

Commentary

# Comments on Prof. Mirowski's "Markets Come to Bits: Evolution, Computation and Markomata in Economic Science"

Kenneth L. Judd\*

*434 Galvez Mall, Hoover Institution, Stanford, CA 94305, United States*

Accepted 21 December 2005

Available online 22 February 2007

---

Prof. Mirowski has written a provocative discussion of new ways to model markets that focus on their algorithmic aspects. Many of his substantive points about the weaknesses of standard theory are widely accepted; in particular, we have little idea of how prices are formed, we agree that economic agents are not infinitely intelligent, and it is clear that the markets in any modern economy take many forms. Prof. Mirowski correctly notes that there has been some movement towards more detailed analyses of markets, such as in the literatures on mechanism design, auction design, "Zero-Intelligence Agent" models, market microstructure, engineering economics, and applications of artificial intelligence. All economists welcome further work on detailed analysis of how markets work and how they evolve over time.

Prof. Mirowski helps the reader greatly by anchoring his presentation in the following definition of a market.

For the purposes of our present argument, we shall define a market as a formal automaton, in the sense of the field of mathematics pioneered by John von Neumann, and now taught as basic computational theory. Intuitively, we shall characterize a particular market as a specialized piece of software, which both calculates and acts upon inputs, comprised of an integrated set of algorithms that perform the following functions:

- Data dissemination and communications, plus rules of exclusion.
- Order routing through time and space.
- Order queuing and execution.

---

\* Tel.: +1 650 723 5866; fax: +1 650 723 1687.

*E-mail address:* [judd@hoover.stanford.edu](mailto:judd@hoover.stanford.edu).

- Price discovery and assignment.
- Custody and delivery arrangement.
- Clearing and settlement, including property rights assignment.
- Record-keeping.

All of these elements are important features of any market and it is natural that future research will focus on them.

I cannot give a broad review of all of Prof. Mirowski's points. Nor will I speculate on the fate of this endeavor. Instead, I will make specific comments that may help markomata theorists pursue and exposit this program.

While I agree with Mirowski's comments about the shortcomings of economic theory, I wish there were more discussion of the relative advantages of markomata theory and conventional economic theory. The tenor of much of the paper is that conventional theory is wrong. Of course, every theory is wrong. The way science proceeds is to use different models for different questions, and try to understand how they fit together. For example, in physics we all know that Newtonian mechanics is wrong and that quantum mechanics is the right theory (or at least better) but that does not make it wrong to ever use Newtonian methods. Also, general relativity and quantum mechanics are incompatible theories but each is considered correct over some domain. Current theoretical physics examines questions of how they interact, such as in Hawking's analysis of black hole radiation, and attempts, such as string theory, to create a unified theory. While such a unified theory would be an intellectual triumph, it will surely not completely replace the use of Newtonian, relativity, or quantum physics.

Therefore, I wonder for what questions will markomata be the preferred theory and for what problems is conventional theory satisfactory? How will the two paradigms interact? As I write this in October 2005, I am also reading about current tax reform proposals. Would markomata theory have something to say about the deduction of state and local taxes? I doubt it. Would markomata theory have something to say about capital gains tax rates? Perhaps it would. Capital gains taxation affects asset market liquidity, the willingness of investors to trade, and the collection of investors that comes to the market. These changes would likely affect the prices generated by any algorithm representing the stock market. More generally, many readers would like to hear some discussion of which economic problems are most in need of a markomata theory.

I also wish that Prof. Mirowski would discuss the potential partnership between conventional economic theory and markomata theory. The tone of this paper is more one of conflict, such as in his reference to "the errors and omissions of previous generations." The fact that past research made only limited efforts to study these issues was not driven by a dogmatic belief that they were unimportant, but rather by the lack of tools that could examine them seriously. The introduction of the computer and the exponential growth in computing power, along with the mathematical literature on computational complexity, now makes it possible to pursue the program he outlines. Of course, there will be resistance to this approach (as there is resistance to computational methods in general) since many (if not most) economists are comfortable with the tools they learned in graduate school and do not want to invest time and effort in learning new ones. New tools and approaches have always been resisted; the only way to overcome this is to solve problems that previous methods have failed to solve.

In his definition of markets, Mirowski advocates the use of automata theory and its concepts. I believe that markomata theory should be more expansive in the mathematical tools it uses. For example, he follows the common practice of calling polynomial-time algorithms tractable, but exponential time and NP-complete algorithms are considered intractable. The practical facts

about algorithms are less clear. For example, Prof. Mirowski states that “Allocational mechanisms based upon inherently intractable algorithms (like most linear programming solution algorithms, which are NP-complete) do not prove durable in the world of real markets.” In August 2005, I attended a conference in memory of Prof. Dantzig; this verdict on the durability of the simplex algorithm would have come as a surprise to the conference participants. While it is true that the time cost of the simplex method grows exponentially in the worst-case, the average case performance is polynomial. Also, we need to remember that these classifications are based on performance as the problem size goes to infinity. This asymptotic index is used in algorithmic theory since it is relatively tractable, but it is often not relevant for real problems. For example, the first polynomial-time algorithm for linear programming, the Khachiyan method, is actually less efficient than the simplex algorithm for most real-world problems. More recently, the Karmarkar algorithm and related interior point methods do give us polynomial-time methods that are useful in real problems. More generally, I believe that markomata theory should be practically oriented, focusing on problems similar to real markets in size and complexity, and not focus exclusively on the abstract concepts used by mathematicians to discuss complexity.

I also doubt that automata theory is the only way to model algorithmic aspects of markets. In fact, Blum et al. (1997) argue that while automata theory has been useful for the foundations of computer science, the inherent discreteness makes it inadequate for scientific computation. I suspect that price setting in a market is more like solving a system of nonlinear equations than the combinatorial tasks, like sorting a list and constructing balanced trees, that arise in computer science. Blum et al. have developed a comparable theory of complexity for problems like solving polynomial equations and implementing Newton’s method. My conjecture is that their approach to computational complexity would also prove useful in markomata theory.

Mirowski lists some implications of markomata. The first one is “prices are expressed as rational numbers”. He also asserts “The fascination with the reals in neoclassical economic theory is yet another accidental artifact of its origins in physics, and as such is eminently disposable.” I do not see this as a major point. I doubt that the fact that prices are expressed as rational numbers with, say, less than  $10^{100}$  significant digits is an important feature of markets. The more relevant fact is that most prices are expressed using fewer than 8 significant digits, but I fail to see how markomata theory can explain that. Approximating the set of rational numbers with its completion is a powerful simplifying tool in science, and I doubt that Prof. Mirowski objects to it in general. For example, quantum theory says that many physical systems, including many related with the physics of jet airplanes, have only a finite number of possible energies, but the fact that the airplane designers ignore this fact does not keep me (or anyone else) from flying. Asserting that the rationality of prices is an important implication of markomata gives sceptics a tempting target for parodying markomata theory and distracts people from the serious aspects of markomata.

Instead, I believe that the most important points are those related to modeling networks of diverse markomata. The interactions among diverse markets could lead to novel and useful insights about the dynamics of economic phenomena.

Economists generally agree that markets and individuals do not display perfect rationality. There have been a variety of approaches to modeling learning and imperfect processing of information. Since individuals are important actors in any market, markomata theory will combine algorithmic representations of markets with algorithmic modeling of economic agents. This combination is a natural one and will be valuable, as similar work has been in the examples he cites.

However, any theory that deviates from perfect rationality should not go too far in the other direction. Mirowski mentions the zero intelligence models in the economics literature. I would

also argue that most models of imperfect rationality assume little intelligence if we measure “intelligence” in terms of amount and rate of computing performed by economic agents. In contrast, the human brain is an enormously powerful computer. A good desktop computer today is a gigaflop machine, whereas the human brain is roughly a million times more powerful. This computing power is needed to process the stream of visual and audio information it receives. If we are not careful, when we simulate a markomata model on our desktop computer, we will be simulating a market where all of the individual agents have had over 99 percent of their brains scooped out! Models of markets should make realistic assumptions about human rationality, but those assumptions should be between the extremes of perfect rationality and zero intelligence. In fact, when we write a paper about boundedly rational economic agents, should not we assume that the agents in our model are intelligent enough to write the paper? Of course, this test would be dangerous since it would trigger the issues uncovered in Gödel’s incompleteness theorem. While humans are not infinitely intelligent, they are quite intelligent and possess a self-awareness absent in most economic models. Humans in general, not just economists, can analyze themselves and think about their environment and their future in ways not captured in simple models.

Prof. Mirowski lays out an ambitious and potentially valuable agenda for future research. I look forward to that work, and believe that it can, in partnership with other tools and methods, help us gain insights into economic phenomena.

## **Reference**

Blum, L., Cucker, F., Shub, M., Smale, S., 1997. *Complexity and Real Computation*. New York: Springer-Verlag.