their homes surprisingly quickly. Only 66% of male and 62% of female loan termination are ascribed to death as opposed to payoff while alive.

1.4 The Model

This section describes a model of post-retirement decision making. We consider optimal consumption, housing and moving decisions for a single retiree. When the retiree decides to move out of her house, transaction costs are incurred.

1.4.1 Preferences

Individual i's plan is to maximize her expected lifetime utility at age t, $t = 64, ..., T$. T is set exogenously and equals 95. In each period she receives utility U from non-durable consumption $C_{i,t}$ and housing services $H_{i,t}$.

The within-period retiree's preference over consumption and housing services are represented by the Cobb-Douglas utility function:

$$U(C_{i,t}, d_{i,t}) = \frac{(C_{i,t}^{1-\omega} H_{i,t}^{\omega})^{1-\gamma}}{1-\gamma} + \varepsilon_{i,t}(d_{i,t}) \tag{1.6}$$

where $C_{i,t}$ denotes consumption, $H_{i,t}$ housing services, ω measures the relative importance of housing services versus numeraire non-durable consumption good, γ is the coefficient of relative risk aversion. Let $d_{i,t}$ be the discrete housing choice, described in the next subsection.

$\varepsilon_{i,t}(d_{i,t})$ represents housing preference shock. It is Extreme Value Type I distributed and it is independent across individuals and time. Individuals move out of their homes for several reasons, which are explained in detail in the survey. They can move out for financial reasons, looking for a smaller or less expensive house; because they desire to live near or with their children or other relatives; due to health problems; for climate or weather reasons; for reasons related to leisure activities or public transportation; due to changes in marital status. We model this unobserved utility from moving as housing preference shock.

When the individual dies, her terminal wealth $TW_{i,t}$ is bequeathed according to a bequest function $b(TW_{i,t})$:

$$b(TW_{i,t}) = \theta_B \frac{TW_{i,t}^{1-\gamma}}{1-\gamma} \tag{1.7}$$

Carroll (2000b) employs a similar bequest function. The degree of altruism is given by the parameter θ_B. In the baseline case, we assume θ_B equal to 1. That is, the retiree has a strong bequest motive. Section 8 revisits the bequest motive.

1.4.2 Choice set

In each discrete period t, the household makes two joint and simultaneous choices, a discrete housing choice and a continuous consumption choice.

Housing is a discrete multi-stage choice. The household chooses whether to move or stay in the house. The household that moves out chooses whether to own or to rent, and the value of the new house[2]. Consistent with our data, homeowners that move could not afford a larger house and renters are only allowed to rent a new house of any value.

First, the household makes the discrete choice of staying or moving out in period t:

$$d_{i,t}^1 = \begin{cases} D_{i,t}^M = 1 & \text{if household } i \text{ moves out of her house in period } t \\ D_{i,t}^M = 0 & \text{otherwise} \end{cases}$$

Second, if she moves out of the house, she makes the binary choice of owning or renting a new house:

[2] As described in Section 6, a complete set of data is available only for three periods. Given this short panel, we do not evaluate the effects of house price on the choice variables. Instead, we set the house price equal to 1. Therefore, house value and housing servings are used indifferently in the paper.

$$d_{i,t}^2 | d_{i,t}^1 = \begin{cases} D_{i,t}^O = 1 & \text{if household } i \text{ owns her house in period } t \\ D_{i,t}^O = 0 & \text{if household } i \text{ rents her house in period } t \end{cases}$$

The third stage decision over housing is the house value. To simplify the computation, we discretize the house value.

$$d_{i,t}^3 | d_{i,t}^1, d_{i,t}^2 = H_{i,t}$$

Therefore, the discrete choice set $d_{i,t}$ is:

$$d_{i,t} = \{d_{i,t}^1, d_{i,t}^2, d_{i,t}^3\}$$

Let $C_{i,t}$ be the continuous choice of consumption.

1.4.3 Housing Expenses

Per period housing expenses ψ are assumed to be a fraction of the house value, deterministic and constant over time. For homeowners, they correspond to a maintenance cost, incurred to keep the house at a constant quality level. For renters, they represent the rental cost. These expenses are denoted by ψ^{own} and ψ^{rent} respectively for homeowners and for renters.

$$\psi_{i,t} = [D_{i,t}^O \psi^{own} + (1 - D_{i,t}^O)\psi^{rent}]H_{i,t}^* \tag{1.8}$$

where $H_{i,t}^* = D_{i,t}^M H_{i,t} + (1 - D_{i,t}^M)H_{i,t-1}$.

If the retiree decides to sell her house at time t and move to another house, she pays or receives the difference in owner-occupied housing wealth, depending on whether the new house value is greater or smaller than the previous house value. In addition, she sustains a one-time transaction cost $\phi(D_{i,t}^O)$. The cost of moving is:

$$M_{i,t} = D^M_{i,t} D^O_{i,t-1}[D^O_{i,t} H_{i,t} - H_{i,t-1} + H_{i,t}\phi(D^O_{i,t})] + D^M_{i,t}(1 - D^O_{i,t-1})(1 - D^O_{i,t})H_{i,t}\phi^{rent} \quad (1.9)$$

The transaction cost equals a fraction $\phi^{own}(\phi^{rent})$ of the value of the new house, i.e.

$$\phi(D^O_{i,t}) = [D^O_{i,t}\phi^{own} + (1 - D^O_{i,t})\phi^{rent}] \quad (1.10)$$

Generally, the transaction cost is larger when buying a new house than when renting it, that is $\phi^{own} > \phi^{rent}$.

1.4.4 The Household's Problem

The state space in period t consists of variables that are observed by the agent and the econometrician $X_{i,t}$ and by variables observed only by the agent $\varepsilon_{i,t}(d_{i,t})$.

$$X_{i,t} = \{A_{i,t}, H_{i,t-1}, D^O_{i,t-1}, Age_{i,t}\}$$

where $A_{i,t}$ is household i's non-housing financial assets at time t, $H_{i,t-1}$ the previous period house value, and $D^O_{i,t-1}$ the previous period housing tenure.

The term $\varepsilon_{i,t}(d_{i,t})$ refers to a vector of unobserved utility components determined by the discrete alternative and it is Type I Extreme Value distributed. Let $\varepsilon_{i,t}$ mean $\varepsilon_{i,t}(d_{i,t})$.

The household maximizes the expected lifetime utility over consumption $C_{i,t}$ and housing $d_{i,t}$:

$$V_{i,t}(X_{i,t}, \varepsilon_{i,t}) = \max_{d_{i,t}, C_{i,t}} E_t \left[\sum_{t=64}^{T} \beta^{t-64}(N(t-1,t)\eta_{it}U(C_{i,t}, d_{i,t})|X_{i,t}, \varepsilon_{i,t}) + b(TW_{i,t}) \right] \quad (1.11)$$

s.t

$$A_{i,t+1} = RA_{i,t} + y - C_{i,t} - \psi_{i,t} - M_{i,t} \tag{1.12}$$

$$C_{i,t} \geq C_{MIN} \tag{1.13}$$

where $\eta_{i,t}$ denotes the probability of being alive at age t conditional on being alive at age $(t-1)$, and let $N(t,j) = (1/\eta_j)\prod_{k=1}^{t}\eta_k$ denote the probability of living to age t, conditional on being alive at age j.

Eq. (1.12) denotes period t retiree i's budget constraint. Let y denote the retiree's income, which includes Social Security, pension and other retiree benefits.

The value function for period t is given by the following expression:

$$V_{i,t}(X_{i,t}, \varepsilon_{i,t}) = \max_{d_{i,t},C_{i,t}} U(C_{i,t}, d_{i,t}) + \varepsilon_{i,t} + \beta\eta_{i,t+1}EV_{i,t+1}(A_{i,t+1}, H^*_{i,t}, D^O_{i,t}, \varepsilon_{i,t+1}|X_{i,t}, C_{i,t})$$

s.t.

$$\begin{aligned} A_{i,t+1} &= RA_{i,t} + y - C_{i,t} - \psi_{i,t} - M_{i,t} && (1.14) \\ H^*_{i,t} &= D^M_{i,t}H_{i,t} + (1 - D^M_{i,t})H_{i,t-1} && (1.15) \\ C_{i,t} &\geq C_{MIN} \end{aligned}$$

The computation of the optimal policy functions is complicated due to the presence of the vector $\varepsilon_{i,t}$. It enters nonlinearly in the unknown value function $EV_{i,t+1}$. Following Rust (1988), we introduce the additivity and the conditional independence assumptions. Thus, $EV_{i,t+1}$ does not depend on $\varepsilon_{i,t}$.

Therefore the Bellman equation can be rewritten as:

$$\begin{aligned} V_{i,t}(X_{i,t}, \varepsilon_{i,t}) &= \max_{d_{i,t}, C_{i,t}} [U(C_{i,t}, d_{i,t}) + \varepsilon_{i,t} + \beta \eta_{i,t+1} E V_{i,t+1}(X_{i,t+1})] \\ &= \max_{d_{i,t}} \left\{ \left[\max_{C_{i,t}} \{U(C_{i,t}, d_{i,t}) + \beta \eta_{i,t+1} V_{i,t+1}(X_{i,t+1}) | d_{i,t}\} \right] + \varepsilon_{i,t} \right\} \end{aligned} \tag{1.16}$$

The solution of period t's problem could be divided in two parts. There is an inner maximization with respect to the continuous choice conditional on the discrete housing choice and an outer maximization with respect to the multi-stage discrete choice.

We assume that there is a measurement error in consumption distributed as a normal with mean 0 and unknown variance σ^2. Given the observed realization of household choices and states $\{C_{i,t}, d_{i,t}, X_{i,t}\}$, the objective is to estimate the preferences denoted as $\theta = \{\gamma, \omega, \sigma\}$. We allow for heterogeneity in the state variables $X_{i,t}$ and $\varepsilon_{i,t}$, but not in the preferences θ.

1.4.5 Inner Maximization

Let $r(X_{i,t}, d_{i,t})$ represent the indirect utility function associated with the discrete choice $d_{i,t}$:

$$r(X_{i,t}, d_{i,t}) = \max_{C_{i,t}} \{U(C_{i,t}, d_{i,t}) + \beta \eta_{i,t+1} V_{i,t+1}(X_{i,t+1}) | d_{i,t}\} \tag{1.17}$$

This function has to be computed for each possible $d_{i,t}$, subject to the contemporary budget constraint and the constraint on consumption.

1.4.6 Outer Maximization

Under the assumption that $\varepsilon_{i,t}$ is distributed as a Type I Extreme Value error, the conditional choice probabilities are given by the following formula:

$$P(j | X_{i,t}, \theta) = \frac{\exp\{r(X_{i,t}, j)\}}{\sum_{k \in d_{i,t}(X_{i,t})} \exp\{r_{i,t}(X_{i,t}, k)\}} \tag{1.18}$$

and $V_{i,t+1}(X_{i,t+1})$ is given by:

$$V_{i,t+1}(X_{i,t+1}) = \ln\left[\sum_{k\in d_{i,t}(X_{i,t})} \exp\{r(X_{i,t},k)\}\right]$$

1.5 Solution Method

We use a recently developed set of tools from Applied Mathematics to estimate an empirical model. Specifically, this set includes the Mathematical Programming with Equilibrium Constraints (MPEC) approach, a flexible polynomial approximation, shape preservation, and the imposition of the Envelope Theorem for calculating the value functions. This is the first paper to use the Envelope Theorem in this way, and the first to use the four tools in combination. Moreover, this is the first example of employing the MPEC approach to solve an empirical structural model with finite horizon dynamic programming.

We illustrate our approach for a simple life-cycle model, underlying its novelty with respect to the conventional approach. The use of a mathematical programming language allows us to rewrite the dynamic programming and estimation problems as a constrained optimization problem that involves the optimization of an objective function subject to equality and inequality constraints. We present the details for the full model in the Appendix.

1.5.1 Simple Life-Cycle Model

For ease of exposition, we assume that there is only one continuous state variable (assets) and one continuous choice variable (consumption).

The backward solution from time T for true value functions is described as follows. The last period value function is known and equal to $V_T(A)$.

In periods $t = 1...(T-1)$ the Bellman equation is:

$$V_t(A) = \max_c \; u(c) + \beta V_{t+1}(RA - c)$$

Given V_{t+1}, the Bellman equation implies, for each asset level A, three equations that determine the optimal consumption, c^*, $V_t(A)$, and $V_t'(A)$:

Euler Equation:

$$u_t'(c^*) - \beta V_{t+1}'(RA - c^*) = 0$$

Bellman equation:

$$V_t(A) = u(c^*) + \beta V_{t+1}(RA - c^*)$$

Envelope Condition:

$$V_t'(A) = \beta R V_{t+1}'(RA - c^*)$$

The backward solution from time T for approximate value functions requires several steps.

We choose a functional form and a finite grid of asset levels. Let $A_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. We will use these grid points for approximating the value functions. Let $\Phi(A; a)$ be the function that we use to approximate the value functions, $V(A)$. If we assume that it is a seventh-order polynomial centered at $\overline{A}$, then

$$\Phi(A; a, \overline{A}) = \sum_{k=0}^{7} a_k (A - \overline{A})^k$$

The time t value function is approximated by

$$V_t\,(A) = \Phi(A; a_t, \overline{A}_t) = \sum_{k=0}^{7} a_{k+1,t}(A - \overline{A}_t)^k \tag{1.19}$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{A}$ on time. We will choose $\overline{A}_t = (A_t^{\max} + A_t^{\min})/2$, the period t average assets. Note that $\overline{A}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, we will drop it as an explicit argument of Φ. So, $\Phi(A; a_t)$ will mean $\Phi(A; a_t, \overline{A}_t)$.

We would like to find coefficients a_t such that each time t Bellman equation, along with the Euler and Envelope conditions, holds with the Φ approximation; that is, for each time $t < T-2$, we want to find coefficients a_t such that for all A

$$\Phi(A; a_t) = \max_c \; u(c) + \beta \Phi(RA - c; a_{t+1})$$

and for time $t = T-1$, we want to find coefficients a_t such that for all A

$$\Phi(A; a_t) = \max_c \; u(c) + \beta V_T(RA - c)$$

We need to approximately solve the Bellman equation. To this end, we need to specify the various errors that may arise in our approximation. We will consider three errors and one side condition.

First, at each time t and each $A_{i,t}$, the absolute value of the Euler equation if consumption is $c_{i,t}$, which we denote as $\lambda_{i,t}^e \geq 0$, satisfies the inequality

$$-\lambda_{i,t}^e \leq u'(c_{i,t}) - \beta \Phi'(RA_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_{i,t}^e \tag{1.20}$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $A_{i,t}$ with consumption $c_{i,t}$ is denoted by λ_t^b and satisfies

$$-\lambda_t^b \leq \Phi(A_{i,t}; a_t) - [u(c_{i,t}) + \beta \Phi(RA_{i,t} - c_{i,t}; a_{t+1})] \leq \lambda_t^b \tag{1.21}$$

Third, the Envelope condition error, λ_t^{env}, satisfies

$$-\lambda_t^{env} \leq \Phi'(A_{i,t}; a_t) - \beta R \Phi'(RA_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_t^{env} \tag{1.22}$$

where $\Phi'(x; a_t)$ is the derivative of $\Phi'(x; a_t)$ with respect to x.

Fourth, because the true value functions are concave, we want our approximate value functions to also be concave. Sometimes we will impose concavity of the approximate value functions on the $A_{i,t}$ grid with the secant condition

$$\Phi(A_{i,t};a_t) \geq \Phi(A_{i-1,t};a_t) + \frac{(\Phi(A_{i+1,t};a_t) - \Phi(A_{i-1,t};a_t))}{(A_{i+1,t} - A_{i-1,t})}(A_{i,t} - A_{i-1,t}) \tag{1.23}$$

With these definitions, the constrained optimization approach to a life-cycle dynamic programming problem can be rewritten as:

$$\min_{a,c,\lambda} \sum_t \sum_i \lambda_{i,t}^e + \sum_t \lambda_t^b + \sum_t \lambda_t^{env} \tag{1.24}$$

subject to:

$$-\lambda_{i,t}^e \leq u'(c_{i,t}) - \beta\Phi'(RA_{i,t} - c_{i,t};a_{t+1}) \leq \lambda_{i,t}^e$$

$$-\lambda_t^b \leq \Phi(A_{i,t};a_t) - [u(c_{i,t}) + \beta\Phi(RA_{i,t} - c_{i,t};a_{t+1})] \leq \lambda_t^b$$

$$-\lambda_t^{env} \leq \Phi'(A_{i,t};a_t) - \beta R\Phi'(RA_{i,t} - c_{i,t};a_{t+1}) \leq \lambda_t^{env}$$

where we choose the value function approximation parameters, a, the consumption choices on the asset grid, c, and the errors, $\lambda \geq 0$, so as to minimize the sum of errors. We may also add the concavity constraint if necessary to attain a concave value function approximation.

There are many variations on this theme. Standard value function iteration ignores the $\sum_t \lambda_t^{env}$ term and imposes $\lambda_{i,t}^e = 0$, both of which we could do here. A more general specification would be

$$\min_{a,c,\lambda} P^e\left(\sum_t \sum_i \lambda_{i,t}^e\right) + P^b\left(\sum_t \lambda_t^b\right) + P^{env}\left(\sum_t \lambda_t^{env}\right)$$

where the P^j parameters are penalty terms. Conventional value function iteration is $P^{env} = 0$ and P^e being "infinitely" larger than P^b.

This setup can be easily extended by also including discrete state variables. This would require to redefine both the a coefficients and the errors λ over the grid points of the discrete state variables.

In sum, given the last period value function, we find simultaneously consumption, saving and the other endogenous variables in each period. Hence, creating a link between past, current and future economic variables, we obtain the only equilibrium that is associated with the optimal consumption and saving decisions in each period. Given the enormous increase in computer speed and progress in algorithms and software for large-scale problems, this technique offers certain advantages. It permits us to keep track of the grid of possible values of the state variables and it is adequate for solving any consumption saving problem of reasonable complexity.

Given our solution for the dynamic programming problem, we can now consider the empirical analysis. Our sample include continuous data on assets and consumption. We assume that the measurement error in consumption is normally distributed with mean 0 and unknown variance σ^2. We can use the Euler equation to recover the predicted value of consumption, denoted as c^{pred}. The probability that household n chooses consumption $c_{n,tp}$ in period tp is:

$$\Pr(c_{n,tp}|A_{n,tp}^{data}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c_{n,tp}^{data} - c_{n,tp}^{pred})^2}{2\sigma^2}}$$

Therefore the log-likelihood is given by:

$$L(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \Pr(c_{n,tp} \mid A_{n,tp}^{data}, \theta) \tag{1.25}$$

The constrained optimization approach to structural estimation with finite horizon dynamic programming is:

$$\begin{aligned} & Max \qquad L(\theta) - P \cdot \Lambda \\ \text{subject to} \quad : & \\ & Euler\ Error \\ & Bellman\ Error \\ & Envelope\ Error \end{aligned} \tag{1.26}$$

where $\Lambda = \sum_t \sum_i \lambda_{i,t}^e + \sum_t \lambda_t^b + \sum_t \lambda_t^{env}$.

The traditional approach to estimate finite horizon dynamic structural models consists in taking a guess of the structural parameters, solving the dynamic programming problem, calculating the log-likelihood and repeating these steps until the log-likelihood is maximized. This can be computationally very demanding. We use the MPEC approach to solve our empirical model. Therefore, the structural estimation of the life-cycle dynamic model simply becomes a problem of optimizing an objective of many variables subject to a set of constraints. The structural parameters and endogenous economic variables are chosen simultaneously and symmetrically. The MPEC approach relies on ideas and methods developed in the statistical and econometric literatures, nevertheless the current econometric literature seems to consider this approach infeasible. Judd and Su (2008) shows that it is feasible if one uses conventional techniques in the mathematical programming literature. We extend their approach presenting the MPEC with finite horizon dynamic programming. The penalty parameter approach introduced in this paper is an example of a nonsmooth exact penalty method. Using an exact penalty function implies that, for certain values of the penalty parameter, a single minimization with respect to the choice variables produces the exact solution of the nonlinear programming problem. For a proof and further reading see Theorem 17.3 in Nocedal and Wright (2000).

Furthermore, we present an example of using a flexible polynomial approximation in an empirical work. The continuous state value function is approximated by a seventh order polynomial as this functional form appears adequate to our analysis. Since this approach could be applied to a wide range of economic problems, the functional form is easily adaptable to new, different, or changing requirements. For example, if necessary for the accuracy of the solution, the functional form could include a specific basis function in addition to the polynomial.

Then, we introduce shape preservation in approximating the value functions. Under the standard assumption of a risk-averse utility, the value function has two main shape features: concavity and monotonicity. If these shape features are deformed by approximation methods, the approximation errors propagate as the number of computations increases. Therefore, the approximate solution is inaccurate. This fact motivates us to introduce additional constraints which guarantee the preservation of the shape characteristics of the value function. Specifically, we introduce the secant condition (Eq. 1.23). This condition is not always necessary; however if needed, it can be easily added to the set of constraints. For further reading on shape preservation methods see Judd (1998).

The fourth innovative aspect is the imposition of the Envelope Condition in our set of constraints. In optimization problems, the Envelope Theorem provides the solution via differentiability techniques; in dynamic programming problems, it is key for characterization, analysis, and computation of the optimal value function from its derivative. By imposing the Envelope Condition, we obtain both a precise characterization of the optimal solution that is appropriate for computation and an explicit expression for the derivative of the value function. Fernandez-Villaverde et al. (2006) shows that economic dynamic models typically lack a closed-form solution; hence, economists numerically approximate the policy functions. It follows that only an approximated likelihood associated with the approximated policy function, instead of the exact likelihood, can be evaluated. Fernandez-Villaverde et al. argue that as the approximated policy function converges to the exact policy function, the approximated likelihood also converges to the exact likelihood. To have an accurate approximation of the policy function, a high order polynomial is required. By introducing both a high order polynomial approximation and the Envelope Condition, our approach generates an accurate approximation for the policy function, which is crucial for structural estimation.

The inequality approach we use is formulated as constraints in a nonlinear programming problem and, to our knowledge, this is the only stable method for dynamic programming problems of this kind.

Finally, AMPL, the mathematical programming language, presents several advantages. AMPL is an extremely easy-to-use modeling language for linear and nonlinear optimization problems involving discrete or continuous variables. It allows the user to easily access the best algorithm on hand for her specific problem. By using the increasing number of solvers for which AMPL interfaces are available, the researcher can compare alternative optimization methods for any application. In this study we use KNITRO, a solver designed for large nonlinear optimization problems which is highly valued for its robustness and efficiency. In addition, when mathematical programming problems are expressed in AMPL, the true analytic derivatives are efficiently computed, invisibly to the user, through automatic differentiation. This significantly improves the speed without any additional cost for the user. Moreover, frequently in economic models, Jacobians and Hessians are sparse. That is, even though they could be large in terms of number of elements, most entries equal zero. The major algorithms and software for constrained optimization problems are based on sparse-matrix methods.

1.6 The Data

The Health and Retirement Study (HRS) is a US panel survey which covers a wide range of topics. In particular, questions on family structure, employment status, demographic characteristics, housing, stocks, bonds, IRA, other financial assets, income, pension, Social Security, and benefits are relevant to our analysis. Questionnaires assessing individual activities and household patterns of consumption are mailed to a subsample of the HRS. The Consumption and Activities Mail Survey (CAMS), the survey including this information on consumption, was first conducted in 2001. The survey is carried out every two years.

We select a group of households that is the potential target segment for a reverse mortgage, according to estimates from the public policy perspective. Our sample includes single and retired homeowners, 62 year old or older. Social Security is the homeowners' main source of income. Pensions and earned interest on financial assets contribute much less as a source of per period income. We eliminate all households with incomplete records or missing information. After these cuts are made, a sample of 175 single households observed for three consecutive periods between 2000 and 2005 remains.

Non-housing financial assets include stocks, bonds, saving accounts, mutual funds, individual retirement accounts (IRAs) and other assets. It does not include the value of any real estate or business. Given that the target segment has almost no debt, focusing on total non-housing financial assets gives nearly identical results as focusing on non-housing financial wealth. Consumption includes vehicles, washing machine, dryer, dishwasher, television, computer, telephone, cable, internet, vehicle finance charges, vehicle insurance, health insurance, food and beverages, dining/drinking out, clothing and apparel, gasoline, prescription and nonprescription medications, health care services, medical supplies, trips and vacations, tickets to movies, sorting events and performing arts, hobbies, contribution to religious, educational, charitable or political organizations, and cash or gifts to family and friends. Housing expenses for homeowners represent the maintenance cost incurred to keep the house at a constant quality, and for renters represent the rental cost.

Table 1.1 shows the descriptive statistics for house value, financial assets, consumption, Social Security income and age for the first year in the panel. Housing represents a significant proportion of the retirees' total assets. The median house value is $70,000. The median ratio of house value to non-housing financial assets is 2.5. Consumption seems to parallel Social Security income. The average per period income is $20,000.

Figure 1.3 - 1.7 illustrates how consumption, Social Security, non-housing financial assets and housing vary with age. Near retirement, the average consumption exceeds the average Social Security income, implying that Social Security income, pensions and liquid savings contribute to finance per period expenses. As the retiree ages, consumption decreases. It is almost completely financed with Social Security income after age 75. Non-housing financial assets represent a fraction of the house value and gradually reduce with age. Housing is constant over time, supporting the thesis that retirees tend not to divest their home equity.

Table 1.2 presents the composition of the financial portfolio. For almost all the retirees in the sample, the financial portfolio does not contain risky assets. Retirees have most of their savings in checking and saving accounts and transportation. About 40% of the retirees have certificates of deposits and approximately 25% have IRAs. Less than 10% have stocks and about 5% have bonds.

In each period, about 10% of the households in our sample move out of their homes. Among those who moved, about 35% decide to rent a new house, while about 65% buy a new house. At the end of the three years of the panel, about 25% of the population moved and about 10% rented a new house. Table 1.3 shows that 50% of the households that move choose to buy a house of equal value and 24% of those households choose to rent a house of equal value. The moving decision does not appear to be strictly related to age. About 50% of the retirees move near or with children or other relatives or friends. About 25% move for financial reasons and the remaining 25% move due to health problems, weather or climate reasons, retirement related area reasons, to have a better location, or other reasons.

1.7 Calibration and Results

The subjective discount rate β is 0.96 and the real interest rate r is 4%. Following Yao and Zhang (2005a), the rental rate is $\psi^{rent} = 6\%$ and maintenance cost is $\psi^{own} = 1.5\%$. Transaction costs are $\phi^{own} = 6\%$ and $\phi^{rent} = 1\%$, respectively, when moving to an owner-occupied house and when moving to a rental house. In the baseline case we assume $\theta_B = 1$. That is, the retiree has a strong bequest motive.

The parameter γ has been estimated using a grid search approach. Given the parameter γ, we use the MPEC approach to estimate ω and σ. Table 1.4 presents the estimation results. We find reasonable estimates of the preference parameters. The coefficient of relative risk aversion is 3.87 and it is similar to other estimates that rely on different methodologies (see Cagetti (2003) and French (2005)). According to the related literature, a small estimate of the coefficient of relative risk aversion means that households save little given their level of assets and their level of uncertainty. On the other side, more risk averse individuals prefer to save more in order to buffer themselves against future risks. Our estimate of 3.87 implies a relatively high coefficient of risk aversion, suggesting that households prefer high levels of precautionary savings. In addition, we obtain an estimate of the preference parameter over housing ω equal to 0.85. To our knowledge, there are no previous structural estimates of this parameter for retirees. Our estimate of ω is consistent with our sample data in which retiree consumption is a small fraction compared to home value. These two estimates together can help explain the retiree behavior. In particular, they show that retirees are highly risk averse and that they significantly value their house as a safe and illiquid asset in which precautionary savings can be locked.

We compute the standard errors using a bootstrap procedure. Resampling was conducted by sampling with replacement across households as is standard practice in panel models. In total, the standard errors are calculated with 100 bootstraps.

1.7.1 Do Reverse Mortgages Pay?

A HUD reverse mortgage is a federally-insured loan against the retiree's home that does not have to be paid back so long as the retiree lives there. We assume that the retiree chooses to receive the proceeds as a single lump sum of cash at the closure of the contract. Following the notation in Section 3, let $G_{i,t}^{RM}$ be the real outstanding debt at time t.

If the retiree decides to move out of the house, she has to repay the minimum between the value of the house and the accumulated debt plus a one-time transaction cost $\phi(D_{i,t}^{O})$. The cost of moving is:

$$M_{i,t} = D_{i,t-1}^{O} D_{i,t}^{M} [D_{i,t}^{O} H_{i,t} - \max(0, H_{i,t-1} - G_{i,t}^{RM}) + H_{i,t} \phi(D_{i,t}^{O})] \tag{1.27}$$

The welfare gain from a reverse mortgage is calculated as the percentage increase in the initial financial assets that makes the household without a reverse mortgage as well off in expected utility terms as the household with a reverse mortgage. For each household in our sample we calculate the expected lifetime utility from closing the reverse mortgage contract in the first year of the panel, the year 2000. Then, we compute the percentage increase in the initial financial assets that generates the same lifetime utility without a reverse mortgage as with a reverse mortgage. We explain our simulation results and we assess the validity of our model in predicting the retirees' behavior in light of the empirical evidence on reverse mortgagees.

We first introduce some notation. We define as "Cash-Poor" those households with initial non-housing financial assets less than \$10,000, "Cash-Medium" those with non-housing financial assets between \$10,000 and \$60,000, and "Cash-Rich" those with non-housing financial assets greater than \$60,000. We consider three house values. Let "House-Poor" denote house value equal to \$40,000, "House-Medium" equal to \$80,000, and "House-Rich" equal to \$120,000.

Table 1.5 and Table 1.6 display the median non-housing financial assets and the median welfare gain. Both are presented as a function of initial non-housing financial assets and house value. The common belief is that a reverse mortgage benefits households with resources tied up in home equity, those defined as house-rich but cash-poor. This simulation shows otherwise. Specifically, house-rich but cash-poor homeowners experience the largest welfare loss from a reverse mortgage equal to a 120% decrease in their initial assets. Additionally, all cash-poor households experience a welfare loss. On the other side, all cash-rich households experience a welfare gain.

This simulation, highlighting the pros and the cons of the contract, could help explain why the reverse mortgage is still a niche product after about 20 years. A reverse mortgage provides liquidity and a form of longevity insurance. The retiree can cash in some of the savings locked in her house and would be able to experience higher levels of consumption than otherwise possible. Furthermore, she can live in the same house while alive, regardless of the amount of the outstanding debt. Reverse mortgages constitute the purchase of a no-exit annuity, an annuity that pays off in the form of the housing services of the current home (implicit rent) provided that the retiree does not permanently exit her home. Since not exiting is partly conditioned on not dying, the no-exit annuity encompasses some longevity insurance. However, closing this contract implies incurring very high up-front costs and facing a new risk, the moving risk.[3] The high up-front costs significantly contribute to the welfare loss for house-poor homeowners. For example, a 62 year old homeowner with a \$40,000 house can borrow about \$20,000. But the cash available at the closure of the contract, after the payment of about \$10,000 in up-front costs, is nearly \$10,000. Moreover, a reverse mortgage is a financial instrument that incorporates an unusual risk, the risk of moving and having to repay the accumulated debt. Empirical evidence supports our finding. Reverse mortgages should be appealing to homeowners that plan to remain in their home for long periods of time. However, reverse mortgagees have exited their homes

[3] In this study, the moving risk is associated with the decrease in the initial financial assets that generates the same lifetime utility without a reverse mortgage as with a reverse mortgage.

surprisingly rapidly, suggesting that an unexpected event happened and forced them to move out. If households have to move, for any exogenous reason, their future well-being, ability to meet unforeseen costs, consumption profile and housing choices could be significantly affected. This is specifically true for households with initially low financial assets. Some of the choices over consumption and housing, available before closing the reverse mortgage contract, are not affordable anymore after. Hence, a precautionary motive appears to be mostly concentrated among cash-poor households.

For a cash-poor homeowner closing a reverse mortgage contract would represent one of the major investing mistakes, namely the lack of diversification. As a rule of thumb, if someone puts all of her eggs in one basket she is taking a much greater risk than if she diversifies. The retiree with initially low financial assets has all her life-savings locked in the house, which is a safe asset under our specification of non-stochastic house price. If she closes a reverse mortgage contract, she reallocates all her savings into a risky financial instrument. While closing a reverse mortgage contract would prevent cash-poor households from diversifying their investments, it would not prevent cash-rich households from spreading their investments around. Consequently, the latter would not experience any welfare loss from the contract. Additionally, the welfare loss for cash-poor retirees comes from not assessing their own level of risk. Essentially, each retiree has to consider how much money she can comfortably afford to lose in the worst case scenario. By closing a reverse mortgage, cash-poor retirees would take on a high risk investment from which they could not escape if they have to move out. Campbell and Viceira (2002) shows that risky assets should be attractive to young households with modest savings and large human wealth relative to financial wealth. However, the attractiveness of risky investments diminishes later in life with the decline in human wealth, which is a relatively safe asset, and the accumulation of financial assets. Consistent with our data, the retirees' financial portfolio consists mostly of safe assets. The house is not only a safe asset, but it is also the main financial asset for the retirees.

1.8 Policy Experiments

The framework presented above allows for many possible policy experiments. In this section we choose the following four: no moving risk, no up-front costs, reduction in current income and no bequest motive. These policy experiments allow us to better identify the risk-expanding and the risk-mitigating aspects of a reverse mortgage.

1.8.1 No Moving Risk

In this subsection we assume that the retiree does not face any moving risk and remains in her house while alive. Hence, reverse mortgages become safe financial instruments. Table 1.7 presents the results. All retirees experience a significant welfare gain. House-rich but cash-poor homeowners have a welfare gain equal to a 72 time increase in their initial financial assets. This result explains the rationale behind reverse mortgage contracts. House-rich but cash-poor homeowners can greatly benefit from the contract if they do not move out of their home.

1.8.2 No Up-front Costs

According to the AARP Survey, many possible reasons could explain the reluctance of older homeowners to tap their home equity: aversion to debt, desire to leave an estate or to use home equity as a last resort for economic or health emergencies (Fisher et al., 2007). However, among homeowners who went through counseling but ultimately chose not to apply for this loan, high costs were the most frequently cited reason for not applying (by 63 percent of non-applicants). In this subsection, we assume zero up-front costs. Table 1.8 shows the simulation results. Compared to the baseline case, the welfare gain is larger, given the larger portion of liquid funds accessible at the closure of the contract. Nonetheless, reverse mortgages remain risky financial instruments unappealing to house-rich but cash-poor homeowners. The welfare loss comes from the fact that the interest rate on the loan exceeds the saving rate.

1.8.3 Reduction in Current Income

Reverse mortgages were originally introduced as financial instruments able to relieve retirees from their financial pressure. In this subsection, we investigate the case of a 10% reduction in current income to assess the importance of the liquidity insurance aspect of these loans. This policy experiment is relevant for two reasons. First, according to the AARP Survey, the median reverse mortgage borrower is house-rich and cash-medium, and her per period income is less than \$20,000. Therefore, we are interested in the welfare gain prediction of our model for this subgroup of homeowners. Second, increases in the living cost and in health care costs, and cutbacks in Social Security or in other employee benefits can expose retirees to reductions in their per period resources available for consumption. Consequently, they might have to adjust to a decreased standard of living in their older years.

In this model, retirees are not allowed to borrow and current consumption is limited by current resources. Thus, a reduction in current income causes a decrease in current consumption. Reverse mortgages, augmenting the resources available to consumption, ease the liquidity problem and generate welfare gains larger than in the baseline case (Table 1.9). The group of households that experience the largest welfare gain are those in the middle right quadrant (Cash-Medium, House-Rich). Therefore, our simulation accounts for the data on reverse mortgage borrowers. The moving risk and the lack of diversification cause welfare losses for cash-poor households.

1.8.4 No Bequest Motive

Leaving an estate is an important reason to save for many retirees. The baseline degree of altruism θ_B is assumed to be equal to 1; hence, the retiree has a strong bequest motive. However, in reality, many households are neither able or eager to leave an estate. In this subsection, the retiree does not receive any utility from leaving an estate and prefers to consume all her assets while alive. That is, we assume $\theta_B = 0$.

The welfare gain without a bequest motive always exceeds the gain in the baseline case (Table 1.10). While in the bequest model the increase in the initial financial assets from closing a reverse mortgage is partly consumed and partly bequeathed, it is entirely consumed in the model without a bequest motive. Given that the retiree receives higher utility from her own consumption than from leaving an estate, the model explains the smaller welfare gain in the baseline case. Nevertheless, similar to the baseline case, all cash-poor households and, particularly, house-rich but cash-poor households, experience a welfare loss from a reverse mortgage. The moving risk and the lack of diversification in the investments are the main causes of this welfare loss.

1.9 Conclusion and Extensions

This paper examines retiree consumption, housing and mobility decisions and provides a plausible explanation for the existence of a niche reverse mortgage market.

Using a structural dynamic life-cycle model, we find that retirees are risk averse and home equity is the most important component of precautionary savings after retirement. Reverse mortgages provide liquidity and a form of longevity insurance, but introduce a new risk, the moving risk. Closing this contract is risky especially for house-rich but cash-poor homeowners. If they have drawn on their home equity through a reverse mortgage, their ability to meet unforeseen costs or move into alternative housing may be limited. Intuitively, a reverse mortgage can be seen as a gamble. Gambling involves a small stake for a large prize. The small stake is the up-front cost that the retiree has to pay to participate in the "reverse mortgage game." The big prize is the use of her own home and the higher consumption that could be enjoyed if the retiree wins, namely if she does not move out. If the retiree moves out while alive, she loses and incurs a significant welfare loss. Gambling can allow someone who is poor to become rich. However, luck plays an important role in this game. These results underline the urgency for further policy analysis directed at designing safe and appealing financial instruments for the elderly which let them liquidate some of their home equity without incurring major risks.

In this paper, we present the first application of a set of mathematical tools to an empirical work, namely the Mathematical Programming with Equilibrium Constraints, a flexible polynomial approximation, shape preservation and the Envelope Condition. Our approach could be fruitfully extended to richer representations of life-cycle consumption behavior and structural estimation problems. Specifically, further extensions of this framework include the estimation of the economic model with Epstein-Zin preferences and the introduction of additional sources of uncertainties, such as out-of-pocket medical expenses, stochastic house price and stochastic interest rate risk . A recursive utility would be of interest since it allows to separate and econometrically evaluate the elasticity of intertemporal substitution and the coefficient of relative risk aversion. Finally, it is possible to use this model to examine how different housing policies can affect the consumption and housing decisions of the elderly.

1.10 Technical Appendix

This Techical Appendix describes the MPEC with dynamic programming (DP) approach in the presence of discrete and continuous choices.

The panel data used in this study involves 3 years and 175 individuals.

The available data are both continuous and discrete.

The continuous data include consumption and non-housing financial assets. The discrete (or discretized) data are the individual's housing tenure (own-rent), her moving decision and her house value. We have additional data on the individuals' demographics, including age.

The MPEC with DP approach simultaneously solves the dynamic programming problem and the maximum likelihood estimation of the preference parameters.

1.10.1 Dynamic Programming with Approximation of the Value Function

The life-cycle model is described as follow.

One continuous state variable: financial assets.

Two discrete state variables: previous period housing tenure and previous period house value.

One continuous choice variable: consumption.

Many discrete choices: Not Move(N), Move to house value h with housing tenure q (Mhq), where q ={Own,Rent}.

1.10.2 Backward Solution from Time T for True Value Functions

In each period, the household chooses whether to stay in her house or to move out. If she moves out, she can either buy or rent a new house and she can choose her new house value. Let the subscripts d^N, d^{Mhq} denote respectively the decision not to move and the decision to move to house value h with housing tenure q. The housing tenure is a binary variable that takes value 1 if the household owns the house.

The last period value function is known and equal to $V_T(A, H, Q)$ where A is the individual's non-housing financial assets, H her previous period house value and Q her previous period housing tenure.

In periods $t = 1...(T-1)$ we define:

$$\begin{aligned} V_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RA - c^*_{d^N} - \psi + y; H, Q) + \varepsilon_t^N \\ V_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RA - c^*_{d^{Mhq}} - \psi - M + y; h, q) + \varepsilon_t^{Mqh} \end{aligned}$$

where M is the transaction cost:

$$M = Q(qh - H + \phi^{own}qh + \phi^{rent}(1-q)h) + (1-Q)(1-q)\phi^{rent}h$$

and ψ is the per period housing expense:

$$\psi = [Q\psi^{own} + (1-Q)\psi^{rent}]H + [q\psi^{own} + (1-q)\psi^{rent}]h$$

c_{d^N} and $c_{d^{Mqh}}$ are the consumption levels respectively if the individual does not move and if she moves to house value h choosing the housing tenure q. y is the household's per period income. η_{t+1} is her survival probability. ε_t^N and ε_t^{Mqh} are Type I Extreme Value errors.

Following Rust, we assume that the additivity and the conditional independence assumptions hold.

To simplify the notation, we introduce the following expressions, which are evaluated at the optimal consumption level:

$$\begin{aligned}
\widehat{V}_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RA - c^*_{d^N} - \psi + y; H, Q) \\
\widehat{V}_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RA - c^*_{d^{Mhq}} - \psi - M + y; h, q)
\end{aligned}$$

The extreme value assumption on ε_t implies that we can reduce the dimensionality of the dynamic programming problem. The Bellman equation is given by the following closed form solution:

$$\begin{aligned}
V_t(A, H, Q) &= \Pr(N|A, H, Q) \cdot \widehat{V}_{d^N,t} + E(\varepsilon_t^N|N) \\
&\quad + \sum_h \sum_q \{\Pr(Mhq|A, H, Q) \cdot \widehat{V}_{d^{Mhq},t} + E(\varepsilon_t^{Mhq}|Mhq)\} \\
&= \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}
\end{aligned}$$

Given V_{t+1}, the Bellman equation implies, for each asset level A, three set of equations that determine the optimal consumption, $c^*_{d^N}$, $c^*_{d^{Mhq}}$, $V_t(A, H, Q)$, and $V_t'(A, H, Q)$:

Euler Equations:

$$\begin{aligned} u'(c^*_{d^N}, H) - \beta\eta_{t+1}V'_{t+1}(RA - c^*_{d^N} - \psi + y; H, Q) &= 0 \\ u'(c^*_{d^{Mhq}}, h) - \beta\eta_{t+1}V'_{t+1}(RA - c^*_{d^{Mhq}} - \psi - M + y; h, q) &= 0 \end{aligned}$$

Envelope Condition:

$$V'_t(A, H, Q) = \Pr(N|A, H, Q) \cdot \widehat{V}'_{d^N,t} + \sum_h \sum_q \Pr(Mhq|A, H, Q) \cdot \widehat{V}'_{d^{Mhq},t}$$

Bellman equation:

$$V_t(A, H, Q) = \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}$$

The time $t = 1...(T-1)$ probabilities of not moving and moving to house value h with housing tenure q are:

$$\Pr(N|A, H, Q) = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(V_t(A, H, Q))}$$

$$\Pr(Mhq|A, H, Q) = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(V_t(A, H, Q))}$$

1.10.3 Backward Solution from Time T for Approximate Value Functions

Let $\Phi(A, H, Q; a)$ and $\Phi_d(A, H, Q; b)$ be the functions that we use to approximate respectively the value functions $V(A, H, Q)$ and the policy functions $c^*_d(A, H, Q)$, with $d = \{d^N, d^{Mhq}\}$. If we assume that they are seventh-order polynomials centered at $\overline{A}$, then

$$\Phi(A, H, Q; a, \overline{A}) = \sum_{k=0}^{7} a_{k,H,Q}(A - \overline{A})^k$$

The time t value function is approximated by

$$V_t\,(A,H,Q) = \Phi(A,H,Q;a_t,\overline{A}_t) = \sum_{k=0}^{7} a_{k+1,H,Q,t}(A-\overline{A}_t)^k$$

The time t policy functions are approximated by

$$c^*_{d,t}\,(A,H,Q) = \Phi(A,H,Q;b_{d,t},\overline{A}_t) = \sum_{k=0}^{7} b_{k+1,H,Q,d,t}(A-\overline{A}_t)^k$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{A}$ on time and the dependence of the policy functions on time is represented by the dependence of the b coefficients and the center $\overline{A}$.

We will choose $\overline{A}_t = (A_t^{\max} + A_t^{\min})/2$, the period t average level of assets. Note that $\overline{A}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, we will drop it as an explicit argument of Φ. So, $\Phi(A,H,Q;a_t)$ will mean $\Phi(A,H,Q;a_t,\overline{A}_t)$.

We would like to find coefficients a_t and $b_{d,t}$ such that each time t Bellman equation, along with the Euler and Envelope conditions, holds with the Φ approximation; that is, for each time $t < T-2$, we want to find coefficients a_t such that for all A

$$\Phi(A,H,Q;a_t) = \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h\sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}$$

where

$$\begin{aligned}
\widehat{V}_{d^N,t} &= u(c^*_{d^N},H) + \beta\eta_{t+1}\Phi_{t+1}(RA - c^*_{d^N} - \psi + y; H, Q; a_{t+1}) \\
\widehat{V}_{d^{Mhq},t} &= u(c^*_{d^{Mhq}},h) + \beta\eta_{t+1}\Phi_{t+1}(RA - c^*_{d^{Mhq}} - \psi - M + y; h, q; a_{t+1})
\end{aligned}$$

and for time $t = T-1$, we want to find coefficients a_t given that

$$\begin{aligned}
\widehat{V}_{d^N,T-1} &= u(c^*_{d^N},H) + \beta\eta_T V_T(RA - c^*_{d^N} - \psi + y; H, Q) \\
\widehat{V}_{d^{Mhq},T-1} &= u(c^*_{d^{Mhq}},h) + \beta\eta_T V_T(RA - c^*_{d^{Mhq}} - \psi - M + y; h, q)
\end{aligned}$$

We need to approximately solve the Bellman equation. To this end, we define various errors.

First, we create a finite grid of asset levels we will use for approximating the value functions. Let $A_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. Then we create a grid of house values. Let $H_{j,t}$ be grid point j in the time t grid. The housing tenure is a binary variable. Let $Q_{z,t}$ be grid point z in the time t grid.

Next we need to specify the various errors that may arise in our approximation. We will consider three errors and one side condition.

First, at each time t and each $A_{i,t}$ and each previous period house value $H_{j,t-1}$ and housing tenure $Q_{z,t-1}$, the absolute value of the Euler equations if consumption is respectively $c^*_{i,j,z,d^N,t}$ and $c^*_{i,d^{Mhq},t}$, which we denote as $\lambda^e_{i,j,z,t} \geq 0$, satisfies the inequality

$$-\lambda^e_{i,j,z,t} \leq u'(c^*_{i,j,z,d^N,t}, H_{j,t-1}) - \beta\eta_{t+1}\Phi'(RA_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \leq \lambda^e_{i,j,z,t}$$

$$-\lambda^e_{i,j,z,t} \leq u'(c^*_{i,d^{Mhq},t}, h_t)$$

$$-\beta\eta_{t+1}\Phi'(RA_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1}) \leq \lambda^e_{i,j,z,t}$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $A_{i,t}$ with consumption $c_{i,j,z,d^N,t}$ and $c_{i,d^{Mhq},t}$ is denoted by $\lambda^b_{j,z,t}$ and satisfies

$$-\lambda^b_{j,z,t} \leq \Phi(A_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) - \ln\left\{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^b_{j,z,t}$$

where

$$\begin{aligned}
\widehat{V}_{i,j,z,d^N,t} &= u(c^*_{i,j,z,d^N,t}, H_{j,t-1}) + \beta\eta_{t+1}\Phi(RA_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \\
\widehat{V}_{i,d^{Mhq},t} &= u(c^*_{i,d^{Mhq},t}, h_t) + \beta\eta_{t+1}\Phi(RA_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1})
\end{aligned}$$

Third, the Envelope condition errors, $\lambda^{env}_{j,z,t}$, satisfies

$$-\lambda^{env}_{j,z,t} \leq \Phi'(A_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) - \{f_{i,j,z,d^N,t} \cdot \Phi'(RA_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y; H_{j,t-1}, Q_{z,t-1}; a_{t+1})$$

$$+ \sum_h \sum_q [f_{i,d^{Mhq},t} \cdot \Phi'(RA_{i,t} - c_{i,d^{Mhq},t} - \psi_{h,q} - M_{j,z,d^{Mhq}} + y; h_t, q_t; a_{t+1})]\} \leq \lambda^{env}_{j,z,t}$$

where

$$f_{i,j,z,d,t} = \Pr(d|A_{i,t}, H_{j,t}, Q_{z,t}) = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})}$$

Fourth, we introduce the policy function errors:

$$-\lambda^{cons}_{i,j,z,d,t} \leq \Phi(A_{i,t}, H_{j,t}, Q_{z,t}; b_t) - c^*_{i,j,z,d,t}(A_{i,t}, H_{j,t}, Q_{z,t}) \leq \lambda^{cons}_{i,j,z,d,t}$$

1.10.4 Empirical Part

In the theoretical DP part we obtain the coefficients used in the approximation of the value function.

In this part, for any individual data of financial asset, previous period house value and age, we calculate the predicted consumption and the probabilities of moving. The individual makes simultaneously the housing decision $d_{n,tp}$ and the consumption decision.

Let $c^{pred}_{n,tp}$ and $c^{data}_{n,tp}$ denote respectively the predicted and the true value of consumption for household n at time tp.

For any given discrete choice on housing $d_{n,tp}$, using the real data on consumption, we calculate the measurement error:

$$\Pr(c_{n,tp}|d_{n,tp}, A_{n,tp}^{data}, H_{n,tp-1}^{data}, Q_{n,tp-1}^{data}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c_{n,tp}^{data} - c_{n,tp}^{pred})^2}{2\sigma^2}}$$

The probability for the discrete choice on housing is given by:

$$\Pr(d_{n,tp}|A_{n,tp}^{data}, H_{n,tp-1}^{data}, Q_{n,tp-1}^{data}) = \frac{e^{\widehat{V}_{d,n,tp}}}{\sum_m e^{\widehat{V}m,n,tp}}$$

Therefore the joint probability of making the discrete housing choice $d_{n,tp}$ and the continuous consumption choice $c_{n,tp}$ is given by:

$$\Pr(d_{n,tp}, c_{n,tp}|A_{n,tp}^{data}, H_{n,tp-1}^{data}, Q_{n,tp-1}^{data}) =$$

$$\Pr(d_{n,tp}|A_{n,tp}^{data}, H_{n,tp-1}^{data}, Q_{n,tp-1}^{data}) \cdot \Pr(c_{n,t}|d_{n,tp}, A_{n,tp}^{data}, H_{n,tp-1}^{data}, Q_{n,tp-1}^{data})$$

The log-likelihood is given by:

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d_{n,tp}, c_{n,tp}|A_{n,tp}^{data}, H_{n,tp-1}^{data}, Q_{n,tp-1}^{data}, \theta)$$

where N denotes the number of individuals in the sample and TP the number of time periods in the panel data.

1.10.5 MPEC

With these definitions, let

$$\Lambda = \sum_t \sum_i \sum_j \sum_z \lambda_{i,j,z,t}^{e} + \sum_t \sum_j \sum_z \lambda_{j,z,t}^{b} + \sum_t \sum_j \sum_z \lambda_{j,z,t}^{env} + \sum_t \sum_i \sum_j \sum_z \sum_d \lambda_{i,j,z,d,t}^{cons}$$

and let P be a penalty parameter.

The MPEC approach to the estimation of the preference parameters is:

$$\underset{\theta,a,c}{Max}\ \mathcal{L}(\theta) - P \cdot \Lambda$$

subject to:

Bellman error

$$-\lambda^{b}_{j,z,t} \le \Phi(A_{i,t}; a_t) - \ln \left\{ \exp(\widehat{V}_{i,i,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t}) \right\} \le \lambda^{b}_{j,z,t}$$

Euler errors

$$\begin{aligned} -\lambda^{e}_{i,j,z,t} &\le u_{c;i,j,z,d^N,t} + \beta \Phi^{+}_{A;i,j,z,d^N,t} \le \lambda^{e}_{i,j,z,t} \\ -\lambda^{e}_{i,j,z,t} &\le u_{c;i,d^{Mhq},t} + \beta \Phi^{+}_{A;i,d^{Mhq},t} \le \lambda^{e}_{i,j,z,t} \end{aligned}$$

Envelope error

$$-\lambda^{env}_{j,z,t} \le \Phi_{A;i,z,t} - \{ f_{i,j,z,d^N,t} \cdot \Phi^{+}_{A;i,j,z,d^N,t} + \sum_h \sum_q [f_{i,d^{Mhq},t} \cdot \Phi^{+}_{A;i,d^{Mhq},t}] \} \le \lambda^{env}_{j,z,t}$$

Policy function error

$$-\lambda^{cons}_{i,j,z,d,t} \le \Phi(A_{i,t}, H_{j,t}, Q_{z,t}; b_{d,t}) - c^{*}_{i,j,z,d,t}(A_{i,t}, H_{j,t}, Q_{z,t}) \le \lambda^{cons}_{i,j,z,d,t}$$

The probability of decision d :

$$f_{i,j,z,d,t} = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})}$$

where Φ^{+} denotes the approximation for the next period value function, as described in the next subsection.

1.10.6 AMPL

Backward Solution from Time T for Approximate Value Functions in AMPL

In order to formulate this problem in AMPL, we need to list every quantity that is computed.

The time-specific asset grids $A_{i,t}$ are fixed.

The parameters are

$$A_{i,t}, \beta, \eta_{i,t}, R, \psi^{own}, \psi^{rent}, \phi^{own}, \phi^{rent}, \theta_B$$

The basic variables of interest are

$$\begin{array}{l} c_{i,j,z,d^N,t},\ \ c_{i,d^{Mhq},t} \\ a_{k,j,z,t},\ \ b_{k,j,z,d,t} \\ \lambda^e_{i,j,z,t},\ \ \lambda^b_{j,z,t},\ \ \lambda^{env}_{j,z,t},\ \ \lambda^{cons}_{i,j,z,d,t} \end{array}$$

AMPL does not allow procedure programming; therefore, we need to define other variables to represent quantities defined in terms of other variables. We first need

$$\begin{aligned}
u_{i,j,z,d^N,t} &\equiv u\left(c^*_{i,j,z,d^N,t}, H_{j,t-1}\right) \\
u_{c;i,j,z,d^N,t} &\equiv u'\left(c^*_{i,j,z,d^N,t}, H_{j,t-1}\right) \\
A^+_{i,j,z,d^N,t} &\equiv RA_{i,t} - c^*_{i,j,z,d^N,t} - \psi_{j,z} + y \\
f_{i,j,z,d^N,t} &= \Pr(N|A_{i,t}, H_{j,t-1}, Q_{z,t-1}) \\
u_{i,d^{Mhq},t} &\equiv u\left(c^*_{i,d^{Mhq},t}, h_t\right) \\
u_{c;i,d^{Mhq},t} &\equiv u'\left(c^*_{i,d^{Mhq},t}, h_t\right) \\
A^+_{i,d^{Mhq},t} &\equiv RA_{i,t} - c^*_{i,d^{Mhq},t} - \psi_{hq} - M_{j,z,d^{Mhq}} + y \\
f_{i,d^{Mhq},t} &= \Pr(Mhq|A_{i,t}, H_{j,t-1}, Q_{z,t-1})
\end{aligned}$$

We next use those variables to build more variables

$$\begin{aligned}
\Phi_{i,j,z,t} &\equiv \Phi(A_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) \\
\Phi_{A;i,j,z,t} &\equiv \Phi'(A_{i,t}, H_{j,t-1}, Q_{z,t-1}; a_t) \\
\Phi^{+}_{i,j,z,d^N,t} &\equiv \Phi(A^{+}_{i,j,z,d^N,t}, H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \\
\Phi^{+}_{A;i,j,z,d^N,t} &\equiv \Phi'(A^{+}_{i,j,z,d^N,t}, H_{j,t-1}, Q_{z,t-1}; a_{t+1}) \\
\Phi^{+}_{i,d^{Mhq},t} &\equiv \Phi(A^{+}_{i,d^{Mhq},t}, h_t, q_t; a_{t+1}) \\
\Phi^{+}_{A;i,d^{Mhq},t} &\equiv \Phi'(A^{+}_{i,d^{Mhq},t}, h_t, q_t; a_{t+1}) \\
\Psi_{i,j,z,d,t} &\equiv \Phi(A_{i,t}, H_{j,t-1}, Q_{z,t-1}; b_{d,t})
\end{aligned}$$

With these variables defined, the Bellman equation error inequality becomes

$$-\lambda^b_{j,z,t} \leq \Phi_{i,j,z,t} - \ln\left\{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})\right\} \leq \lambda^b_{j,z,t}$$

where

$$\begin{aligned}
\widehat{V}_{i,j,z,d^N,t} &= u_{i,j,z,d^N,t} + \beta\eta_{t+1}\Phi^{+}_{i,j,z,d^N,t} \\
\widehat{V}_{i,d^{Mhq},t} &= u_{i,d^{Mhq},t} + \beta\eta_{t+1}\Phi^{+}_{i,d^{Mhq},t}
\end{aligned}$$

the Euler equation error inequalities become

$$\begin{aligned}
-\lambda^e_{i,j,z,t} &\leq u_{c;i,j,z,d^N,t} + \beta\Phi^{+}_{A;i,j,z,d^N,t} \leq \lambda^e_{i,j,z,t} \\
-\lambda^e_{i,j,z,t} &\leq u_{c;i,d^{Mhq},t} + \beta\Phi^{+}_{A;i,d^{Mhq},t} \leq \lambda^e_{i,j,z,,t}
\end{aligned}$$

and the Envelope error inequality becomes

$$-\lambda^{env}_{j,z,t} \leq \Phi_{A;i,j,z,t} - \{f_{i,j,z,d^N,t} \cdot \Phi^{+}_{A;i,j,z,d^N,t} + \sum_h \sum_q [f_{i,d^{Mhq},t} \cdot \Phi^{+}_{A;i,d^{Mhq},t}]\} \leq \lambda^{env}_{j,z,t}$$

The probability of decision d :

$$f_{i,j,d,t} = \frac{\exp(\widehat{V}_{i,j,z,d,t})}{\exp(\widehat{V}_{i,j,z,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t})}$$

The policy function errors are

$$-\lambda^{cons}_{i,j,z,d,t} \leq \Psi_{i,j,z,d,t} - c^{*}_{i,j,z,d,t} \leq \lambda^{cons}_{i,j,z,d,t}$$

Empirical Part in AMPL

We consider individuals in our sample such that $Age^{data}_{n,tp} = 1...(T-2)$.

Let $A^{data}_{n,tp}$, $Age^{data}_{n,tp}$, $H^{data}_{n,tp-1}$ and $Q^{data}_{n,tp-1}$ denote the data on non-housing financial assets, age, previous period house value and previous period housing tenure for household n in year tp in the panel data. Given these data, the variables of interest are:

$$c^{pred}_{d^N,n,tp} = \Psi_{d^N}(A^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; b_{Age^{data}_{n,tp}})$$

$$c^{pred}_{d^{Mhq},n,tp} = \Psi_{d^{Mhq}}(A^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; b_{Age^{data}_{n,tp}})$$

$$u^{pred}_{d^N,n,tp} \equiv u\left(c^{pred}_{d^N,n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}\right)$$

$$u_{c;d^N,n,tp} \equiv u'\left(c_{d^N,n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}\right)$$

$$A^{+}_{d^N,n,tp} \equiv RA^{data}_{n,tp} - c^{pred}_{d^N,n,tp} - \psi(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) + y$$

$$f^{pred}_{d^N,n,tp} = \Pr(N | A^{data}_{n,t}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp})$$

$$u^{pred}_{d^{Mhq},n,tp} \equiv u\left(c^{pred}_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}\right)$$

$$u_{c;d^{Mhq},n,tp} \equiv u'\left(c_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}\right)$$

$$A^{+}_{d^{Mhq},n,tp} \equiv RA^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) - M(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) + y$$

$$f^{pred}_{d^{Mhq},n,tp} = \Pr(Mhq|A^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp})$$

We next use those variables to build more variables

$$\Phi^{data}_{n,tp} \equiv \Phi(A^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}})$$

$$\Phi^{data}_{A;n,tp} \equiv (\Phi^{data})'(A^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}})$$

$$\Phi^{+}_{d^{N},n,tp} \equiv \Phi^{data}(A^{+}_{d^{N},n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})$$

$$\Phi^{+}_{A,d^{N},n,tp} \equiv \Phi'(A^{+}_{d^{N},n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})$$

$$\Phi^{+}_{d^{Mhq},n,tp} \equiv \Phi(A^{+}_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})$$

$$\Phi^{+}_{A;d^{Mhq},n,tp} \equiv \Phi'(A^{+}_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})$$

$$\begin{aligned}\widehat{V}^{pred}_{d^{N},n,tp} &= u(c^{pred}_{d^{NM},n,tp}, H^{data}_{n,tp}) \\ &\quad + \beta\Phi(RA^{data}_{n,tp} - c^{pred}_{d^{NM},n,tp} - \psi(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) + y; H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})\end{aligned}$$

$$\begin{aligned}\widehat{V}^{pred}_{d^{Mhq},n,tp} &= u(c^{pred}_{d^{Mhq},n,tp}, H^{choice}_{n,tp}) + \beta\Phi(RA^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) \\ &\quad - M(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) + y; H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})\end{aligned}$$

The probabilities of not moving and moving are:

$$f^{pred}_{d^{N},n,tp} = \Pr(H_{d^{N},n,tp}|A^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{N},n,tp})}{\exp(\widehat{V}^{pred}_{d^{N},n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}$$

$$f^{pred}_{d^{Mhq},n,tp} = \Pr(H_{d^{Mhq},n,tp}|A^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}{\exp(\widehat{V}^{pred}_{d^{N},n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}$$

The measurement error in consumption is normally distributed with mean 0 and variance σ^2:

$$\Pr(c_{n,tp}|d_{n,tp}, A^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(c^{data}_{n,tp} - c^{pred}_{d,n,tp})^2}{2\sigma^2}\right)$$

Figure 1·1: Survival Curves of HECM Loans for Single Males, Single Females, and Couples (Bowen Bishop and Shan, 2008)

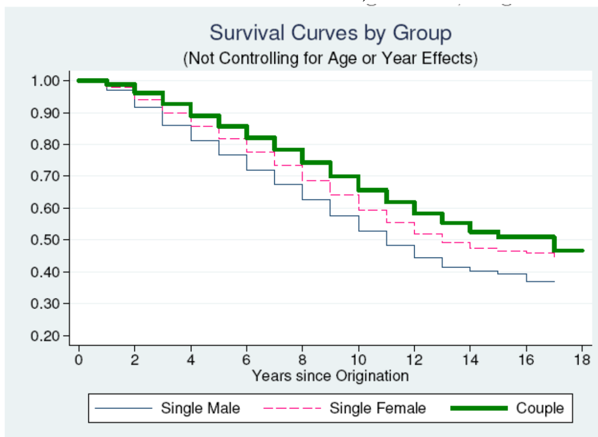


Table 1.1: Descriptive Statistics

	Percentiles			Min	Max	Mean
	25%	50%	75%			
H	\$40,000	\$70,000	\$92,000	\$ 2,500	\$170,000	\$71,000
A	\$5,000	\$17,500	\$63,000	\$0	\$276,548	\$45,950
H/A	0.86	2.5	7.5	0.11	1500	23.4
C	\$6,270	\$9,774	\$15,090	\$800	\$84,380	\$13,873
ss	\$6,972	\$9,468	\$11,340	\$0	\$ 24.701	\$9,087
Age	69	74	79	64	86	74

Figure 1·2: Termination Hazard Rates of HECM Loans for Single Males, Single Females, and Couples (Bowen Bishop and Shan, 2008)

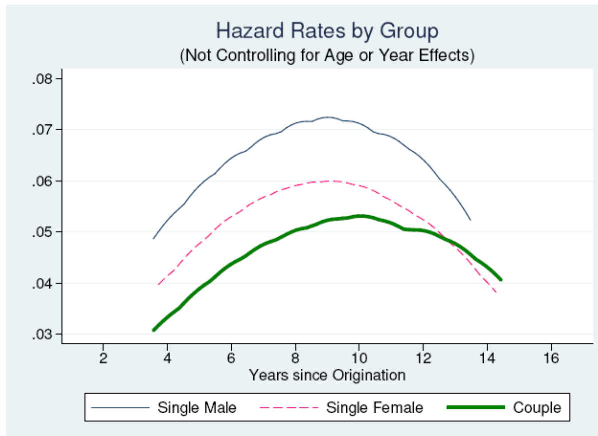


Figure 1·3: Consumption

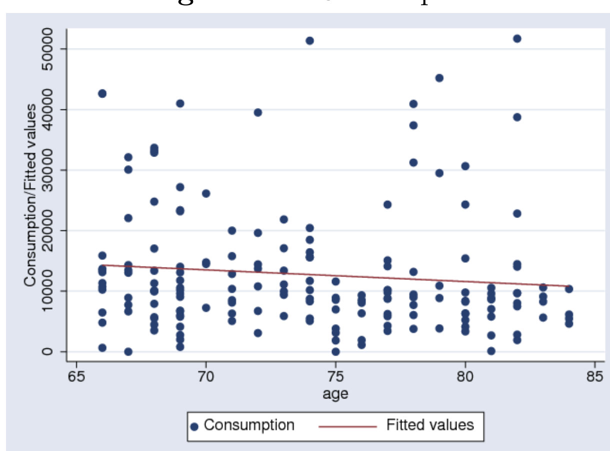


Table 1.2: Financial Portfolio Composition

	Percentiles			Min	Max
	25%	50%	75%		
Stocks	\$0	\$0	\$0	\$ 0	\$125,000
Chck	\$300	\$2,500	\$9,000	\$ 0	\$100,000
Cds	\$0	\$0	\$4,000	\$ 0	\$200,000
Tran	\$700	\$4,000	\$8,500	\$ 0	\$30,000
Bonds	\$0	\$0	\$0	\$ 0	\$80,000
IRA	\$0	\$0	\$1000	\$ 0	\$137,000
Debt	\$0	\$0	\$0	\$ 0	\$7,000

Figure 1·4: Social Security Income

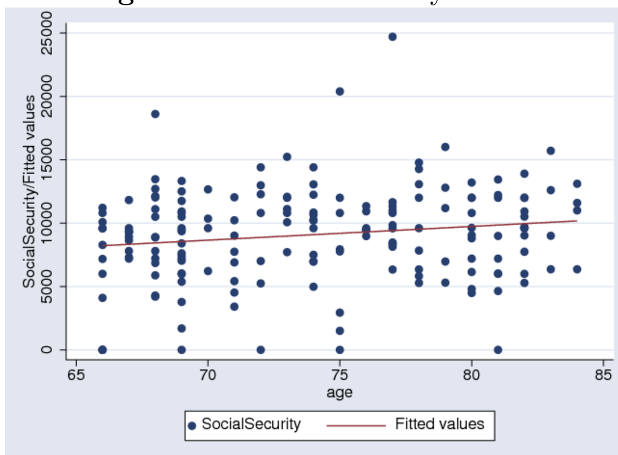


Figure 1·5: Non-housing Financial Assets

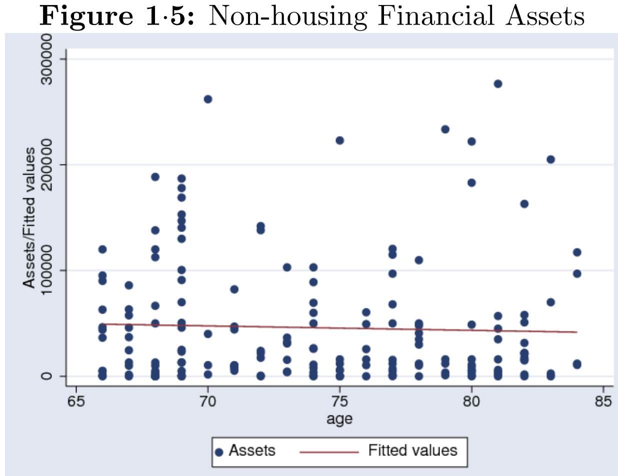


Table 1.3: Housing Choices when Moving

Housing Choices	(1)	(2)	(3)	(4)	(5)
Percentage of Households	50%	24%	13%	6.5%	6.5%

where:
(1) Buy a house of equal value
(2) Rent a house of equal value
(3) Buy a smaller house
(4) Rent a smaller house
(5) Rent a larger house

Figure 1·6: Housing

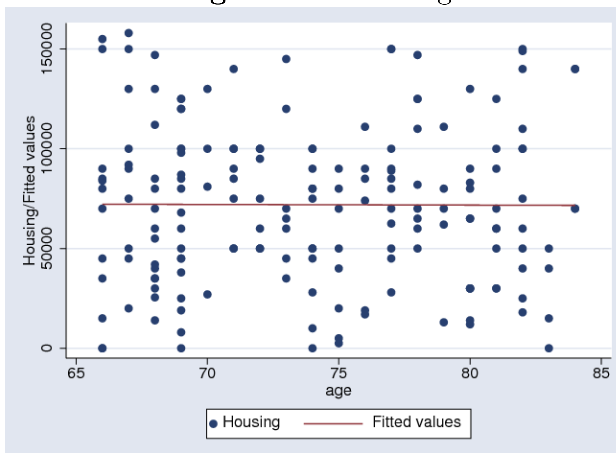


Table 1.4: Structural Estimation Results

Description	Parameter	Estimate
Coefficient of relative risk aversion	γ	3.87 (0.61)
Preference parameter over housing	ω	0.85 (0.04)
s.d. of measurement error in consumption	σ	0.87 (0.07)

Table 1.5: Median Non-Housing Financial Assets

		HOUSE	
	House-Poor	House-Medium	House-Rich
FINANCIAL ASSETS			
Cash-Poor	$1,000	$2,000	$2,250
Cash-Medium	$16,000	$28,000	$46,000
Cash-Rich	$120,000	$103,000	$135,250

Table 1.6: Median Welfare Gain

		HOUSE	
	House-Poor	House-Medium	House-Rich
FINANCIAL ASSETS			
Cash-Poor	-59%	-64%	-120%
Cash-Medium	-27%	30%	24%
Cash-Rich	85%	20%	47%

Table 1.7: Median Welfare Gain, No Moving Risk

	HOUSE		
	House-Poor	House-Medium	House-Rich
FINANCIAL ASSETS			
Cash-Poor	3,550%	7,804%	7,173%
Cash-Medium	243%	418%	496%
Cash-Rich	28%	115%	35%

Table 1.8: Median Welfare Gain, No Upfront Costs

	HOUSE		
	House-Poor	House-Medium	House-Rich
FINANCIAL ASSETS			
Cash-Poor	-40%	9%	-120%
Cash-Medium	17%	171%	219%
Cash-Rich	103%	98%	71%

Table 1.9: Median Welfare Gain, 10% Cut in Current Income

	HOUSE		
	House-Poor	House-Medium	House-Rich
FINANCIAL ASSETS			
Cash-Poor	-30%	-22%	-121%
Cash-Medium	-4%	184%	208%
Cash-Rich	94%	138%	23%

Table 1.10: Median Welfare Gain, No Bequest

	HOUSE		
	House-Poor	House-Medium	House-Rich
FINANCIAL ASSETS			
Cash-Poor	-21%	-30%	-120%
Cash-Medium	40%	190%	27%
Cash-Rich	103%	122%	64%

Chapter 2

Marrying for Money

2.1 Introduction

Ostensively, people marry for love. In fact money may be the driving factor. Marriage engenders economizing of resources, pooling risks and sharing wealth. How important are these factors? Does it pay to get married? This paper provides an economic view of marriage and determines what it is worth.

Across many different countries, marriage has historically been viewed as a source of financial security, as noted by Gallagher and Waite (2000). The life uncertainties make it hard to decide how much to spend for the current consumption and how much to save against unpredictable events. Spouses' explicit agreement, as a part of their marriage vow, of mutual support against life's unforeseen events acts as a kind of insurance policy. Of course, people can buy insurance from private companies. However, since these companies have to cover the costs of running the business, the private insurance policies are much more expensive than the same level insurance that results from marriage. In addition, within the family, there exists a level of trust and information that alleviates the key problems associated with the provision of insurance by public markets. Among those, moral hazard, adverse selection and deception. Important risks for which the market "public" problems can be particularly severe are the risk of job loss or change in earnings and the risk of disability. In particular, the public market is not always able to determine the extent to which an individual actually suffered an earning loss or became disabled. The family's role in providing insurance to family members has been explicitly considered in Schultz (1974), Becker (1973, 1981), Becker et al. (1977), Kotlikoff and Spivak (1981).

This paper is concerned with marriage as an implicit insurance contract against the risk of earning loss, of disability and of running out of consumption resources because of greater than average longevity. This contract is made ex-ante by completely selfish individuals who obtain utility only from their own consumption. Each individual works between age 30 and age 65 and then retires. We assume that each individual faces earning uncertainty in the working period and uncertain medical expenses after retirement. Along the entire life length, the date of death is uncertain. The labor income process, the medical expenditure process and the survival probabilities are ex-ante known. We calculate the spouses' gain from pooling their uninsurable risks of labor income, health expenditure and longevity through marriage. The prospect of bad realizations in future earnings, of out-of-pocket medical expenses, or of a longer life than average can influence the individuals' decisions about how fast to consume over time, about their wedding, and about their spouse. Throughout the paper, we simulate the gain from marriage as a result of the risk-sharing arrangements offered within the family. In the final paragraph we introduce the economies of shared living, to make our analysis more realistic. Our finding shows that even though the economies of shared living are the main determinant of the marriage gain, the risk-sharing arrangements play an important role. Focusing on risk sharing, we find that a 30-year-old representative spouse enjoys about a 15% higher consumption than if she stayed single. The marriage gain increases during the working period until age 45, mainly due to risk pooling opportunities against job loss risk, and then declines as the individual approaches retirement. After retirement, the gain from marriage increases again, reaching a peak at age 75, as families can self-insure against unexpected out-of-pocket medical expenses, nursing home costs and uncertain dates of death. We find that women experience higher marriage gain than men at each age. This reflects the fact that while both partners agree on equal consumption in an equally weighted marriage contract, the husband has lower survival probabilities and therefore is more likely to die and bequest first.

We compare the gain from marriage at different ages for both men and women with different education levels. We can explain the existing 'schooling homogamy' among highly educated people as a consequence of the behavior of rational individuals that at each age are likely to choose their spouse to maximize their expected lifetime utility and consumption. Love and affection can be important, but "there are fewer Cinderella marriages these days," as noted by Coontz (2005). "Men are less interested in rescuing a woman from poverty. They want to find someone who will pull her weight." From the vast literature on marriage and assortative mating, Mare (1991), Pencavel (1998), and Mancuso and Pencavel (1999) deal specifically with schooling homogamy. In our model, a 30-year-old male college graduate who marries a woman without high school faces a decrease in his consumption by about 4% than if he stayed single, while he can experience a consumption increase by about 7% from marrying a college graduate. For a 30-year-old female college graduate the marriage gain is about 7.5% and 21% respectively from marrying a man without high school and a college graduate. As noted before, the difference in male and female marriage gain reflects only the difference in survival probabilities for identical education levels.

The model cannot be solved analytically; therefore we develop a dynamic programming model in which single individuals and families face longevity, earning, and health uncertainty. The simulation analysis is conducted in partial equilibrium, and factor prices (wages and interest rates) are assumed constant over time. Partial equilibrium analyses can overstate the associated general equilibrium results but can give a first impression of the gain from marriage when individuals face multiple uncertainties.

The paper is organized as follows.

Section 1 introduces our life-cycle model for a single individual and for a couple. Section 2 describes the gain from marriage calculation. Section 3 discusses the numerical solution and Section 4 the calibration. Section 5 presents the simulation results. Section 6 concludes.

2.2 A Life-Cycle Model

2.2.1 The Individual's Consumption Plan

We consider a representative individual that faces earning, life span and health expenditure uncertainty. Time is discrete and each period t corresponds to one year. The retirement age is set exogenously and equal to age 65. T is the maximum longevity and it is set equal to 95. For age $t = \{30, \ldots, 65\}$, the individual faces earning shocks $Y_t^i \in E_t^i = \{Y_t^{i,\min}, ..., Y_t^{i,\max}\}$. At the beginning of each period, before observing the current period earning shock , the representative individual chooses her consumption $\{C_t^i\}_{t=30}^{65}$. This choice is conditioned on her history, which includes her initial endowment of wealth and accumulated assets. At the end of each period, the individual observes the current period earning shock. For the age $t = \{66, \ldots, 95\}$ the individual faces longevity and health shocks. Her wealth is determined by the accumulated assets and the associated interests, less medical expenses. The representative individual choice of her consumption $\{C_t^i\}_{t=66}^{95}$ is conditioned on her history, accumulated assets and medical expenses.

The individual i's choice problem is to decide on the path of her control variable $\{C_t^i\}_{t=30}^{95}$ in order to maximize the expected discounted sum of her current and future utility, which can be written as follows:

$$\max_{\{C_t^i\}_{t=30}^{95}} E \sum_{t=30}^{95} \beta^{t-30} \left(\prod_{j=0}^{t-2} Q_j^i \right) u(C_t^i) \tag{2.1}$$

where $\beta = 1/(1+\rho)$ is the rate of time preference and Q_t^i is individual i's probability of survival from period zero until period t. We assume that individual i's preferences are represented by a time-separable iso-elastic utility function:

$$u(C_t^i) = \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \tag{2.2}$$

2.2.2 The Family's Consumption Plan

We consider a representative family, composed of two individuals who both work from the age of 30 until the age of 65, and then they retire. For age $t = \{30, \ldots, 65\}$, each family member faces earning shocks $Y_t^{f_i} \in E_t^{f_i} = \{Y_t^{f_i,\min}, ..., Y_t^{f_i,\max}\}$, with $i = \{1, 2\}$, while for the age $t = \{66, \ldots, 95\}$ each family member faces health shocks. During their entire life, the individuals face longevity uncertainty. At the beginning of each period the representative family chooses her consumption $\{C_t^f\}_{t=30}^{95}$, which is the sum of both family member's consumption $\{C_t^{f_1}, C_t^{f_2}\}_{t=30}^{95}$. The choice is conditioned on the family's history, which includes each member's initial endowment of wealth and the accumulated assets. At the end of the period, each family member observes her current period earning shock. For age $t = \{66, \ldots, 95\}$ the family's wealth is determined by accumulated assets and associated interests, less medical expenses.

The family's choice problem is to decide on the path of her control variables $\{C_t^{f_1}, C_t^{f_2}\}_{t=30}^{95}$ in order to maximize the expected discounted weighted sum of each family member's current and future utility.

Both family members have the same iso-elastic utility function:

$$u(C_t^{f_i}) = \frac{(C_t^{f_i})^{1-\gamma}}{1-\gamma} \qquad for \qquad i = 1, 2 \tag{2.3}$$

The current period family's consumption choice problem is described as follows:

$$\max_{\{C_t^{f_1}, C_t^{f_2}\}} Q_t^{f_1} Q_t^{f_2} [u(C_t^{f_1}) + \theta u(C_t^{f_2})] + Q_t^{f_1}(1 - Q_t^{f_2}) u(C_t^{f_1}) + (1 - Q_t^{f_1}) Q_t^{f_2} u(C_t^{f_2}) \tag{2.4}$$

subject to:

$$C_t^f = C_t^{f_1} + C_t^{f_2} \tag{2.5}$$

where $Q_t^{f_1}$ and $Q_t^{f_2}$ are respectively the male and female survival probabilities. $C_t^{f_1}$ and $C_t^{f_2}$ are the husband's and the wife's consumptions. C_t^f is the family's consumption. θ is the differential weight applied to the wife's expected utility. Throughout the paper we assume that θ equals 1, that is an equal consumption (equal weighting) marriage contract.

We rewrite this problem as follows:

$$\max_{\{C_t^f\}} Q_t^{f_1} Q_t^{f_2} U(C_t^f) + Q_t^{f_1}(1 - Q_t^{f_2}) u(C_t^f) + (1 - Q_t^{f_1}) Q_t^{f_2} u(C_t^f) \tag{2.6}$$

given that:

$$U(C_t^f) = \max_{\{C_t^{f_1}, C_t^{f_2}\}} (u(C_t^{f_1}) + \theta u(C_t^{f_2})) \qquad \textit{subject to} \qquad C_t^f = C_t^{f_1} + C_t^{f_2} \tag{2.7}$$

The FOCs associated with (2.7) are:

$$C_t^{f_1} = \frac{C_t^f}{1 + \theta^{1/\gamma}} \qquad \textit{and} \qquad C_t^{f_2} = \frac{C_t^f \theta^{1/\gamma}}{1 + \theta^{1/\gamma}} \tag{2.8}$$

It follows that:

$$U(C_t^f) = \frac{(C_t^f)^{1-\gamma}}{1 - \gamma} (1 + \theta^{1/\gamma})^{\gamma} \tag{2.9}$$

2.2.3 The Labor Income Process

We consider the labor income process, as described in Cocco, Gomes and Maenhout (2005).

Before retirement ($t < 65$), age-t labor income is exogenously given by the sum of a deterministic component and two random components. The deterministic component $f(t)$ is a function of age and is calibrated to capture the hump shape of income over the life cycle. The random components are one permanent v_t^i and one transitory ε_t^i.

$$\log(Y_t^i) = f(t) + v_t^i + \varepsilon_t^i \tag{2.10}$$

The process for the permanent random component v_t^i is a random walk, as described by the following equation[1]:

$$v_t^i = v_{t-1}^i + u_t^i \tag{2.11}$$

The permanent shock u_t^i and is distributed as $N(0, \sigma_u^2)$ and is uncorrelated with ε_t^i. We assume that the transitory shock is distributed as $N(0, \sigma_\varepsilon^2)$.

A more realistic labor income process should take into account also catastrophic shocks, as described in Gomes, Kotlikoff, and Viceira (2007), according to which the transitory shock is distributed as:

$$\begin{cases} N(0, \sigma_\varepsilon^2) & \textit{with probability } (1-\pi) \\ \ln(0.1) & \textit{with probability } \pi \end{cases} \tag{2.12}$$

Assuming this process for the transitory shock, we include in the model the probability of a large negative income shock as in Heaton and Lucas (1997), Caroll (1992, 1997), Deaton (1991).

2.2.4 Medical Expenses

We assume that medical expenses include both out-of-pocket health care expenditures $G(t, Z_t^i)$ and potential nursing home costs $D(t, Z_t^i)$.

$$M_t = G(t, Z_t^i) + s_t^i D(t, Z_t^i) \tag{2.13}$$

[1] Caroll (1997), Gourinchas and Parker (2002) used the same assumption about the permanent random component. Hubbard, Skinner and Zeldes (1995) estimate a general first-order autoregressive process for three categories of education of the family head: less than twelve years of schooling (no high school degree), between twelve and fifteen years (with high school degree) and sixteen years or more (college degree). They find a value for the autocorrelation coefficient close to one.

The process for out-of-pocket health care expenses for the retired individual is taken from Scholz, Seshandri, and Khitatrakun (2006)

$$\begin{aligned} G(t, Z_t^i) &= \beta_0 + \beta_1 Age_t + \beta_2 Age_t^2 + \eta_t \\ \eta_t &= \rho\eta_{t-1} + \zeta_t \\ \zeta_t &\sim N(0, \sigma_\zeta^2) \end{aligned} \tag{2.14}$$

where Age_t is the age of the individual at time t, η_t is an AR(1) error term and ζ_t is white noise.

The process for potential nursing home costs is taken from Luo (2006). Each period the representative individual has a certain probability of incurring nursing home admission. s_t^i denotes if the individual is under nursing home services. Let $s_t^i \in \{0, 1\}$ be a binary variable which takes value 1 in the case of nursing home admission at time t.

$$s_t^i = \begin{cases} 1, & with\ probability\ \delta(t, h_t^i, Z_t^i) \\ 0, & with\ probability\ (1 - \delta(t, h_t^i, Z_t^i)) \end{cases} \tag{2.15}$$

where h_t^i is a discrete variable which represents the individual's health status. $h_t^i \in \{good, fair, poor\}$ follows a Markov process.

The probability of nursing home admission is assumed to be a function of the individual's age, health status and personal characteristics, and subject to the logistic distribution:

$$prob(s_t^i = 1) = \frac{\exp(\vartheta' x_t^i)}{1 + \exp(\vartheta' x_t^i)} \tag{2.16}$$

Where $prob(s_t^i = 1)$ is $\delta(t, h_t^i, Z_t^i)$, ϑ is a vector of estimated coefficients and x_t^i is the vector of independent variables.

Luo (2006) runs a regression to calibrate nursing home cost $D(t, Z_t^i)$. She finds that the lenght of nursing home stay conditional on entry does not depend on age, health status or personal characteristics. Following Luo (2006), we assume that a nursing home admission implies an expected cost based on the average length of stay in a year, which is 6.37 months. This cost is then calculated following Palumbo(1999):

$$D = (1-n)n(c^{ac} + c^{nh}) + n\left(\frac{n}{2}\right)(c^{ac} + c^{nh}) - Medicare\,Benefit \tag{2.17}$$

Where D is the total cost sustained in year t, n denotes the fraction of the year spent in a nursing home, c^{ac} is the average cost of one year of acute care received in a nursing home, c^{nh} is the average cost of one year of nursing home care. We assume that the individuals do not receive $MedicareBenefit$.

2.2.5 The Individual's Optimization Problem

In order to take into account the different sources of uncertainty, we split the individual's economic problem into three time periods: the after retirement period $t = \{66, \ldots, 95\}$, the last working year $t = 65$ and the working period $t = \{30, \ldots, 64\}$. We solve it recursively, under the assumption that $A_{96} = 0$.

For the age $t = \{66, \ldots, 95\}$, the individual faces longevity and health uncertainty. The individual's recursive problem can be written as:

$$V_t^i(A_t^i, \eta_{t-1}^i, h_{t-1}^i) = \max_{C_t^i} \left\{ u(C_t^i) + \beta Q_{t+1}^i E_t V_{t+1}^i(A_{t+1}^i, \eta_t^i, h_t^i) \right\} \tag{2.18}$$

subject to

$$A_{t+1}^i = (1+r) \cdot (A_t^i - C_t^i - M_t^i) \tag{2.19}$$

$$0 \leq C_t^i \leq A_t^i \qquad and \qquad A_{96}^i = 0 \tag{2.20}$$

where $V_t^i(A_t^i, \eta_{t-1}^i, h_{t-1}^i)$ is period t maximum expected utility for individual i. The state variables h_{t-1}^i and η_{t-1}^i are respectively individual i's period $(t-1)$ health status and period $(t-1)$ out-of-pocket shock. $A_t^i \in \{A^{i,\min}, ..., A^{i,\max}\}$ is period t individual beginning-of-period wealth. M_t^i is the current period medical expenses. $Q_{t+1}^i$is the probability of survival from period zero until period $(t+1)$ and r is the interest rate. We impose a non-borrowing constraint, according to which the individual's current consumption cannot exceed the current available wealth.

In the last working year $t = 65$ there are no medical expenses. We assume that after retirement income does not depend on earnings in the last working year. Therefore, the permanent component of the labor income does not affect the value function in the next period. The level of assets is the only state variable

$$V_t^i(A_t^i) = \max_{C_t^i} \left\{ u(C_t^i) + \beta Q_{t+1}^i E_t V_{t+1}^i(A_{t+1}^i, \eta_t^i, h_t^i) \right\} \tag{2.21}$$

subject to

$$A_{t+1}^i = (1+r) \cdot (A_t^i - C_t^i) \tag{2.22}$$

$$0 \leq C_t^i \leq A_t^i \tag{2.23}$$

For $t = \{30, \ldots, 64\}$, the individual faces longevity and earning uncertainty. The state variables are A_t^i and v_{t-1}^i, where v_{t-1}^i is period $(t-1)$ permanent random component of the individual's labor earning.

The Bellman equation for the dynamic problem associated with the individual's choice problem is given by:

$$V_t^i(A_t^i, v_{t-1}^i) = \max_{C_t^i} \left\{u(C_t^i) + \beta Q_{t+1}^i E_t V_{t+1}^i(A_{t+1}^i, v_t^i)\right\} \tag{2.24}$$

subject to

$$A_{t+1}^i = (1+r) \cdot (A_t^i - C_t^i) + Y_t^i \tag{2.25}$$

$$0 \leq C_t^i \leq A_t^i \tag{2.26}$$

2.2.6 The Family's Optimization Problem

We split the family's economic problem into three time periods: after retirement $t = \{66, \ldots, 95\}$, the last working year $t = 65$ and the working period $t = \{30, \ldots, 64\}$. We solve it recursively, under the assumption that $A_{96} = 0$.

For the age $t = \{66, \ldots, 95\}$, the family's recursive problem can be written as:

$$\begin{aligned} V_t^f(A_t^f, \eta_{t-1}^{f_1}, \eta_{t-1}^{f_2}, h_{t-1}^{f_1}, h_{t-1}^{f_2}) \quad = \quad & \max_{C_t^f}\{u(\phi C_t^f) \\ & +\beta Q_{t+1}^{f_1} Q_{t+1}^{f_2} E_t V_{t+1}^f(A_{t+1}^f, \eta_t^{f_1}, \eta_t^{f_2}, h_t^{f_1}, h_t^{f_2}) \\ & +\beta Q_{t+1}^{f_1}(1 - Q_{t+1}^{f_2}) E_t V_{t+1}^{f_1}(A_{t+1}^f, \eta_t^{f_1}, h_t^{f_1}) \\ & +\theta\beta Q_{t+1}^{f_2}(1 - Q_{t+1}^{f_1}) E_t V_{t+1}^{f_2}(A_{t+1}^f, \eta_t^{f_2}, h_t^{f_2})\} \end{aligned} \tag{2.27}$$

subject to

$$A_{t+1}^f = (1+r) \cdot (A_t^f - C_t^f - M_t^{f_1} - M_t^{f_2}) \tag{2.28}$$

$$0 \leq C_t^f \leq A_t^f \qquad and \qquad A_{96}^f = 0 \tag{2.29}$$

where $V_t^f(A_t^f, \eta_{t-1}^{f_1}, \eta_{t-1}^{f_2}, h_{t-1}^{f_1}, h_{t-1}^{f_2})$ is the period t maximum expected weighted utility of the two family members and $V_t^{f_i}(A_t^f, \eta_{t-1}^{f_i}, h_{t-1}^{f_i})$ for $i = 1, 2$ is the maximum expected utility of family member i if he or she survives alone until period t and is obtained from equation (2.3). We introduce the parameter ϕ in order to take into account the economies of shared living.

In the last working year $t = 65$ there are no medical expenses and the permanent component of the labor income does not affect the value function in the next period. Therefore the level of assets is the only state variable.

$$\begin{aligned} V_t^f(A_t^f) &= \max_{C_t^f}\{u(\phi C_t^f) \\ &\quad +\beta Q_{t+1}^{f_1} Q_{t+1}^{f_2} E_t V_{t+1}^f(A_{t+1}^f, \eta_t^{f_1}, \eta_t^{f_2}, h_t^{f_1}, h_t^{f_2}) \\ &\quad +\beta Q_{t+1}^{f_1}(1 - Q_{t+1}^{f_2}) E_t V_{t+1}^{f_1}(A_{t+1}^f, \eta_t^{f_1}, h_t^{f_1}) \\ &\quad +\theta\beta Q_{t+1}^{f_2}(1 - Q_{t+1}^{f_1}) E_t V_{t+1}^{f_2}(A_{t+1}^f, \eta_t^{f_2}, h_t^{f_2})\} \end{aligned} \tag{2.30}$$

subject to

$$A_{t+1}^f = (1 + r) \cdot (A_t^f - C_t^f) \tag{2.31}$$

$$0 \leq C_t^f \leq A_t^f \tag{2.32}$$

For the age $t = \{30, \ldots, 64\}$, each family member faces life span and earning uncertainty. Let $S_t^f = (A_t^f, v_{t-1}^{f_1}, v_{t-1}^{f_2})$ denote the state vector of family f at age t, where $A_t^f \in \{A^{f,\min}, ..., A^{f,\max}\}$ is the family beginning-of-period wealth, $v_{t-1}^{f_i}$ for $i = \{1, 2\}$ period $(t - 1)$ permanent random component of each family's member labor earning and period $(t - 1)$ health shock.

The Bellman equation for the dynamic problem associated with the family's choice problem is given by:

$$V_t^f(A_t^f, v_{t-1}^{f_1}, v_{t-1}^{f_2}) = \max_{C_t^i}\{u(\phi C_t^i) + \beta Q_{t+1}^{f_1} Q_{t+1}^{f_2} E_t V_{t+1}^f(A_{t+1}^f, v_t^{f_1}, v_t^{f_2}) + \beta Q_{t+1}^{f_1}(1 - Q_{t+1}^{f_2}) E_t V_{t+1}^{f_1}(A_{t+1}^f, v_t^{f_1}) + \theta\beta Q_{t+1}^{f_2}(1 - Q_{t+1}^{f_1}) E_t V_{t+1}^{f_2}(A_{t+1}^f, v_t^{f_2})\} \tag{2.33}$$

subject to

$$A_{t+1}^f = (1+r)\cdot(A_t^f - C_t^f) + Y_t^{f_1} + Y_t^{f_2} \tag{2.34}$$

$$0 \leq C_t^f \leq A_t^f \tag{2.35}$$

2.3 Gain From Marriage

The gain from marriage is measured as the percentage increase in the single individual's initial wealth that is required to make her as well off as if she were a member of a family. The welfare calculations are done in the form of consumption-equivalent variations: for each rule chosen respectively by the single individual and by the married individual we determine the constant consumption stream that makes the individual as well off in expected utility terms as the consumption stream that can be financed by the consumption rule.

For the single individual, we first solve the optimal consumption problem, denoting the optimal consumption stream by $\{C_t^R\}_{t=30}^{95}$. The subscript R indexes the individual optimal consumption rule followed. The expected lifetime utility from implementing $\{C_t^R\}_{t=30}^{95}$ is as follows:

$$V^R = E\sum_{t=30}^{95}\beta^{t-30}\left(\prod_{j=0}^{t-2}Q_j^i\right)\frac{(C_t^R)^{1-\gamma}}{1-\gamma} \tag{2.36}$$

V^R represents the maximal lifetime utility for an individual who uses the consumption rule $\{C_t^R\}_{t=30}^{95}$ throughout her life. We can then compute the equivalent consumption stream $EC^R \equiv \{\overline{C}^R\}_{t=30}^{95}$ that leaves the individual indifferent between EC^R and between the consumption stream attained from implementing the consumption rule $\{C_t^R\}_{t=30}^{95}$.

$$V^R = E\sum_{t=30}^{95}\beta^{t-30}\left(\prod_{j=0}^{t-2}Q_j^i\right)\frac{(\overline{C}^R)^{1-\gamma}}{1-\gamma} \tag{2.37}$$

By comparing the last two equations, we obtain:

$$\overline{C}^R = \left[\frac{(1-\gamma)V^R}{\sum_{t=30}^{95}\beta^{t-30}\left(\prod_{j=0}^{t-2}Q_j^i\right)}\right]^{\frac{1}{1-\gamma}} \tag{2.38}$$

We proceed analogously in the case of the family. We first calculate the optimal family consumption rule $\{C_t^{f,R}\}_{t=30}^{95}$ and the associated expected lifetime utility from implementing the rule. We then obtain the equivalent consumption stream $EC^{f,R} \equiv \{\overline{C}^{f,R}\}_{t=30}^{95}$ that leaves the married individual indifferent between $EC^{f,R}$ and the consumption stream attained from implementing the consumption rule $\{C_t^{f,R}\}_{t=30}^{95}$.

The gain from marriage for each individual i is computed as the percentage gain in equivalent consumption when choosing to get married and adopting the consumption rule $\{C_t^{f,R}\}_{t=30}^{95}$ rather than staying single and adopting the consumption rule $\{C_t^R\}_{t=30}^{95}$.

$$Gain\,from\,marriage = \frac{\overline{C}^{f,R}-\overline{C}^R}{\overline{C}^R} = \frac{(V^{f,R})^{\frac{1}{1-\gamma}}-(V^R)^{\frac{1}{1-\gamma}}}{(V^R)^{\frac{1}{1-\gamma}}} \tag{2.39}$$

2.4 Numerical Solution

The dynamic programming problem is solved by iterating on the value function. For the retired single individual, the state space is composed of four variables. These are age t, asset A, out-of-pocket medical expenses η, health status h. For the retired couple the state space involves six variables: age t, family asset A, each family member's out-of-pocket medical expenses η and health status h. We 'discretize' the state space. We construct an equally spaced 20-point grid for the assets, a 10-point grid for each individual's out-of-pocket medical expenses and a 3-point grid for each individual's health status. We approximate the density function for the innovation to the out-of-pocket medical expenditure using gaussian quadrature methods. The transition matrix for the health shock is taken from Luo (2006). For the working single individual the state space is composed by three variables: age t, asset A, permanent component of the labor income v. For the working couple the state space is composed of four variables: age t, family asset A and each family member's permanent component of the labor income v. The density function for both innovations to each individual's labor income process is approximated using gaussian quadrature to perform the numerical integration. We assume that husband and wife labor income shocks are uncorrelated. The model is solved using backward induction. We start at age T, assumed to be 95, and we compute the value function for the single individual $V_T^i(A_T^i, \eta_T^i, h_T^i)$ and for the couple $V_T^f(A_T^f, \eta_T^{f_1}, \eta_T^{f_2}, h_T^{f_1}, h_T^{f_2})$ associated with all the possible states in the discretized set. We move backward and solve for the decision rule for the assets, and hence for consumption, in all the periods, starting from age 30.

2.5 Calibration

2.5.1 Parameters

This section summarizes how we calibrate the model to the U.S. economy [2]

[2] Even though the product between β and $(1+r)$ is greater than 1, the Euler inequality $\beta(1+r)\frac{u'(c_{t+1})}{u'(c_t)} \leq 1$ is satisfied.

Table 2.1: Calibrated Parameters

Description	Parameter	Value
Discount factor	β	0.96
Risk aversion	γ	2
Coefficient for economies of shared livine	ϕ	1
Interest rate	r	0.065
Riskfree Rate	R_F	2%
Differential weight applied to family member f_2 utility	θ	1

Table 2.1 shows the key parameters used in our experiments.

2.5.2 Survival Probabilities

The survival probabilities are the male and female survival rates reported in the Social Security Administration Actuarial Study No.116 for the year 1999.

2.5.3 Labor Income Process

The labor income process analyzed is described in Gomes, Kotlikoff, and Viceira (2007). Their analysis is based on Cocco, Gomes and Maenhout (2005), who estimate age profiles for three different education groups: households without high school education, household with high school education and college graduates. In our baseline simulations, we take the weighted average of the three. The values of σ_u^2 and σ_ε^2 are 10.95% and 13.89% respectively. In simulating the marriage gain for different education levels we use the three distinct processes described in Cocco, Gomes and Maenhout (2005). The probability of a large negative income shock π equals 2%.

2.5.4 Medical Expenses

The process for out-of-pocket medical expenses for the retired individual is taken from Scholz, Seshandri, and Khitatrakun (2006). They estimate age profiles for two different education groups, with and without college degree. In the baseline case we take the weighted average of the two. The values of ρ and σ_ϑ^2 are respectively 0.838 and 0.5635. In our simulations of the marriage gain by education level we use the two distinct processes. The process for nursing home expenses is taken from Luo (2006). The probability of nursing home admission $\delta(t, h_t^i, Z_t^i)$ results from a logistic regression of nursing home admission. The health transition matrix is given by:

$$T_{k,j} = \begin{pmatrix} 0.823 & 0.142 & 0.035 \\ 0.327 & 0.486 & 0.187 \\ 0.13 & 0.296 & 0.574 \end{pmatrix}$$

Where $k = 1, 2, 3$ represents $h_{t-1}^i = good$, $h_{t-1}^i = fair$, $h_{t-1}^i = poor$ respectively and $j = 1, 2, 3$ represents $h_t^i = good$, $h_t^i = fair$, $h_t^i = poor$ respectively.

Rivlin and Wiener (1988) estimated the average daily nursing home costs and the average monthly acute care expenses and obtain c^{ac} and c^{nh} to be equal to \$14,381 and \$616.

2.6 Results

This section presents the results. Table 2.2 has six panels, each of which shows the marriage gain at different ages. In particular, Panel A reports the gain from marriage in the baseline scenario. In Panel B we assume that both family members have identical survival probabilities. Panel C and Panel D consider changes in the volatility to shocks respectively to permanent labor income and to out-of-pocket medical expenses. Panel E shows the marriage gain for women and men. Panel F presents the marriage gain when the wife is five years older than her husband.

Panel A presents the baseline results. The labor income process and the out-of-pocket medical expenditure process are a weighted average of the processes estimated respectively by Cocco, Gomes, and Maenhout (2005) and Scholz, Seshandri, and Khitatrakun (2006). The single individual's survival probabilities Q_{t+1}^i in equations (2.18), (2.21), (2.24) are a weighted average of the male and female survival probabilities. The gain from marriage curve exhibits an inverted hump-shaped pattern, with two humps. Starting from a value equal to 14.49% at age 30, it grows until age 45, when it falls until age 65. It increases again and reaches a peak at age 75. The first hump reflects the risk-sharing opportunities provided by marriage against earning uncertainty, the second hump shows the risk-sharing opportunities offered against uncertain medical expenses. Along the entire length of life the individuals face longevity uncertainty and the death-risk-pooling opportunities can be quite important, especially at old ages. As a matter the fact, as one becomes older, the uncertainty about the date of death is much greater. The baseline analysis shows that for 45-year-old representative individual pooling risks through marriage is equivalent to about a 30% increase in her consumption than if stayed single. For a 75-year-old the marriage gain is equivalent to increasing one's consumption by about 20%.

Panel B reports the results when we assume that both family members have identical survival probabilities. The results are close to the baseline case; however the marriage gain is slightly higher when both family members have male survival probabilities than when they have female survival probabilities. The difference in the gains between these two cases is particularly relevant at the end of the individuals' life, given that male survival probabilities are lower than female.

Panel C shows the marriage gain as we vary the volatility to shocks to permanent income σ_u. With an increase in the volatility to these shocks, each period earning becomes more uncertain and therefore the risk-sharing opportunities provided by marriage are more relevant. A 45-year-old representative individual can increase her consumption by an amount equivalent to 41% than if she stayed single. This corresponds to about a 13% higher gain than in the baseline scenario.

Panel D shows the marriage gain when we vary the volatility of shocks to out-of-pocket medical expenses. Symmetrical to the previous case, an increase in the volatility of these shocks makes the after-retirement cash on hand more uncertain and increases the after-retirement marriage gain. In particular, a 75-year-old representative individual has a marriage gain equal to 26.94% when σ_ζ equals 0.865, which is about 6% higher than in the baseline case.

Gender is another relevant variable. Panel E reports the marriage gain respectively for men and women. We assume the baseline labor income and out-of-pocket medical expenses processes. However, the single survival probabilities Q^i_{t+1} for $t = 30, \ldots, 95$, in equations (2.18), (2.21), (2.24) are now the male survival probabilities in the former case and the female survival probabilities in the latter. Under our assumption of equal age family members, the wife has a marriage gain higher than the husband does at each age. As a matter the fact, marrying an individual with a higher survival rate (the wife) to one with a lower survival rate (the husband) and assuming an equal consumption (equal weighting) marriage contract, would leave the former slightly better off and the latter slightly worse off than in the baseline case. The difference between male and female marriage gain, even though present at young ages, is particularly important at the end of life, as the husband is much more likely to die and bequest first. The risk-pooling opportunities through marriage guarantee the 30-year-old wife a 20.96% increase and the 30-year-old husband a 7.32% increase of their respective consumption than if they stayed single. For a 75-year-old woman the marriage gain is 58.24%, while the husband faces a loss of 9.71%.

Panel F shows the marriage gain for each spouse under the assumption that the wife is five years older than the husband. We assume the baseline labor income and out-of-pocket medical expenses processes. In addition, we assume that both spouses work for 35 years and then they retire. In this case, the husband faces a positive and significant after-retirement marriage gain. The wife's marriage gain decreases with respect to the baseline female gain, but it is still positive. In addition, the two spouses' gains are closer to each other and therefore both spouses' incentive to leave the marriage contract declines.

2.6.1 The Gain From Marriage by Gender and Education

Our model allows us to calculate the gain from marriage according to the spouse's gender and education. In simulating the marriage gain for different education groups, we use the labor income and the out-of-pocket medical expenses processes as estimated respectively by Cocco,

Table 2.2: Gain From Marriage for the Representative Individual

	Age				
	30	45	60	75	90
A.Baseline	14.49%	28.68%	23.85%	20.77%	2.48%
B.Identical Survival Probabilities					
Male Survival Probabilities	14.55%	28.31%	22.66%	37.47%	12.57%
Female Survival Probabilities	12.49%	24.49%	19.12%	19.08%	6.8%
C. Labor Shocks					
σ_u=13%	19.75%	41.55%	25.11%	17.4%	1.21%
σ_u=7%	10.54%	18.37%	22.87%	28.93%	5.12%
D. Health Shocks					
σ^2_ς=0.865	15.08%	28.68%	24.1%	26.94%	2.85%
σ^2_ς=0.26	14.49%	28.67%	23.72%	15.64%	1.92%
E. Gender					
Female (Baseline)	20.96%	37.64%	37.92%	58.24%	29.11%
Male (Baseline)	7.32%	17.81%	7.19%	-9.71%	-25.62%
F. Different Age					
Female, 5 years older than her husband	16.04%	29%	25.41%	23.41%	3.22%
Male, 5 years younger than his wife	10.17%	21.88%	14.79%	16.17%	11.35%

Gomes and Maenhout (2005) and by Scholz, Seshandri, and Khitatrakun (2006). When the gender is taken into consideration, we use distinct survival probabilities in equations (2.18), (2.21), and (2.24).

Our results can be useful in explaining the existing schooling homogamy among college graduates (i.e., the correlation between the wife's and the husband's schooling), even though for each individual, regardless of her education level and gender, the best partner is a college graduate. In fact, college graduates have the highest potential earnings and lowest potential medical expenditure. There is a vast literature on marriage and assertive mating, but we limit the review to two recent papers that deal specifically with schooling homogamy. U.S. Census and Current Population Survey (CPS) data are exploited in both of them. Mare (1991) analyzes the probability that spouse education levels differ at various education levels using a "crossing model". He finds that between the 1930s and the 1970s the association between spouses' schooling grew, while in the 1980s it was stable or decreased He motivated this trend as a consequence of the time gap between marriage and schooling. During the period 1930-1960, the gap decreased following a decline in age for first marriage and an increase in educational attainment. On the other side, it increased in the 1970s and 1980s because of an increase in age at marriage. Mare observes that highly educated individuals are likely to marry individuals with the same schooling level. Unlike Mare, Pencavel (1998) finds an increase in schooling homogamy during the period 1960-1990. He explains this different result considering the availability of data from the 1990 Census. He attributes his finding to the increase in labor force participation of wives. In an assortative mating framework, this could lead to an increased emphasis placed by the husband on the wife's potential earnings, and therefore on her education.

In particular, we find that each individual, regardless of her educational level and gender, obtains a higher marriage gain from marrying a more educated individual and a lower marriage gain from marrying a less educated individual than in the baseline case. The greater the difference in the level of education of the two spouses, the greater the marriage gain for the less educated spouse while the lower the gain for the more educated one. On the other side, by marrying an 'educationally homogamous' individual, one obtains a gain close to the baseline gain. As follows, we report two examples. The first considers a marriage between a female without high school education and a college graduate. The second considers the marriage between a female college graduate and a male without high school education. Our results show that in both cases the college graduate would be better off from marring an 'educationally homogomous' individual.

A 30-year-old female without high school education has a marriage gain of 43.71% by marrying a college graduate. On the other side, a 30-year-old male college graduate faces a loss of about 4% by marrying a woman without high-school education. Therefore, the model predicts that a 30-year-old male college graduate is more likely to look for a wife with the same education level, which guarantees him an increase in his consumption by about 7% than if he stayed single.

A 30-year-old man without high school education has a marriage gain of about 30% from marrying a college graduate, which is 22% higher than the male baseline gain. However a female college graduate has a marriage gain of 8.51%, which is 12% lower than the baseline female gain. The woman is significantly better off when marrying a college graduate, in which case the marriage gain is about 21%.

We also find that, at young ages, the gap between the lowest marriage gain, which the spouse obtains from marrying a less educated person, and the highest gain, which is obtained from marrying a more educated one, is quite relevant. In particular, at age 30, it is in the range 10-20% for both men and women and reflects the expectations of the spouse's future earnings. However, as individuals age, the gap decreases significantly and for 75-year-old individuals is in the range between 5 and 10%.

Table 2.3 shows that women experience the highest marriage gain at the end of their life. It is about 55%, 50% and 40% respectively for a 75-year-old without high school education, with

Table 2.3: Gain for Women for Different Education Level

	Age				
	30	45	60	75	90
Female without High School					
Husband without High School	25.95%	39.07%	37.23%	61.73%	30.58%
Husband with High School	30.31%	43.35%	39.1%	62.43%	30.93%
Husband with College	43.71%	52.77%	45.84%	87.78%	53.94%
Female with High School					
Husband without High School	18.26%	31.47%	35.00%	60.16%	29.06%
Husband with High School	22.11%	35.3%	36.80%	60.85%	29.47%
Husband with College	34.38%	43.94%	43.01%	68.92%	30.70%
Female with College					
Husband without High School	8.51%	26.25%	29.19%	44.45%	24.17%
Husband with High School	11.76%	29.70%	31.12%	44.96%	24.52%
Husband with College	20.78%	37.63%	36.44%	49.71%	25.54%

high-school education and with college education. This important marriage gain for a female reflects, not just the risk-sharing opportunities against uncertain medical expenses, but mostly the death-risk-sharing opportunities. The husband's promise to leave his possessions to the wife together with the two spouses' shared accumulated assets act as a kind of insurance policy. In particular, the agreement to share until death and bequeath future support acts as an annuity. This allows the wife to increase her consumption than if she stayed single. On the other side, for 75-old-year men without college education the marriage gain is close to zero, while male college graduates face a loss of about 20%, regardless of their wives' education level. Every 90-year-old man faces a marriage loss. Given the expectation of dying before his partner, the man would be better off to consume at a faster rate at the end of his life. However, when participating in an equal weighted marriage contract the man is forced to consume at a slower rate than the optimal one and this decreases his end-of-life utility.

Table 2.4: Gain for Men for Different Education Level

	Age				
	30	45	60	75	90
Male without High School					
Wife without High School	12.85%	19.61%	6.87%	-3.67% -23.44%	
Wife with High School	16.82%	23.42%	8.42%	-1.18%	-23.19%
Wife with College	29.2%	31.83%	13.71%	1.85%	-22.48%
Male with High School					
Wife without High School	6%	12.4%	2.32%	-11.5%	-27.44%
Wife with High School	9.49%	15.86%	6.29%	-4.66%	-23.92%
Wife with College	20.85%	23.54%	11.39%	0.29%	-23.20%
Male with College					
Wife without High School	-4.02%	7.81%	0.47%	-22.39%	-30.89%
Wife with High School	-1%	10.87%	1.81%	-22.12%	-30.7%
Wife with College	7.10%	17.89%	6.43%	-20.21%	-30.64%

2.7 The Economies of Shared Living

The needs of a household grow with each additional member, but, because of economies of scale in consumption, not in a proportional way. Needs for electricity, housing space, etc. will not be two times as high for a family with two members as for a single person. Equivalence scales are used to assign each household type in the population a value in proportion to its needs. The most commonly used scales are the following. The *OECD equivalence scale* assigns a value of 1 to the first household member, of 0.7 to each additional adult, and 0.5 to each child. The *OECD-modified equivalence scale* assigns a value of 1 to the head of the household, of 0.5 to each additional adult member, and 0.3 to each child. The *Square root scale* divides household income by the square root of the household size. There is no generally accepted method for determining equivalence scales and the choice of a particular equivalence scale is a function of technical assumptions about economies of scale in consumption. In our model, the economies of shared living are introduced by multiplying the two family members' joint consumption by the parameter ϕ. In the baseline scenario, ϕ equals 1, which implies that the needs of a household

Table 2.5: Gain From Marriage and Economies of Shared Living

	Age				
	30	45	60	75	90
ϕ =1.25	39.80%	55.72%	47.61%	33.17%	17.83%
ϕ =1.4	54.34%	70.65%	69.43%	38.13%	26.99%
ϕ =1.5	63.98%	81.1%	69.31%	43.75%	32.81%
ϕ =1.75	87.14%	104%	88.62%	52.39%	49.78%

grows with each additional members in a proportional way. That is, two family members have double the needs of a single individual. At the other extreme, if two people can live as cheap as one individual, then the parameter ϕ takes value 2. Since there is no generally accepted equivalence scale, we consider the marriage gain when the parameter ϕ takes the intermediate values 1.25, 1.4, 1.5, and 1.75. All the other variables are as in the baseline scenario.

Table 2.5 reports the marriage gain for different values of ϕ. As ϕ increases, the marriage gain increases proportionally.

2.8 Conclusion

This paper provides a quantitative analysis of the economic gain from marriage. We present the family as an institution which provides risk-pooling opportunities against three main uninsurable risks: the risk of death, the risk of job loss, and the risk of health expenditure. We show that the risk-sharing advantage from marriage is associated with the specific characteristics of the spouse. The opportunity of pooling risks through marriage acts not only as a kind of insurance policy but also as an incentive for college graduates to marry 'schooling homogamous' individuals. In general, any individual, regardless of her gender and education, obtains the highest increase in her consumption from marrying a highly educated individual. This results from the fact that the education level significantly affects the labor income process and the medical expenses process and, therefore, indirectly affects the marriage gain. We assume that the two family members' labor income processes are uncorrelated. A more realistic model should take into account the possibility of correlation between the spouses' permanent labor income shocks.

Our main goal is to simulate the marriage gain as a result of the risk-sharing opportunities provided by the institution of the family. Therefore, in our study we do not take into account the economies of shared living until the last paragraph. Our main finding is that risk-sharing opportunities can be quite important, even though the economies of shared living are the dominant factors in determining the gain from marriage.

Our model presents two main limitations. Firstly, it simulates the marriage gain in absence of private and public insurance markets. The Social Security System, Medicare, other welfare programs, and private insurance markets can significantly reduce the economic incentive for family formation. Secondly, we abstract from the study of the effect of the US different tax treatment toward married and single individuals on the gain from marriage. Tax bonuses, calculated as the difference between the income tax that a couple pays and the amount that the single individual pays, undoubtedly increase the marriage gain. Both of these mechanisms are left to further research. Even with these limitations, this paper provides strong support for the life-cycle model as a good tool for the simulation of the marriage gain at different ages. More importantly, it shows that family insurance is particularly significant, even in small families.

Chapter 3

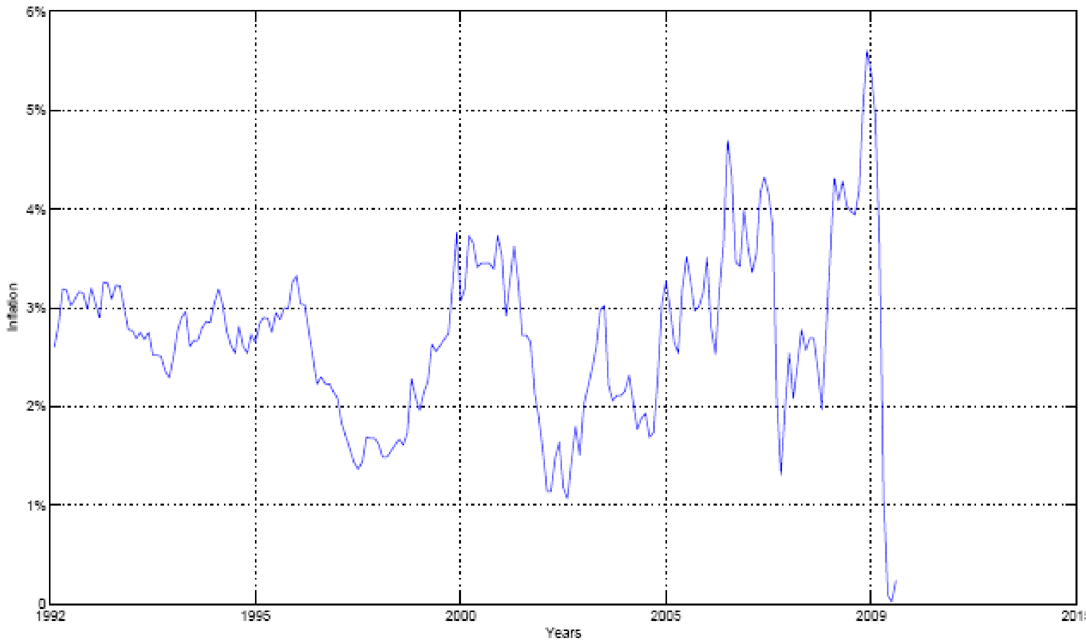
Does it Pay to Pay Off Your Mortgage?

3.1 Introduction

Many retirees enter their retirement with a mortgage. Specifically, according to the 2005 Health and Retirement Study (HRS), about one in four retirees has a mortgage. Moreover, nearly one third of the U.S. retirees with mortgage have enough financial resources to pay off their mortgage. This paper addresses the question of whether a retiree should use her financial resources to pay off her mortgage or should keep the mortgage and invest. The mortgage payoff dilemma is ordinary and complicated at the same time. It is ordinary, because one in four American retirees faces the tradeoff between paying an extra dollar off the outstanding mortgage and investing that extra dollar by participating in financial markets. It is complicated because involves financial considerations of uncertainty in inflation and return on risky assets, borrowing constraints, taxation, risk aversion and evaluation of alternative investments.

The decision to pay off a mortgage involves a cost-benefit analysis. The cost of a mortgage is equal to the interest that the retiree pays on the mortgage debt, which equals the return on the riskfree asset plus a premium to compensate the lender for the risk of default. The benefit of keeping a mortgage is the expected return on alternative investments, namely stocks and bonds. The return on riskfree assets is lower than the cost of a mortgage, while the expected return on risky assets exceeds the cost of a mortgage. If the total expected return on other investments is lower than the cost of a mortgage, the retiree is better off prepaying the mortgage. The optimal decision about other investments depends on the initial level of financial assets and on risk aversion.

Figure 3·1: Inflation from 1992 to 2009



Moreover, when considering whether to pay off a mortgage, the retiree should evaluate the riskiness associated with the future real value of debt and investments. Figure 3.1 shows the inflation rate between year 1992 and 2009. Focusing on the recent years, we note that in mid-2002, after experiencing a low of just over one percentage point (1.07%), the inflation rate started rising, indicating that the disinflationary period had ended. Specifically, the inflation rate began a six year upward trend, with an increase in consumer prices mainly due to the increase in monetary supply. Then, a rise in the prices of food, energy and oil drove the inflation rate up to a peak of 5.6% in mid-2008. However, at the end of 2008, the oil bubble bursts and prices deflated to a level equal to the previous low of 1.07%. In January 2009, the annual inflation rate was 0.03%. Given these fluctuations in inflation, one important question is how the inflationary risk affects homeowner's decision to pay off her mortgage.

A fall in inflation boosts retirees' real incomes. However, falling prices make debt more expensive. In turn, inflation reduces the real cost of mortgage debt. Therefore, retirees that prepay their mortgage eliminate the inflationary risk associated with the uncertainty of the real value of future mortgage payments.

In this paper, we consider homeowners that enter their retirement with a different level of outstanding mortgage debt, financial assets and aversion to risk. These homeowners have enough financial assets to pay off their mortgage. The goals of this paper are the following. First, to find the characteristics of a homeowner that benefits from paying off her mortgage. Second, to evaluate how portfolio composition changes when a homeowner chooses to pay off her mortgage compared to when she keeps her mortgage. Third, to evaluate the relation between inflationary risk and the homeowner mortgage payoff decision.

In this paper, we build a dynamic model of optimal consumption and portfolio choice in retirement. The retiree makes her portfolio and consumption choices in an environment with stochastic inflation and stochastic return on a risky asset. The retiree can choose to keep the mortgage and invest or to pay off the mortgage. After comparing these two choices, we calculate the welfare gains from prepayment.

We find that those homeowners with more initial wealth are better off from prepaying their mortgage, whereas those with less initial wealth are worse off prepaying. The welfare gains to the wealthy can run as high as 4 percent of their initial assets. These gains reflect the fact that the nominal mortgage interest rate exceeds the nominal return rate one can earn on safe assets. This holds for those with low initial wealth, but such homeowners are typically liquidity constrained, so paying off a mortgage comes at cost of less consumption smoothing. Moreover, by paying off a mortgage, the homeowners eliminate the inflationary risk, namely the risk associated with future uncertainty over their real mortgage repayments. We also show that more risk averse homeowners tend to tilt their portfolio towards safe assets. For them, the expected return from their investments decreases and the welfare gain from paying off a mortgage increases. Our results are supported by empirical evidence. An analysis of the HRS retirees shows that less wealthy retirees, with few years before loan termination, never choose to prepay their debt. On the other hand, the wealthiest retirees, with many years before loan termination, often choose to prepay the debt.

This paper is organized as following. In section 2, we present the related literature. In section 3, we describe the model. In section 4, we describe the results. In section 5, we compare my results with the HRS data. This comparison provides evidence of the strengths and limitations of the model. Section 6 concludes.

3.2 Literature review

There is a vast academic literature on housing and on portfolio and mortgage choice. Gomes, Kotlikoff, and Viceira (2008) study optimal consumption, asset accumulation, and portfolio decisions in a life-cycle model with a flexible labor supply and calculate the welfare costs of constraining portfolio allocations over the life cycle. Campbell and Cocco (2003) study household decision between a fixed-rate (FRM) and an adjustable-rate (ARM) mortgage using a life-cycle model that features stochastic inflation and a stochastic interest rate. They find that a nominal FRM has a risky real capital, while an ARM has a stable real capital value but short term variability in required real payments. Cocco (2005) studies the portfolio choice including housing. He finds that investment in housing plays an important role in explaining the cross-sectional change in the composition of wealth and the level of stockholdings observed in the data. Gomes and Michaelides (2005) build a life-cycle model with realistically calibrated uninsurable labor income risk and moderate risk aversion which simultaneously matches stock market participation rates and asset allocation decisions conditional on participation. Their model features Epstein-Zin preferences, a fixed stock market entry cost, and moderate heterogeneity in risk aversion. Cocco, Gomes and Maenhout (2005) build a realistically calibrated life cycle model of consumption and portfolio choice with non-tradable labor income and borrowing constraints. Since labor income substitutes for riskless asset holdings, the optimal share invested in equities is roughly decreasing over a lifetime. Flavin and Yamashita (2002) focus on the impact of the portfolio constraint imposed by the consumption demand for housing, called "the housing constraint," on the household's optimal holdings of financial assets. Yao and Zhang (2005) examine the optimal dynamic portfolio decisions for investors who acquire housing services from either renting or owning a house. They show that when investors are indifferent between owning and renting, they choose different portfolio allocation when owning a house versus when renting a house. Investors, who own a house, hold a lower equity proportion in their net worth (bonds, stocks, and home equity), reflecting the substitution effect of home equity for risky stocks. However, when owning, investors hold a higher equity proportion in their liquid portfolios (bonds and stocks). This reflects the diversification benefit afforded the homeowners who can use home equity to buffer financial and labor-income risks. Koijen, Hemert, and Nieuwerburgh (2007) show that the bond risk premium affects mortgage choice. Namely, if the bond risk premium is high, fixed-rate mortgage payments are high and, therefore, the adjustable-rate mortgage is more appealing. They confirm this fact empirically. Gomes, Kotlikoff, and Viceira (2007) build a life cycle model with earnings, lifespan, investment return, and future policy uncertainty to measures the excess burden from delayed resolution of policy uncertainty. The uncertain policies concern the level of future Social Security benefits and the marginal tax rate. Green and Shoven (1986) analyze the effects of interest rates on mortgage prepayments. They show that the cash flow from the fixed interest rate mortgages is constant, if the mortgage is not prepaid. An increase in the interest rate generates nominal capital gains for the homeowner and, therefore, she is less likely to prepay.

This paper builds also on the literature on optimal asset allocation choice that takes into account the tradeoff between savings in taxable versus tax-deferred accounts (TDA). Amromin, Huang, and Sialm (2007) show that it can become a tax arbitrage to decrease mortgage prepayment and augment TDA contributions due to the tax deductibility of mortgage interest and tax-exemption of qualified retirement savings. Poterba, Shoven, and Sialm (2000) compare two asset location strategies for retirement savers by using data on actual returns on taxable bonds, tax-exempt bonds, and a small sample of equity mutual funds over the 1962-1998 period. Poterba (2002) describes how taxation influences household decisions about portfolio structure and asset trading.

We extend the current literature by investigating the optimal consumption, asset allocation and portfolio decision when households choose to prepay their mortgage and when they keep their mortgage and invest. We also compute the welfare gains from paying off the mortgage.

3.3 The model

3.3.1 Preferences

We model the after-retirement consumption and financial asset choices of a homeowner who lives until period T, with T equal to 95. In each period t, $t = 62, ..., T$, the homeowner chooses real consumption of all goods other than housing, C_t. We assume preference separability between housing and consumption. Since we do not allow the homeowner to move out of her house, the utility from housing services is constant and we can omit housing from the objective function. After time T, the homeowner bequests her terminal wealth, W_{T+1}. Preferences are described by

$$U_{it}(C_{it}) = E_t \sum_{t=0}^{T} \beta^t \frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta^{T+1} \frac{W_{it}^{1-\gamma}}{1-\gamma} \tag{3.1}$$

where β is the time discount factor, γ is the coefficient of relative risk aversion.

3.3.2 Inflation

Homeowners make their consumption and financial assets decisions in an environment characterized by stochastic inflation. As in Campbell and Cocco (2003), the nominal price level at time t is P_t. Lowercase letters denote log variables, therefore $p_t = log(P_t)$ and the log inflation rate is $\pi_t = p_{t+1} - p_t$. We do not consider one-period uncertainty in realized inflation, therefore the expected inflation at time t is equal to the inflation realized from t to $t + 1$. We assume that inflation follows an AR(1) process:

$$\pi_t = \mu(1-\phi) + \phi\pi_{t-1} + \epsilon_t \tag{3.2}$$

where ϵ_t is a normally distributed white noise with mean zero and variance σ_ϵ^2.

3.3.3 Financial assets and credit markets

There are three financial assets. The first financial asset is a riskless asset, called Treasury bills, with gross real return R_F. The second financial asset is a risky asset, called stocks, with gross real return R_t. We follow Cocco (2005), and the log return on the risky assets R_t is assumed to be:

$$log(R_t) = \mu_R + \iota_t \tag{3.3}$$

where $\mu_R > 0$ is the expected log return and ι_t is the innovation to log returns and is assumed to be distributed as $N(0, \sigma_\iota^2)$. B_t and S_t denote the dollar amount that the homeowner has in bills and stocks at date t, such that

$$S_t \geq 0, B_t \geq 0, \forall t. \tag{3.4}$$

This implies that the homeowner cannot short-sell any of these financial assets. The third financial asset is a mortgage. We assume that any homeowner has a thirty-year mortgage loan, but we allow homeowners to have different outstanding mortgage loans in the initial period. Each homeowner can choose to continue her scheduled mortgage payments until loan termination. The annual real mortgage payment, M_{it}, is:

$$M_{it} = \frac{X_M D_{it} + \Delta D_{it+1}}{P_t} \tag{3.5}$$

where D_{it} is the nominal principal amount of the original loan outstanding at date t, ΔD_{it+1} is the component of the mortgage payment at date t that pays down principal instead of interest.

The nominal interest on a mortgage X_M is assumed to be equal to the riskfree plus a constant premium:

$$X_M = R_F + \theta \tag{3.6}$$

The mortgage premium θ includes inflation plus a compensation to the mortgage lender for the default risk.

In the initial period, each homeowner can choose to pay off her mortgage debt. In this case, the initial mortgage payment is equal to the real value of the outstanding mortgage debt plus interest. In any following period, the mortgage payments are set equal to zero. After paying off her mortgage, we do not allow the homeowner to borrow against the value of her house and we do not allow her to refinance her mortgage in the future. These simplifications of our model allow us to focus on the homeowner's decision about her current loan.

3.3.4 The retiree's optimization problem

In each period, the retiree chooses consumption and portfolio composition among liquid assets. She receives a non-stochastic income Y_t, which is the sum of social security, pension and other retiree benefits. The tax code is modeled assuming a linear taxation rule. The retiree's income is taxed at a constant rate τ and mortgage interests are tax deductible at the same rate τ.

The date t budget constraint is given by

$$W_{it} + Y_{it}(1 - \tau) = C_{it} + S_{it} + B_{it} + M_{it} - \tau X_M D_{it}/P_t \tag{3.7}$$

W_{it} is the financial wealth and evolves over time according to the following expression:

$$W_{t+1} = R_t S_{it} + R_F B_{it} - M_{it} + \tau X_M D_{it}/P_t \tag{3.8}$$

Following Epstein and Zin (1989), we consider the following value function:

$$V_t(C_{it}) = [C_{it}^{1-\gamma} + \beta E V_{t+1}^{1-\gamma}(C_{it+1})]^{\frac{1}{1-\gamma}} \tag{3.9}$$

3.4 Solution technique

The model cannot be solved analytically. We use numerical techniques to solve it (Judd, 1998). Given its finite horizon, the model can be solved by using backward induction. The state space involves one continuous state variable, non-housing financial wealth W_t, and two discrete state variables, risky asset return R_t and price level P_t. The control variables are consumption C_t, the amount invested in the risky asset S_t, and the amount invested in the safe asset B_t. We approximate the value function by using a n Chebyshev nodes z and n Chebyshev basis function T, with n equals to 35.

$$\hat{V}_t = \sum_{i=0}^{n} a_i T_i(z) \tag{3.10}$$

We use Gaussian quadrature methods (Tauchen and Hussey, 1991) to approximate the density functions for the random variables, namely the innovation to log inflation and to the risky asset return. In the last period, for each combination of the state variables, we calculate the terminal wealth. In this terminal period, the utility function coincides with the value function. In each previous period $t = 62...T-1$, the household chooses the amount to invest in risky and safe assets, and consequently consumption, to maximize the expected lifetime utility for a given state vector.

3.5 Calibration

In the baseline case the discount rate β is 0.98 and the coefficient of relative risk aversion γ is 5. We will consider how changes in the value of parameter γ affect household portfolio composition and welfare gains in prepaying the debt.

We divide the sample of HRS retirees according to their house value distribution. We consider retirees with a house value less than $250,000 in year 2000, who represent about 95% of the retirees in the sample. We considered two house values, namely $80,000 and $150,000. Consistent with our data, we assume that all the retirees have closed a thirty-year mortgage contract. In

Table 3.1: Calibrated Parameters

Description	Parameter	Value
Discount factor	β	0.98
Risk aversion	γ	5
House size ($ thousand)	H	80, 150
Tax rate	τ	0.20
Riskfree Rate	R_F	2%
Mean of stock return	μ_R	10%
Standard deviation of the log stock return	σ_ι	0.1674
Nominal interest rate on mortgage	X_t	7%
Mean log inflation	μ_π	0.046
Standard deviation of log inflation	σ_π	0.039
Autoregressive parameter	ϕ_π	0.754

this way, we can easily compute the mortgage payment and the outstanding debt at different ages. The tax rate τ is 20%.

Following Cocco (2005), the riskfree rate is 2% per year. The compensation to the mortgage lender for the default risk is 2%. Therefore, the nominal interest rate on the mortgage X_t is 7%. The annual mean return on risky assets is 10% and the standard deviation of the log stock return is 0.1674 (Campbell, Lo, and MacKinlay, 1997). The inflation is calibrated following Campbell and Cocco (2003), who use consumer price index data from 1962 to 1999 to estimate equation (2). The mean and the standard deviation of log inflation are 4.6% and 3.9%, and the annual autoregressive coefficient is 0.754. The following table contains the calibrated parameters.

3.6 Results

Some retirees have enough financial assets to pay off their outstanding mortgage debt. Should they use their financial assets to pay down their mortgage or keep the mortgage and seek a higher return from other investments? We use our model to provide an answer to this question. We are particularly interested in the effects of mortgage size, inflationary risk and risk aversion on retiree behavior and welfare. Accordingly, we consider two house values, namely $80,000 and $150,000 which are expressed in 2000 US dollars. Retirees can have five, ten or fifteen years before loan termination. Therefore, the outstanding debt is \$12,000, \$24,000 or \$36,000 for the \$80,000 house value and \$25,000, \$50,000 and \$75,000 for the \$150,000 house value. The welfare gain from paying off the mortgage is calculated as a percentage increase in the initial financial assets that generate the same expected lifetime utility when keeping it as when paying it off immediately.

The following tables summarize our results. Homeowners with house value of \$80,000 and an outstanding mortgage of \$12,000 or \$24,000 should pay off the mortgage only if their non-housing financial assets exceed \$150,000. Furthermore, homeowners with a mortgage of \$36,000 should pay off the mortgage only if their non-housing financial assets exceed \$200,000. Homeowners with house value of \$150,000 and a mortgage of \$25,000 or \$75,000 should not prepay their mortgage, regardless of their initial level of non-housing financial assets, while homeowners with a mortgage of \$50,000 should pay off the mortgage only if their non-housing financial assets exceed \$200,000.

When deciding whether to pay off her debt, each retiree should compare the borrowing rate versus the expected return rate. With an average inflation rate equal to 3%, the real interest on the mortgage is about 4%. The real return on the safe asset is 2%, while the mean return on the risky asset is 10%. Under the assumption that retirees choose their portfolio allocation optimally, they would be better off prepaying their mortgage if the expected return on other investments is lower than the real cost of the mortgage. The amount of financial resources that each retiree chooses to invest in risky assets with higher expected return depends on her initial level of non-housing financial assets, future uncertainties and risk aversion. In the baseline case, we show that less wealthy retirees, who prepay their mortgage, reduce their investments in both risky and safe assets. The prepayment comes at cost of less consumption smoothing. Instead, if they keep their mortgage, they can invest optimally in risky and safe assets and smooth consumption over time. This explains why less wealthy retirees are better off by keeping a mortgage. On the other hand, more wealthy retirees, who prepay their mortgage, experience welfare gains. In the baseline scenario, the welfare gains to the wealthy can be as high as 4% of their initial financial assets. These gains reflect the fact that the nominal mortgage interest rate exceeds the nominal return rate one can earn on safe assets.

Moreover, homeowners that prepay mortgage debt benefit from the elimination of the inflationary risk associated with the uncertainty in future real mortgage payments. A fall in inflation boosts homeowners' real incomes, so that homeowners can benefit from a boost in their purchasing power. However, deflation increases the real cost of mortgage debt. On the other hand, inflation decreases the real cost of mortgage debt.

The degree of risk aversion partially affects retiree consumption and portfolio choices, as well as welfare gain from paying off mortgage debt. More risk averse retirees optimally tilt their portfolio toward safe assets. In an extreme case, risk averse retirees choose only to invest in safe assets. The real return from their investment is 2%, while the real cost of borrowing is about 4%. Thus, these retirees would certainly benefit from paying off their mortgage. However, in reality, very few wealthy people invest only in safe assets. In the following policy experiments, we considered the case of γ equal to 3 and equal to 9. When γ is equal to 3, retirees are less risk averse compared to the baseline case and increase their investment in risky assets. Therefore, the expected return from their portfolio is larger and the welfare gains are generally smaller. When γ is equal to 9, retirees are more risk averse compared to the baseline case and decrease their investment in risky assets. The welfare gains from paying off a mortgage are generally larger compared to the baseline case. Given the increase in the share invested in safe assets, with a return lower than the cost of a mortgage, paying off a mortgage becomes the best alternative for a larger number of households. Finally, we note that even though there are differences in the welfare gains when using different degrees of risk aversion, these differences are generally small.

We simulate the lives of 100 homeowners who respectively choose to pay off their mortgage

Table 3.2: Welfare Gain - House Value \$80,000

	5 years (\$12,000)	10 years (\$24,000)	15 years (\$36,000)
Financial Wealth			
\$100,000	-3% (-\$3,000)	-8% (-\$8,000)	-35% (-\$35,000)
\$150,000	-0.6% (-\$1,000)	-2% (-\$3,500)	-10% (-\$15,000)
\$200,000	0% (\$0)	0% (\$0)	-1% (-\$2,000)
\$250,000	0.6% (%1,500)	3% (\$7,500)	4% (\$10,000)

Table 3.3: Welfare Gain - House Value \$150,000

	5 years (\$25,000)	10 years (\$50,000)	15 years (\$75,000)
Financial Wealth			
\$100,000	-30% (-\$30,000)	-50% (-\$50,000)	-
\$150,000	-13% (-\$20,000)	-15% (-\$23,000)	-33% (-\$50,000)
\$200,000	-7.5% (-\$15,000)	-4% (-\$7,500)	-12.5% (-\$25,000)
\$250,000	-4% (-\$10,000)	1% (\$3,000)	-3% (-\$7,000)

(straight line) or to keep their mortgage and invest (dotted line). The graphs show that average consumption over the remaining lifetime is almost the same in both cases; however average consumption is slightly reduced in the initial years for homeowners that prepay their mortgage. Moreover, homeowners that pay off the mortgage tend, on average, to invest more in risky assets. All homeowners decrease their portfolio share invested in risky assets as they age.

3.7 Data

We compare the results with data from the HRS panel study. We focus on homeowners with house value less than \$250,000 in year 2000. Table 3.8 shows that about one third of the homeowners with a mortgage have enough financial assets to pay off their mortgage. Among this group of homeowners, only one third choose to prepay the mortgage before loan termination. We identify homeowners that prepay the mortgage as those homeowners that have more than eight years

Table 3.4: Welfare Gain - House Value \$80,000 and γ=3

	5 years (\$12,000)	10 years (\$24,000)	15 years (\$36,000)
Financial Wealth			
Low Inflation			
\$100,000	-4% (-\$4,000)	-10% (-\$10,000)	-40% (-\$40,000)
\$150,000	-2% (-\$3,000)	-2% (-\$3,000)	-10% (-\$15,000)
\$200,000	-1% (-\$2,000)	-0.2% (-\$500)	-2% (-\$4,000)
\$250,000	0.6% (\$1,500)	3% (\$8,000)	2% (\$5,000)

Table 3.5: Welfare Gain - House Value \$150,000 and γ=3

	5 years (\$25,000)	10 years (\$50,000)	15 years (\$75,000)
Financial Wealth			
\$100,000	-30% (-\$30,000)	-50% (-\$50,000)	-
\$150,000	-13% (-\$20,000)	-20% (-\$30,000)	-53% (-\$80,000)
\$200,000	-10% (-\$20,000)	-4% (-\$8,000)	-15% (-\$30,000)
\$250,000	-6% (-\$15,000)	1% (\$3,000)	-3% (-\$8,000)

Table 3.6: Welfare Gain - House Value \$80,000 and γ=9

	5 years (\$12,000)	10 years (\$24,000)	15 years (\$36,000)
Financial Wealth			
\$100,000	-2% (-\$2,000)	-6% (-\$6,000)	-40% (-\$40,000)
\$150,000	0% (\$0)	-2% (-\$3,000)	-10% (-\$10,000)
\$200,000	0.5% (\$1,000)	0.5% (\$1,000)	0.5% (\$1,000)
\$250,000	1% (\$2,500)	4% (\$10,000)	6% (\$15,000)

Table 3.7: Welfare Gain - House Value \$150,000 and γ=9

	5 years (\$25,000)	10 years (\$50,000)	15 years (\$75,000)
Financial Wealth			
\$100,000	-30% (-\$30,000)	-50% (-\$50,000)	-
\$150,000	-13% (-\$20,000)	-13% (-\$20,000)	-7% (-\$10,000)
\$200,000	-7.5% (-\$15,000)	-4% (-\$8,000)	-10% (-\$20,000)
\$250,000	-4% (-\$10,000)	1% (\$3,000)	0% (\$0)

before loan termination and have declared a mortgage in the year 2000 and no mortgage in the year 2006.

Table 3.8: Summary Statistics for the entire sample

	(1)	(2)	(3)	(4)	(5)	(6)
Number of Homeowners	12,337	3,850	1,011	277	335	241

where:
(1) All homeowners
(2) Homeowners with mortgage in year 2000
(3) Homeowners with financial assets greater than their mortgage
(4) Homeowners with scheduled mortgage payments for less than eight years
(5) Homeowners with scheduled mortgage payments for more than eight years that do not prepay their mortgage
(6) Homeowners with scheduled mortgage payments for more than eight years that prepay their mortgage

Consistent with the previous simulation analysis, we divide the homeowners in two groups according to their house value. The first group includes those with house value equal to $80,000 in 2000 US dollars. The second group includes those with house value equal to $150,000 in 2000 US dollars. We also make three subgroups according to the homeowners' behavior with respect to mortgage. The first subgroup includes homeowners that have less than eight years of scheduled mortgage payments. The second group includes homeowners with more than eight years before loan termination that choose not to prepay their mortgage. The third subgroup includes homeowners with more than eight years before loan termination that choose to prepay their mortgage.

First let's consider homeowners with a $80,000 house. The homeowners that have less than eight years of mortgage payments have on average a mortgage debt of $7,500 and non-housing financial assets equal to $85,000. They never choose to prepay their mortgage debt. This empirical evidence is consistent with our baseline results, according to which this subgroup of homeowners should prepay only if their non-housing financial assets exceed $100,000. Homeowners with more than eight years before loan termination have non-housing financial assets equal to $160,000 on average. Homeowners that choose to prepay have, on average, a smaller mortgage and larger investments in stocks and safe assets. Homeowners with financial assets greater than $150,000 that are more risk averse find it optimal to pay off their mortgage, while homeowners that are less risk averse find it optimal to pay off their mortgage only if their non-housing financial assets exceed $200,000.

Homeowners with average house value $150,000 and less than eight years of mortgage have, on average, non-housing financial assets equal to $180,000 and, consistent with our simulated results, never choose to prepay the mortgage. Homeowners that choose to prepay the mortgage have non-housing financial assets significantly larger compared to homeowners that choose to continue their mortgage payments until loan termination.

Our model's results are consistent with the behavior of many retirees. Our model is, therefore, able to reproduce several aspects we can observe in the HRS data. Moreover, inflationary risk and risk aversion provide further insights about retiree behavior.

Table 3.8 and Table 3.9 contain some summary statistics about the HRS retirees' portfolio composition.

Table 3.9: Summary Statistics - House Value $80,000

	(4)	(5)	(6)
Number of Homeowners	116	124	103
Mortgage	7,500	40,000	34,000
Non housing financial assets	85,000	160,000	160,000
Stocks	42,000	85,000	90,000
Checking Account	22,000	22,000	36,000
Married or Partnered	85%	84%	90%
Birth Year	1938	1938	1935

Table 3.10: Summary Statistics - House Value $150,000

	(4)	(5)	(6)
Number of Homeowners	153	198	124
Mortgage	19,000	85,000	74,000
Non housing financial assets	180,000	300,000	440,000
Stocks	110,000	190,000	290,000
Checking Account	38,000	50,000	52,000
Married or Partnered	90%	87%	93%
Birth Year	1938	1938	1936

3.8 Conclusion

In this paper we studied the welfare gains from paying off a mortgage for retirees. Retirees make consumption and portfolio choices in an environment characterized by stochastic inflation and stochastic return on risky assets. Our model shows that wealthier homeowners are better off prepaying their mortgage, whereas less wealthy homeowners are worse off prepaying. The welfare gains to the wealthy can run as high as 4 percent of their initial assets. These gains reflect that the nominal mortgage interest rate exceeds the nominal return one can earn on safe assets. This holds for those with low initial wealth, but such homeowners are typically liquidity constrained, so paying off their mortgage comes at a cost of less consumption smoothing. Moreover inflation and degree of risk aversion affect behavior and welfare gains from prepaying a mortgage.

This paper can be fruitfully extended in several directions. First, we can consider a more realistic taxation system, which takes into account the progressivity of the tax code, and evaluate how modifications in the treatment of the mortgage tax deduction affect homeowner behavior and welfare gains. Second, we can introduce stochastic out-of-pocket expenses in a model characterized by borrowing constraints to determine the effects on our results. Third, we can extend our analysis to longer life-time periods, namely younger homeowners that receive a stochastic per-period income to determine under which conditions they benefit from prepaying their mortgage debt.

References

Altig, D., A. J. Auerbach, L. J. Kotlikoff, K. A. Smetters, and J. Walliser. Simulating Fundamental Tax Reform in the United States, *American Economic Review* 2001; 91: 574-595.

Aguirregabiria, V. and P. Mira. Swapping the Nested Fixed Point Algorithm: A Class of Estimators for Discrete Markov Decision Models, *Econometrica* 2002; 70: 1519-1543.

Amromin, G., J. Huang, and C. Sialm. The tradeoff between mortgage prepayments and tax-deferred retirement savings, *Journal of Public Economics* 2007; 91(10): 2014–2040.

Ando, A., and F. Modigliani. The "Life-Cycle" Hypothesis of Saving: Aggregate Implications and Tests, *American Economic Review* 1963; 53: 55-84.

Attanasio, O., J. Banks, C. Meghir, and G. Weber. Humps and Bumps in Lifetime Consumption, *Journal of Business and Economic Statistics* 1999; 17(1): 22-35.

Auerbach, A., and L. J. Kotlikoff. *Dynamic Fiscal Policy* 1987; Cambridge University Press.

Becker, G. A theory of marriage: Part I, *Journal of Political Economy* 1973 ; 81(4): 813-846.

Becker, G. *A treatise on the family* 1981; Harvard University Press

Becker, G., E. M. Landes, and R. T. Michael. An Economic Analysis of Marital Instability, *Journal of Political Economy* 1977; 85(6): 1141-87.

Bowen Bishop, T., and H. Shan. Reverse Mortgage: A Closer Look at HECM Loans, *working paper* 2008.

Cagetti, M. Wealth Accumulation Over the Life Cycle and Precautionary Savings, *Journal of Business and Economic Statistics* 2003; 21(3): 339-353.

Campbell, J. Y., and J. F. Cocco. Household Risk Management and Optimal Mortgage Choice, *Quarterly Journal of Economics* 2003; 118: 1149-1194.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay. *The Econometrics of Financial Markets* 1997; Princeton University Press.

Campbell, J. Y., and L. M. Viceira. *Strategic Asset Allocation* 2002; Oxford University Press.

Caroll, C. D. The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence, *Brookings Papers on Economic Activity* 1992; 61-156.

Carroll, C. D. Buffer-Shock Saving and the Life-Cycle/Permanent Income Hypothesis, *Quarterly Journal of Economics* 1997; 112: 1-55.

Carroll, C. D., and L. H. Summers. Consumption Growth Parallels Income Growth: Some New Evidence, in *National Savings and Economic Performance* 1991; ed. by B. Bernheim and J. B. Shoven, University of Chicago Press.

Cocco, J. F. Portfolio Choice in the Presence of Housing, *Review of Financial Studies* 2005; 18(2): 535-567.

Cocco, J. F., F. Gomes, and P. Maenhout. Consumption and Portfolio Choice over the Life Cycle, *Review of Financial Studies* 2005; 18(2): 491-533.

Coontz, S. *Marriage, a History: From Obedience to Intimacy, or How Love Conquered Marriage.* 2005; Penguin Group, New York (USA).

Davidoff, T., and G. Welke. Selection and Moral Hazard in the Reverse Mortgage Market, *working paper* 2007.

Deaton, A. Saving and Liquidity Constraints, *Econometrica* 1991; 59(5): 1221-1248.

Epstein, L. G., and S. E. Zin. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica* 1989; 57(4): 937–69.

Fernandez-Villaverde, J., J. F. Rubio-Ramirez, and M. S. Santos. Convergence Properties of the Likelihood of Computed Dynamic Models, *Econometrica* 2006; 74(1): 93-119.

Fisher, J. D., D. S. Johnson, J. T. Marchand, T. M. Smeeding, and B. Boyle Torrey. No Place Like Home: Older Adults and Their Housing, *Journal of Gerontology* (2007); 62B: S120-S128.

Friedman, M. *A Theory of the Consumption Function* 1957; Princeton University Press.

Flavin, M., and T. Yamashita. Owner-Occupied Housing and the Composition of the Household Portfolio, *American Economic Review* 2002 ; 92(1): 345-362.

French, E. The Effect of Health, Wealth, and Wages on Labor Supply and Retirement behaviour, *Review of Economic Studies* (2005); 72: 395-427.

Gallagher, M. and Waite, L. J. *The Case for Marriage. Why Married People are Happier, Healthier, and Better Off Financially* 2000; Broadway Books, New York.

Gomes, F., Kotlikoff, L. J., and Viceira, L. M. The Excess Burden of Government Indecision, *working paper* 2007.

Gomes, F., Kotlikoff, L. J., and Viceira, L. M. Optimal Life-Cycle Investing with Flexible Labor Supply: A Welfare Analysis of Life-Cycle Funds, *American Economic Review* 2008; 98(2): 297– 303.

Gomes, F., and A. Michaelides. Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence, *Journal of Finance* 2005; 60(2): 869–904..

Gourinchas, P. O., and J. A. Parker. Consumption over the Life Cycle, *Econometrica* 2002; 70(1): 47-89.

Green, J., and J. B. Shoven. The Effects of Interest Rates on Mortgage Prepayments, *Journal of Money, Credit and Banking* 1986; 18(1): 41–59.

Heaton, J., and D. Lucas. Market Frictions, Savings Behavior, and Portfolio Choice, *Macroeconomic Dynamics* 1997; 1: 76-101

Hotz, J., and R. A. Miller. Conditional choice probabilities and the estimation of dynamic models, *Review of Economic Studies* 1993; 60: 497-529.

Hubbard, G., J. S. Skinner, and S. Zeldes. The importance of Precautionary Motives for Explaining Individual and Aggregate Saving, *Canegie-Rochester Conference Series on Public Policy* 1994; 40: 59-125.

Hubbard, G., J. S. Skinner, and S. Zeldes. Precautionary Saving and Social Insurance, *Journal of Political Economy* 1995; 103: 360-399.

Hurd, M. Mortality Risk and Bequests, *Econometrica* 1989; 57: 779-813.

Judd, K. L. *Numerical Methods in Economics* 1998; MIT Press.

Judd, K. L., and C. L. Su. Constrained Optimization Approaches to Estimation of Structural Models, *working paper* 2008.

Koijen, R. S., O. V. Hemert, and S. V. Nieuwerburgh. Mortgage Timing, *NBER Working Papers 13361* 2007.

Kotlikoff, L. J. Health expenditure and Precautionary Savings, in Laurence J. Kotlikoff *What determine savings?* 1989; MIT Press.

Kotlikoff, L. J., and A. Spivak. The Family as an Incomplete Annuities Market, *Journal of Political Economy* 1981; 89(21): 372-391.

Kotlikoff, L. J., A. Spivak, and L. H. Summers. The Adequacy of Savings, *American Economic Review* 1982; 72(5): 1056-69.

Kotlikoff, L. J., and L. H. Summers. The Role of Intergenerational Transfers in Aggregate Capital Accumulation, *Journal of Political Economy* 1981; 86: 706-732.

Kotlikoff, L. J., S. Johnson, and W. Samuelson. Can People Compute? An Experimental Test of the Life Cycle Consumption Model, *Essays on Saving, Bequests, Altruism, and Life-Cycle Planning* 2001; MIT Press.

Kutty, N. K. The Scope for Poverty Alleviation among Elderly Home-owners in the United States through Reverse Mortgage, *Urban studies* 1998; 35(1): 113-29.

Luo, J. Uncertain Medical Expenses and Portfolio Choice over the Life Cycle, *job market paper* 2006.

Mancuso, D. and Pencavel, J. Assortative Mating by Schooling and the Timing of Marriage, *manuscript* 1999.

Mare, R. D. Five Decades of Educational Assortative Mating, *American Sociological Review* 1991; 56(1): 15-32.

Mayer, C. J., and K. V. Simons. Reverse Mortgages and the Liquidity of Housing Wealth, *Journal of the American Real Estate and Urban Economics Association* 1994; 22(2): 235-55.

Merrill, S. R., M. Finkel and N. K. Kutty. Potential Beneficiaries from Reverse Mortgage Products for Elderly Homeowners: An Analysis of American Housing Survey Data, *Journal of the American Real Estate and Urban Economics Association*1994; 22(2): 257-99.

Meyer, J. W., and A. Speare. Distinctively Elderly Mobility: Types and Determinants, *Economic Geography* 1985; 61(1): 79-88.

Modigliani, F., and R. Brumberg. Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data, in *Post-Keynesian Economics* 1954; ed. by K. K. Kurihara, Rutgers University Press, New Brunswick, 388-436.

Munnell, A. H., M. Soto, and J. Aubrey. *Do People Plan to Tap Their Home Equity in Retirement*? 2007; Report No. 7-7. Boston: Center for Retirement Research at Boston College.

Nishiyama, S., and K. Smetters. Consumption Taxes and Economic Efficiency with Idiosincratic Wage Shocks, *Journal of Political Economy* 2005; 113(5): 1088-1115;

Nocedal, J., and S. J. Wright. *Numerical Optimization* Second Edition 2000; Springer.

Palumbo, M. Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle, *Review of Economic Studies* 1999; 66: 395-422.

Pencavel, J. Assortative Mating by Schooling and the Work Behavor of Wives and Husbands, *American Economic Review* 1998; 88(2): 326-329.

Poterba, J. M. Taxation, risk-taking, and household portfolio behavior, in *Handbook of Public Economics* 2002; ed. by A. J. Auerbach, and M. Feldstein.

Poterba, J. M., J. B. Shoven, and C. Sialm. Asset Location for Retirement Savers, *NBER Working Papers 7991* 2000.

OECD Social Policy Division http://www.oecd.org/els/social

Rasmussen, D. W., I. F. Megbolugbe and B. A. Morgen. Using the 1990 Public Use Microdata Sample to Estimate Potential Demand for Reverse Mortgages, *Journal of Housing Research* 1995; 6(1): 1-24.

Rivlin, A. M., and J. M. Wiener. *Caring for the Disabled Elderly: Who Will Pay?* 1998; The Brookings Institution.

Rosalsky, M. C., R. Finke, and H. Theil. The downward bias of asymptotic standard errors of maximum likelihood estimates of non-linear systems, *Economics Letters* 1984; 14(2-3): 207-211.

Rust, J. Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher, *Econometrica* 1987; 55: 999-1033.

Scholz, H. K., A. Seshandri, and S. Khitatrakun. Are Americans Saving Optimally for retirement?, *Journal of Political Economy* 2006; 114(4).

Schultz, T. W. *Economics of the Family: Marriage, Children and Human Capital* 1974; University of Chicago Press (for NBER).

Stucki, B. *Use Your Home to Stay at Home: Expanding the Use of Reverse Mortgages for Long-Term Care* 2005; Washington, DC: National Council on Aging.

Tauchen, G., and R. Hussey. Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models, *Econometrica* 1991; 59(2): 371–96.

U.S. Department of Health, Education, and Welfare, Social Security Administration. Bell Felicitie C. and M. L. Miller. Life Table for the United States Social Security Area 1900-2100 Actuarial Study No.116.

Yao, R., and H. H. Zhang. Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints, *Review of Financial Studies* 2005; 18(1): 197-239.

Zeldes, S. P. Consumption and Liquidity Constraints: An Empirical Investigation, *Journal of Political Economics* 1989; 97(2): 305-346.

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