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Computation and economic theory: Introduction

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The recent advances in computational science present economists with new opportunities and methods for studying economic questions. Economists are exploiting some of those tools and demonstrating their promise. This symposium collects some recent work which shows where computational methods can take us.

This symposium examines both extensions of familiar problems as well as some new directions for computational methods. Most economists are familiar with the applied general equilibrium literature and dynamic generalizations. Today computation is being used to solve an ever-increasing variety of problems using an expanding range of techniques. The symposium covers a wide variety of applications as well as a variety of computational techniques. These papers solve for equilibria in asset markets with asymmetric information, in incomplete asset markets, in markets with heterogeneous beliefs. They also solve dynamic mechanism design problems, equilibria of auctions, and Nash equilibrium of dynamic duopoly. The increase in computer speed is one reason why we see these new applications, but an equally important reason is the development of new algorithms. Many of the papers in this symposium contribute to the algorithm development literature.

The symposium begins with an analysis of insider trading in Bernardo [3]. Insider trading has been a popular topic since the provocative arguments of Manne [16]. Bernardo presents an analysis of insider trading that models both the incentives of the executives with the superior information and an asset market where traders with superior information can execute profitable trades without revealing all of their information. This is a difficult problem with no functional form assumptions that produce a closed-form solution. Ausubel[1] and Bernardo and Judd [4] developed numerical approaches to solving asset market problems with asymmetric information. Bernardo's paper computes equilibrium in a model where executives' actions are unobserved and they may trade on inside information. He finds the commonsense result that firms generally do not want their executives engaging in insider trading since that option would distort their incentives and decisions.

Hart [7] showed that general equilibrium theory with incomplete asset markets differs substantially from the Arrow-Debreu theory for complete markets. The same is true for the computational side of general equilibrium analysis. The Scarf algorithm and later homotopy methods (such as Eaves [6]) can solve Arrow-Debreu models of general equilibrium but they do not apply to markets with incomplete asset markets. The next two papers illustrate techniques for incomplete asset market models. Schmedders [21] solves an asset design problem. He finds the call option that a profit-maximizing monopolistic market maker would offer when asset markets are incomplete. Schmedders' paper relies on the algorithm in Schmedders [22] which takes a homotopy approach for solving general equilibrium models with incomplete asset markets.

Kubler [13] extends the numerical literature for solving general equilibrium with incomplete asset markets. His algorithm allows for transaction costs, taxes, cash-in-advance constraints, and other market frictions. He proves that his algorithm is globally convergent for a generic set of economies. Kubler and Schmedders use programs incorporating HOMPACK [26], a high-quality professionally written software package for implementing homotopy methods. This is a good example of how economists can benefit from using the large stock of software available in the public domain written by professional numerical analysts. Kubler also demonstrates that his algorithm is practical, capable of solving nontrivial problems quickly.

Motolese [17] examines equilibrium in dynamic markets with heterogeneous beliefs. Arrow-Debreu general equilibrium does not require common beliefs but most dynamic models make that assumption, partly since this makes the analysis tractable. However, the hypothesis of common beliefs is unreasonable. Motolese applies the Rational Beliefs Equilibrium concept of Kurz [14] to examine the nonneutrality of monetary policy. These models present substantial numerical challenges since heterogeneous beliefs imply that portfolios can fluctuate wildly. Motolese uses spline-collocation methods to construct an effective algorithm.

Judd and Guu [12] introduce techniques from bifurcation theory to examine models of incomplete asset markets. They produce Taylor series expansions around the case of no risk. Since stocks and bonds are perfect substitutes when all assets are safe, they cannot use the implicit function theorem, and instead use analytic bifurcation methods. The result is essentially a mean-variance-skewness-etc. theory of asset demand and equilibrium pricing. This follows Samuelson's [20] approach to asset demand but shows that the bifurcation method is a more powerful way to proceed. The bifurcation approach is particularly interesting since it handles the complete and incomplete asset market cases in the same way. The results look like a conventional theoretical analysis in the style of Jones' [9] classic "hat calculus" analysis of international trade. However, the formulas involved so much algebra that only a computer could juggle the thousands of terms involved in their derivations. This paper shows the potential that computer algebra systems, such as Mathematica, Macsyma, and Maple offer economists for solving qualitative economic problems.

Computation is becoming more important in applications of game theory. The basic literature on solving normal form games goes back to Lemke and Howson [15], Wilson [27], and Rosenmüller [19]. Herings and Peeters [8] improves on these algorithms by developing a smooth, differentiable technique which allows homotopy methods to more quickly find equilibria. They relate their approach to the tracing procedure. They also use their algorithm in an experiment to find computation time for the average game.

There has been much recent empirical work on auctions, work which requires efficient computational methods for computing equilibria. Bajari [2] presents some algorithms for solving auctions and applies them to asymmetric conditions related to collusion. His techniques are important for empirical work where algorithm speed is very important as well as for theoretical studies of the impact of collusion.

The dynamic mechanism design literature is an excellent example of theory and computation teaming up to solve a problem. In general, an optimal dynamic contract would need to keep track of the entire history, making the problem a dynamic programming problem with a large and growing state space. Spear and Srivastava [24] showed that one could sometimes reduce the problem to a one-dimensional dynamic programming problem. Unfortunately, they could not solve that simpler problem analytically. However, the reduction to one state variable made it possible for Phelan and Townsend [18] to numerically compute optimal contracts. Sleet and Yeltekin [23] extends these techniques to examine optimal employment contracts when layoffs are permitted and when commitment on the part of the parties is less than perfect.

Dynamic models are increasingly used in modelling oligopoly, but few models have closed-form solutions. Vedenov and Miranda[25] apply projection methods to solve continuous-time, continuous-state dynamic games. Methods for static games do not apply since these dynamic games require the solution of partial differential equations. Vedenov and Miranda show that projection methods originally developed for fluid dynamics and other engineering problems can quickly produce results with high accuracy.

These papers all show the benefits of a partnership between economic theory and computational methods.[1] Economic theory is good at proving existence theorems and qualitative features of equilibrium. It is not good at telling us what is quantitatively important and what is of second- or third-order importance. Some of the problems discussed in these papers have special cases with closed-form solutions but those cases often have undesirable features or make very restrictive assumptions. Computation allows a researcher to explore territory and address issues which existing theory on its own cannot.

Theorists are sometimes skeptical about computational results since computation produces, at best, examples instead of a theorem. This is where algorithm

[1] These issues are more extensively discussed in Judd [10] and Judd [11].

efficiency is important. One or two examples can never say much, but patterns which hold through dozens or hundreds of examples are difficult to ignore. Efficient algorithms can produce many examples quickly and help the researcher find robust patterns. Efficient algorithms are also more reliable since we can demand more of them in terms of accuracy. For example, Vedenov and Miranda [25] use efficient projection methods and show that their solutions satisfy equilibrium conditions to more than six digits. Bernardo [3] also uses projection methods and attains three- and four-digit accuracy. That kind of accuracy gives us confidence in the numerical approximation. Furthermore, it is doubtful if real-life economic agents actually solve their problems with six- or four-digit accuracy. The numerical results are ε-equilibria for small ε and are as plausible a prediction of behavior as the exact equilibrium.

This symposium also attempts to deal with another problem facing economists who want to use computational methods. The treatment of computer code has been a serious problem hindering the development of computational methods in economics. Computer code (like data) is generally not included in published papers since there is not enough room. In fact, there is seldom any substantive description of the computational details. Authors generally do not include code in their submissions and referees typically do not ask to see code. Many authors discard their code after publication, and many who do save their code refuse to distribute it in a useful form. Since code is not meant to be seen by the reader of a paper, authors make little effort in making it understandable.

These problems make it difficult to replicate published work. Since replication is the hallmark of scientific work, this is potentially a serious problem. The JMCB replication project showed that replication is, in fact, a serious problem in economics papers. Dewald et al. (1986) presents the results of the JMCB replication project and showed how sloppy practices have undermined the quality of much economic research. They found that "inadvertent errors in published empirical articles are a commonplace rather than a rare occurrence." In particular they note the role of computer code, observing that "Many .. researchers utilize programs which they or their research assistants have written in FORTRAN, Pascal, or other languages; interpretation and evaluation of these programs is difficult at best – and impossible at worst – without considerable skill, experience, and the cooperation of the original programmer." They note: "Our findings suggest that the existence of a requirement that authors submit to the journal their programs and data along with each manuscript would significantly reduce the frequency and magnitude of errors."

The lack of care regarding code also hinders follow-up research. Current practices make it difficult for others to use and extend published computational work since they must often first duplicate the coding effort behind published papers. The original authors may like this since it gives them an advantage in doing extensions of their early work. However, it cuts off the original insights which others may be able to bring to the work.

These cavalier attitudes towards computer code ignores its importance. Computer code plays the same role in computational work as proofs play in economic theory. It is difficult to trust the statement of a theorem if one cannot see the proof. Similarly, it is difficult to be confident in computational results if one cannot see the computer code producing those results. Proofs are also valuable for ongoing research since they reveal techniques and strategies which will later help solve related problems. Authors of theory papers are required to show the techniques and tools they use to solve their problems. They are not granted any secrecy privileges on the content of their proofs. There is no evidence that this free exchange of information has hindered development of economic theory; in fact, it has surely helped. Similarly, computer code can be adapted to solve a variety of problems beyond its original specific application. Therefore, access to computer code would be valuable to many readers who could improve on it.

This symposium and the editors of *Economic Theory* will try to set an example of how code should be treated. Many of the authors in this symposium are making available their code (to the extent permitted by copyright laws) by posting it on the *Economic Theory* web page. Computer code is particularly valuable in the case of many of the papers in this symposium. A reader may be interested in the solutions for tastes and technological specifications not examined a paper. With the computer code, he could easily change those parameters or, to some extent, change utility and production functions, and rerun the code. The fact that *Economic Theory* has a web page accessible to economists around the world is another example of how computer technology can help economic research.

The role of computation in economics will continue to grow as computing power and algorithms improve. However, these tools must be used with care. The papers in this symposium show how a careful integration of mathematics, economics, and computer programming can produce robust and reliable analyses of important economic questions.

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