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Reply to "Asset trading volume in infinite-horizon economies with dynamically complete markets and heterogeneous agents: Comment" ☆

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Abstract

In a comment, Peter Bossaerts and William R. Zame [2006. Finance Research Letters. This issue] claim that the main result of our paper [Judd, K.L., Kubler, F., Schmedders, K., 2003. The Journal of Finance 58, 2203–2217], namely the no-trade theorem for the dynamic Lucas infinite horizon economy with heterogeneous agents, is an artifact of the assumption that asset dividends and individual endowments follow the same stationary finite-state Markov process. In this reply, we clarify our assumptions and contrast them with the examples in Bossaerts and Zame.

We are grateful to Peter Bossaerts and William R. Zame for their comment (Bossaerts and Zame, 2006) on our paper (Judd et al., 2003). They make an excellent observation that sharpens our understanding of the no-trade theorem. However, while their remarks are formally correct, we disagree with their key assertion that "the more natural heterogeneous agent version of the [Lucas] model should require only that aggregate endowments be stationary but allow for individual endowments that are not stationary."

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In Section II of their note, Bossaerts and Zame (henceforth BZ) correctly point out that their Eq. (2) implies that there is a strong mathematical sense in which there will be trade in equilibrium for 'generic individual endowments' (in the sense of being an open and dense set in the space of all bounded sequences), although prices and consumptions depend only on the current exogenous shock. This is an interesting result which eluded us when we wrote our paper.

However, to describe an infinite horizon economy, one needs to specify preferences and all agents' individual endowments. This is not possible if one does not make strong assumptions on individual endowments, since most arbitrary infinite sequences of endowments simply cannot be described otherwise. The assumption that they follow a Markov chain is of course not the only possible way to do this but it turns out that it is much more general than BZ claim. In fact, their example in Section III fits perfectly well into the framework of our paper. Contrary to the assertion made by BZ, we can describe their economy as having the same finite state Markov process for asset dividends and individual endowments.

1. The numerical example

In their example, BZ let individual endowments depend on the underlying state $s \in \mathcal{S} = \{1, 2\}$ and on whether the date is even or odd. As a result there are four state/time-parity possibilities denoted by $(1, \mathrm{E})$, $(2, \mathrm{E})$, $(1, \mathrm{O})$, and $(2, \mathrm{O})$, where E (O) denotes an even (odd) date t. We can just code these four combinations of states in $\mathcal{S}$ and time parities as four states $y \in Y = \{1, 2, 3, 4\}$, where $y = 1$ corresponds to $(1, \mathrm{E})$, $y = 2$ to $(2, \mathrm{E})$, $y = 3$ to $(1, \mathrm{O})$, and $y = 4$ to $(2, \mathrm{O})$. The resulting transition matrix is then

$$\Pi = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}.$$

Both individual endowments and dividend processes follow a Markov process governed by this transition matrix. Specifically, we can write

$$e_1 = (4, 6, 2, 9), \qquad e_2 = (3, 9, 5, 6), \qquad d^1 = (1, 0, 1, 0), \qquad d^2 = (0, 1, 0, 1).$$

We see that in the BZ example individual endowments do follow a Markov chain. What makes the BZ economy special is that the Markov transition matrix contains zero elements. Each of the four states has only two immediate successors and the two independent securities suffice to achieve dynamically complete asset markets. As we emphasize in our paper there will be usually trade in such an economy. We stress (Judd et al., 2003, p. 2213, line 2) that assumptions on the Markov transition matrix are crucial for the no-trade result:

> "Theorem 2 [The No-Trade Theorem] assumes that Π has no zero entries. While most Π have no zero entries (in the sense of genericity), some natural dividend processes have sparse Π matrices. For example, if dividends follow a simple random walk then from each state $y \in Y$ there are only two states $z \in Y$ for which $\Pi_{yz} \neq 0$. In this case only two assets are needed to dynamically complete the market [...]. Theorem 2 does not apply and there will be trade in this case. This is clear since constant holdings of two assets can span only a two-dimensional space of consumption plans but the efficient allocation will lie in a larger

space. The techniques of Theorem 2 can still be used to determine that volume as long as the consumption allocation is efficient."

The BZ example falls into the special category of economies with zero entries in the transition matrix Π. And that property is the cause for trade in this economy. In the space of all possible Markov transition matrices, this is of course not robust and in this space, there is no trade for generic specifications of the economy.

In Judd et al. (2004) we compute equilibria for two economies with transition matrices that have zero elements. There is trade in both economies. Schmedders (2005) proves that there is trade for a generic class of economies with equi-cautious HARA utility functions that have transition matrices with zero entries. The proof could be easily adapted to more general classes of economies.

2. Finite history dependence

In Section IV of their note BZ argue that individual endowments depending on the whole history will result in portfolios depending on the whole history as well. This is a correct observation—however when applied researchers consider history dependence, they often assume that endowments depend on finite histories. For this case, one can again rewrite the process as a Markov chain.

For economies with a Markov transition matrix Q for states in $\mathcal{S}$ but with endowments and dividends depending on a history of shocks in $\mathcal{S}$ of fixed finite length $T+1$ the described reformulation into our framework with zero entries in the transition matrix still works. Any state $(s, s_{-1}, s_{-2}, \ldots, s_{-T}) \in \mathcal{S}^{T+1}$, where $s_{-t} \in \mathcal{S}$ denotes the state t periods ago can be coded as a state $y \in Y = \{1, 2, \ldots, |\mathcal{S}|^{T+1}\}$. The element $\Pi_{yy'}$ for $y = (s, s_{-1}, s_{-2}, \ldots, s_{-T})$ and $y' = (s', s'_{-1}, s'_{-2}, \ldots, s'_{-T})$ of the resulting transition matrix Π is zero unless $s'_{-1} = s$, $s'_{-2} = s_{-1}$, $s'_{-3} = s_{-2}$, $\ldots$, $s'_{-T} = s_{-T+1}$. In this last case $\Pi_{yy'} = Q_{ss'}$. The transition matrix Π is an $|\mathcal{S}|^{T+1} \times |\mathcal{S}|^{T+1}$ matrix. Each row $\Pi_{y\cdot}$ of Π contains the $|S|$ elements of the corresponding row $Q_{s\cdot}$ of Q and zeros otherwise.

This reformulation reveals that portfolios only depend on the history of the last T shocks $s, s_{-1}, \ldots, s_{-T+1}$ but not on the shock from T periods ago, s_{-T}. Each state $(s, s_{-1}, s_{-2}, \ldots, s_{-T+1}, z)$ for $z \in \mathcal{S}$ has the same transition probabilities across all $|\mathcal{S}|$ possible successor states. The proof of the no-trade theorem shows that this property implies that the portfolio all such states $(s, s_{-1}, s_{-2}, \ldots, s_{-T+1}, z)$ with $z \in \mathcal{S}$ must be identical. That is, the equilibrium portfolio is independent of the state $z \in \mathcal{S}$ from T periods ago.

3. Conclusion: Stationary vs Markovian processes

If one acknowledges that in any version of the Lucas model one needs to make some assumption on individual endowments, the observations of BZ and our remarks seem to center on the issue of stationary and Markov processes.

It appears to us that large parts of the literature are rather sloppy in making the distinction between the two types of processes. However, it is important to note that many non-stationary processes, like the one underlying the BZ example, are Markov processes. There are also many other papers in the literature, which assume that dividend and endowment processes are Markovian, not that they must be stationary. In the textbook definition, a stochastic process is said to be stationary if the distribution of x_t converges to a distribution. Our no-trade theorem assumes that all elements in the transition matrix are positive; therefore, each state can be reached from each other state in one period. This assumption implies that the Markov chain is aperiodic; hence, it has a unique stationary distribution. Bossaerts and Zame's example is a case where the Markov chain has many zero elements; moreover, their Markov chain is periodic, and, hence, is not stationary in the standard sense. In both instances one can construct an underlying Markov process. Our genericity assumption then rules out the ones with zero elements.

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