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Asset market equilibrium with general tastes, returns, and informational asymmetries☆

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Abstract

This paper develops a general computational approach for solving rational expectations equilibrium in asset markets with asymmetric information. Our approach can be applied to models with arbitrary specifications of tastes, return distributions, and information structures. We demonstrate our methodology by examining a variation of the canonical Grossman and Stiglitz (1980, American Economic Review 70, 393–408) model of endogenous information acquisition in which traders have constant relative risk aversion (CRRA) preferences and stock returns are lognormally distributed. We find that the results in Grossman and Stiglitz are not robust to changes in the parametric assumptions. The speed and accuracy displayed by our computational methods indicates that more complex problems are tractable. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Much of the literature analyzing asset markets with asymmetrically informed traders makes very special and simple assumptions about preferences, the distribution of returns, and the information asymmetries to derive closed-form expressions of equilibrium. Furthermore, many analyses add a group of traders, called 'noise' or 'liquidity' traders, whose actions are insensitive to prices and their information content. The focus on closed-form solutions and the inclusion of noneconomic behavior substantially limits the generality of the analyses and the range of questions which can be addressed. In this paper, we present a computational approach which can examine a far broader range of models and address many more questions.

Grossman (1976) used the combination of negative exponential utility functions, normally distributed payoffs, and normally distributed prediction errors to obtain closed-form solutions in a trading model with N competitive, privately informed traders. In this setting, all private information is summarized by a single statistic, allowing a single price to also be a sufficient statistic for all private information. Therefore, even when information is private, there is a market-clearing price rule which conveys the total content of all information. While the result is interesting, it is too good. For example, Jordan (1983) has shown that the strong revelation properties of the Grossman model is atypical among such economies, calling into question the wisdom of basing a theory on such a special example.

Grossman and Stiglitz (1980) developed a model in which traders have incentives to acquire private information even if it is partially revealed in equilibrium. To avoid the strong aggregation properties of the Grossman (1976) model they introduced 'noise traders' into their analysis. The key fact is that noise traders generate shocks to aggregate demand which are unrelated to the asset's payoff. This causes traders to wonder, for example, if a high price is due to some traders having good information or due to unexpected 'noisy' demand. Since this decomposition matters, there is no one-dimensional sufficient statistic which conveys all information. This 'noise trader' approach has been extensively used in analyses of asset markets with asymmetrically informed traders.

There are many undesirable features to this approach. The addition of 'noise traders' limits the type of analyses which can be conducted. For example, welfare analyses of such markets is difficult because the noise traders do not have well-defined preferences.[1] All of the assumptions in the Grossman–Stiglitz approach are special, not just the noise traders. The assumption of exponential

[1] For example, Ausubel (1990b) and Bernardo (1999) argue that the costs and benefits of insider trading cannot be analyzed rigorously except in models where all traders rationally use all available information to maximize well-specified objectives.

utility implies that a trader's holding of risky assets is unrelated to his wealth, an unreasonable assumption making dynamic, general equilibrium extensions (De Long et al., 1990,1991; Wang, 1994) of this model unrealistic. The assumption that the informed trader's prediction error is normal with known variance and that all traders know the true unconditional variance of the return is also very special. For example, it rules out asymmetric information about the variance of an asset's return, whereas in reality traders often disagree about an asset's riskiness as well as its expected return. The assumption of normally distributed asset payoffs is problematic because it implies unlimited liability, allows for negative consumption in equilibrium, and excludes assets, such as options, whose payoffs are not normally distributed. As emphasized by Admati (1989): 'For tractability, all of the models of noisy rational expectations equilibrium… assume exponential utility functions and normal distributions. Although these models capture many important phenomena, this limitation should be noted. Needless to say, tractable models with different parametric assumptions are sorely needed'.

In this paper, we develop a general computational approach to rational expectations modeling that can handle many more specifications of tastes, return distributions, and information structures. The numerical approach used here is based on the projection method described in Judd (1992). There the projection method was applied to conventional numerical problems arising in a simple stochastic growth model, and related methods have been refined to solve a variety of symmetric information economic problems. Unfortunately, no methods have been developed to analyze general models of asset markets with asymmetric information. The key novel difficulty is that the endogenous price law we want to compute appears in both the trader's decision rules and the information set used in computing each trader's conditional expectations. This feedback from equilibrium prices to traders' expectations imply that the equations defining a rational expectations equilibrium do not fit into any of the conventional categories of partial or ordinary differential equations or integral equations we usually see in economics and mathematics and their numerical literatures. Furthermore, Radner (1979) showed that rational expectations equilibria are generically fully revealing when there are only a finite number of states and prices are from R^n.[2] These results are also important for developing a numerical approach. Some might say that we could approximate a continuous model using an approximate model with a finite number of states. Radner's

[2] The reasons for these strong results are clear: when the price space is large relative to the complexity of the world, the full information price is (generically) different for each state, and prices can reveal all information. However, most would argue that the dimension of the information space exceeds the dimension of the price space, making it unlikely that reasonable price functions could convey all information.

results tell us that the discretized version would generically have a fully revealing equilibrium, thereby eliminating its value as an approximation to partially revealing rational expectations equilibrium.

The focus of this paper is to present projection methods for solving models with asymmetric information, discuss the problems and limitations of this approach, and present various diagnostics which can be used to test the quality of the numerical solution. We do not prove existence, leaving that problem for theoretical analysis. If there does not exist an equilibrium then the diagnostics would likely detect the nonexistence. In any case, our method did produce ε-equilibria for economically small ε for many interesting examples, and we advocate that no numerically computed equilibrium should be accepted unless it can be verified to be a ε-equilibrium for economically small ε.

We present our numerical methodology in the context of the Grossman and Stiglitz (1980) model of endogenous information acquisition. We replace their assumptions of negative exponential utility functions and normally distributed returns with the assumptions of constant relative risk aversion (CRRA) utility functions and lognormally distributed returns. Our assumptions preclude closed-form expressions of equilibrium and allow us to demonstrate the flexibility of our numerical approach. We show that the results in Grossman and Stiglitz are not robust to this alternative parameterization of their model. For example, Grossman and Stiglitz found that the informativeness of the price system is invariant to the variance of liquidity trades. In their model, an increase in liquidity trade variance reduces the informativeness of prices for a fixed proportion of informed traders. However, an increase in liquidity trade variance increases the *equilibrium* proportion of informed traders and does so to the extent that it *exactly* offsets the former effect on the informativeness of prices. In our model, however, we find the surprising result that an increase in the liquidity trade variance increases the equilibrium informativeness of prices: thus, increased information acquisition more than offsets the static effect of liquidity trade variance on the informativeness of equilibrium prices. Grossman and Stiglitz also derived a similar invariance result with respect to the residual uncertainty (after private information is observed) of the stock return. In our example, we find that greater residual uncertainty decreases the equilibrium informativeness of prices. While the equilibrium amount of information acquisition increases when residual uncertainty increases, it does not completely counteract the static effect that residual uncertainty has on the informativeness of the price system.

We also derive a novel comparative static result relating the equilibrium proportion of informed traders and the informational efficiency of prices to the wealth of potentially informed traders. In Grossman and Stiglitz, traders have negative exponential utility functions which exhibit no wealth effects. In our model, however, we show that increasing wealth decreases the proportion of informed traders but increases the informativeness of stock prices. This may seem surprising but there is a simple intuition. Wealthier traders act more aggressively on their information because they have decreasing absolute risk aversion. Thus, for a fixed proportion of informed traders, prices become more informative and the marginal benefit of acquiring information (beyond what can be inferred from the price) falls. In equilibrium, fewer traders acquire information but the impact on price informativeness due to the aggressive trading behavior of the informed more than offsets this.

Finally, we derive comparative statics results relating unconditional expected stock returns to the underlying parameters of the model. We find that expected stock returns are increasing in risk aversion, decreasing in wealth, increasing in residual payoff variance, and largely invariant to liquidity trade variance. This is another interesting exercise that is difficult to carry out in the standard exponential-normal models in which expected returns are not well-defined because normally distributed payoffs allow negative prices in some states. This is an important concern if one is interested in applying asymmetric information models to stock market return data (e.g., Campbell and Kyle, 1993; Spiegel, 1998). In contrast, computing expected stock returns is a straightforward exercise with our distributional assumptions.

We find that our computational approach quickly produces excellent approximate solutions. We test our approach on some numerically nontrivial benchmark cases where we know the true solution. In those cases, our numerical solutions correctly identify the known equilibrium prices and demands to four and five significant digits. To measure the quality of our approximations for cases where we do not know the solution, we take the bounded rationality approach described in Judd (1992) and compute norms of Euler equation errors implicit in our approximate solutions. We find small Euler equation errors for our approximate solutions, demonstrating that our solutions are ε-equilibria for small ε, on the order of a dollar mistake for each \$1,000,000 traded. We argue that such an ε-equilibria is as plausible a prediction of what real, boundedly rational, traders would do as is any exact equilibria. Approximate rational expectations concepts have also been developed in Anderson and Sonnenschein (1982) and Allen (1985a). The notion we adapt is motivated by computational considerations, but corresponds to an equilibrium where traders are not infinitely rational, but approximate the information content of prices using standard econometric methods. Our approach for evaluating the quality of our approximations corresponds to the tests an econometrician living in our model would use to evaluate statistical models to decide if some alternative is economically significantly better. We also find that projection methods applied to asymmetric information models produce far smaller errors than some other methods in the numerical asset pricing literature.

The speed displayed by these methods when applied to nontrivial problems indicates that more complex problems are tractable. Also, in this paper we use only elementary and general numerical methods; future versions which use problem-specific applications of asymptotic methods and more sophisticated integration methods will increase speed by at least a couple orders of magnitude. We believe that our numerical procedure for computing approximate equilibria will make it possible to examine many interesting problems that are otherwise intractable.

The remainder of the paper is organized as follows. Section 2 describes a version of the Grossman and Stiglitz (1980) model with CRRA preferences and lognormally distributed returns. The numerical methodology is developed in the context of this model in Section 3. We then discuss the results of our model. Section 4 tests our numerical algorithm on problems where we know the true solution. We then develop a methodology for checking the quality of approximate solutions in cases where we do not know the true solution. Section 5 concludes.

2. A model of endogenous information acquisition

In this section we consider an adaptation of the canonical Grossman and Stiglitz (1980) model of information acquisition. We replace their assumptions of negative exponential utility functions (implying constant absolute risk aversion) and normally distributed returns with the assumptions of constant relative risk aversion (CRRA) utility and lognormally distributed returns. In the next section, we will develop our general numerical methodology within the context of this model and we will demonstrate that the results of the Grossman and Stiglitz model are not robust to our alternative parameterization.

Consider a one period (two date) economy in which risk-averse traders allocate their wealth between a stock and a bond at date 0, and consume the proceeds at date 1. The bond is in perfectly elastic supply and pays a certain amount R dollars at date 1 for every dollar invested at date 0 while the stock pays a random amount $\tilde{Z}$ dollars per share at date 1. The stock payoff is lognormally distributed with $\tilde{Z} = \exp(\tilde{S} + \tilde{\varepsilon})$, where $\tilde{S}$ is normally distributed with mean μ and variance σ_S^2 and $\tilde{\varepsilon}$ is normally distributed with mean zero and variance σ_ε^2. The bond price is normalized to one, thus the price of the stock is expressed in units of bonds. We also normalize the supply of the stock to one.

There are two types of traders in the economy. The first type, the 'potentially informed', can acquire information about the stock payoff at a cost c. Following Grossman and Stiglitz, we assume that if information is acquired the random variable $\tilde{S}$ is observed precisely. The 'potentially informed' have identical cash endowments, W, identical share endowments, $\bar{\theta}$, and identical CRRA preferences of the form $u(c) = 1/(1+\gamma)c^{1+\gamma}$ where $\gamma < 0$ is the parameter of relative risk aversion. The proportion of such traders who choose to acquire information, denoted by λ, is determined endogenously. The second group of traders submit random demands $\tilde{x}$ which are normally distributed with mean zero and variance σ_x^2. Such 'liquidity traders' are used throughout the literature to ensure that the equilibrium price does not perfectly reveal the private information about $\tilde{S}$. Introducing noise into equilibrium prices is critical for any model of information acquisition because without it no trader would have the incentive to acquire costly information when it could be observed at no cost by observing the equilibrium price. Although there are many problems with using such a device, we ignore them in this example so that we can directly compare the effects of our parametric assumptions to those in the Grossman and Stiglitz model.[3] Finally, the random variables $\tilde{S}$, $\tilde{\varepsilon}$, and $\tilde{x}$ are mutually independent.

Consider a trader who has chosen to acquire information $\tilde{S} = s$ at cost c. The informed trader solves

$$\max_{\theta_{\mathrm{I}}} \mathrm{E}\left[\frac{1}{1+\gamma}\tilde{c}_{\mathrm{I}}^{1+\gamma}\,\middle|\,\tilde{S} = s\right],$$

where $\tilde{c}_{\mathrm{I}} = \theta_{\mathrm{I}}\tilde{Z} + [W - c - (\theta_{\mathrm{I}} - \bar{\theta})p]R$ represents the random date-1 consumption when the informed trader holds θ_{I} shares of stock after trade at date 0 and the share price is p dollars per share.

The first-order condition for the choice of θ_{I} is given by

$$\mathrm{E}[\tilde{c}_{\mathrm{I}}^{\gamma}(\tilde{Z} - pR)\,|\,\tilde{S} = s] = 0. \tag{1}$$

For the informed traders, the price p contains no payoff-relevant information that is not contained in s. For the uninformed traders, the information set is the equilibrium price alone. Let $P(s, x)$ be the asset price function which depends on the realizations of the random variables $\tilde{S}$ and $\tilde{x}$. Then the uninformed trader solves

$$\max_{\theta_{\mathrm{U}}} \mathrm{E}\left[\frac{1}{1+\gamma}\tilde{c}_{\mathrm{U}}^{1+\gamma}\,\middle|\,P(s,x) = p\right],$$

where $\tilde{c}_{\mathrm{U}} = \theta_{\mathrm{U}}\tilde{Z} + [W - (\theta_{\mathrm{U}} - \bar{\theta})p]R$ represents their random date-1 consumption.

The first-order condition for the choice of θ_{U} is given by

$$\mathrm{E}[\tilde{c}_{\mathrm{U}}^{\gamma}(\tilde{Z} - pR)\,|\,P(s,x) = p] = 0. \tag{2}$$

[3] We have also solved a similar model without liquidity traders which yields the same results. That model replaces the liquidity traders with a group of traders whose average risk tolerance is not known with certainty. The introduction of another source of uncertainty ensures that the equilibrium prices are not fully revealing. However, because all traders have well-defined preferences complete welfare analyses can be conducted in such a model. This is not true of liquidity trader models because liquidity traders do not have utility functions.

A rational expectations equilibrium is a collection of price and demand rules such that θ_{I} satisfies the first-order condition (1), θ_{U} satisfies the first-order condition (2), and the market for the stock clears in every state of the world

$$\lambda\theta_{\mathrm{I}}(s) + (1-\lambda)\theta_{\mathrm{U}}(P(s,x)) + x = 1 \quad \forall s, x. \tag{3}$$

Since the utility functions are concave, the second-order conditions are automatically satisfied. The equilibrium proportion of informed traders, λ, is determined by requiring that the expected utility achieved by those who choose to gather information, net of information gathering cost c, be equal to the expected utility achieved by those who do not. Finally, the assumption of rational expectations implies that the trader computes expectations using the distribution of $\tilde{Z}$ conditional on the trader's private information and the correct equilibrium relation between prices and information.

3. Numerical methodology

We cannot in general compute rational expectations equilibria. In fact, as demonstrated in Kreps (1977), there may not exist a rational expectations equilibrium. Allen (1985a,b) and Anderson and Sonnenschein (1982) have formulated notions of approximate equilibria and proven existence theorems for these notions. Since our computational approach does not exactly correspond to either of these approaches, we will present our own definition of ε-rational expectations, which we are attempting to compute.

Definition 1. $(P(s,x),\ \theta_{\mathrm{I}}(s),\ \theta_{\mathrm{U}}(p))$ is an ε-rational expectations equilibrium if and only if for all states in a set of probability $1-\varepsilon$:

1. for all trader types i with information sets $\mathscr{F}_i$, decisions are nearly optimal, that is

$$\left|\frac{\mathrm{E}[u_i'(c_i)(Z - P(s,x)R) \,|\, \mathscr{F}_i]}{\mathrm{E}[u_i'(c_i)(W + \bar{\theta}P(s,x)) \,|\, \mathscr{F}_i]}\right| \leqslant \varepsilon$$

   and
2. markets nearly clear, that is, $|\theta_{\mathrm{I}}(s) + \theta_{\mathrm{U}}(P(s,x)) + x - 1| \leqslant \varepsilon$ for all s, x.

Our notion of ε-equilibria asserts that the market *nearly* clears and that traders *nearly* optimize in *nearly* all states. Our 'nearly optimal' criterion is a ratio which expresses the optimality error in terms of the fraction of wealth. It is necessary to take this ratio if we want to make the concept of approximate equilibria independent of inessential transformations of u and changes in the units of wealth. If we do not focus on relative quantities, anything can be an ε-equilibrium for arbitrarily small ε just by replacing each u_i with u_i/M for very large M. This is a detail which arises when one does numerical work since we actually want to state the magnitude of ε, not just examine the limit as ε goes to zero. The near market clearing condition is similarly expressed in relative terms for the same reasons.

This definition may also include the asymptotically fully revealing approximate equilibria in Jordan (1982). However, we do not want to approximate these equilibria since they utilize unintuitive functions with large (asymptotically infinite) variation to reveal information. To avoid this, we approximate the price function $P(s,x)$ and the trading functions $\theta_{\mathrm{I}}(s)$ and $\theta_{\mathrm{U}}(p)$ with smooth functions of low variation. In many respects, our approximate equilibria most resembles the method used in Ausubel (1990a,b) but is more flexible.

3.1. Formulating the equilibrium approximation

We now formulate the numerical approximation for the equilibrium functions. Generally speaking, our method approximates equilibrium by finitely parameterizing $P(s,x)$, $\theta_{\mathrm{I}}(s)$, and $\theta_{\mathrm{U}}(p)$ and imposing a finite number of the conditions implicit in our definition of equilibrium. We now turn to the precise details in our model. We begin by approximating the price law with the polynomial

$$\hat{p}(s,x;\, a) = \sum_{j=0}^{N_p} \sum_{k=0}^{N_p} a_{jk} H_j(s) H_k(x),$$

where H_j denotes the degree j Hermite polynomial, and N_p represents the total degree of the polynomial approximation. We use Hermite polynomials because these polynomials are mutually orthogonal with respect to the normal density with mean zero.[4] We also use the complete set of polynomials, rather than the full tensor product, to reduce the number of unknowns.[5]

[4] See Judd (1992) for a discussion of the advantages of orthogonal bases in projection methods.

[5] The set of *complete polynomials of degree N* over R^m is defined to be

$$\mathscr{P}_N \equiv \left\{ x_1^{i_1} \cdots x_m^{i_m} \,\middle|\, \sum_{\ell=1}^{m} i_\ell \leqslant N,\ i_\ell \geqslant 0,\ \ell = 1,\dots,m \right\}.$$

An alternative is the *tensor product of degree N* over R^m:

$$\mathscr{T}_N \equiv \{ x_1^{i_1} \cdots x_m^{i_m} \,|\, 0 \leqslant i_\ell \leqslant N,\ \ell = 1,\dots,m \}.$$

The use of complete polynomials generally results in little loss of accuracy as compared to the full tensor product basis but has the advantage of many fewer unknown parameters. We use the tensor product notation in the paper to avoid clutter.

An informed trader's demand rule will be a function of their private information s. The informed learn nothing from the equilibrium price that is not already privately observed, thus we postulate the following approximation to the informed's demand function:

$$\hat{\theta}_{\mathrm{I}}(s;\, b^{\mathrm{I}}) = \sum_{m=0}^{N_\theta} b^{\mathrm{I}}_m H_m(s).$$

The demand policies for the uninformed traders, however, will be a function of the equilibrium price alone thus we postulate the following approximation:

$$\hat{\theta}_{\mathrm{U}}(\hat{p}(s,x;\, a);\, b^{\mathrm{U}}) = \sum_{m=0}^{N_\theta} b^{\mathrm{U}}_m H_m(\hat{p}(s,x;\, a)).$$

Our goal then is to determine the unknown a_{jk}, b^{I}_m, and b^{U}_m coefficients. Their number depends on the choice of N_θ and N_p. We will (for reasons stated below) let $N_\theta = N_p = 3$, resulting in eighteen unknown coefficients if complete polynomials are used: ten for the price function and four for each of the two demand functions. To determine the unknown coefficients we impose projection conditions on the traders' first-order conditions and market clearing. The total number of conditions will equal the number of unknown coefficients, hoping that they are sufficient to fix the unknown coefficients. In this paper, our approach is, using an econometric term, to exactly identify the unknown coefficients by imposing an equal number of projections.

3.2. Computing conditional expectations

Numerical implementation of the conditional expectation conditions implied by (1) and (2) is the most challenging aspect of this problem. Suppose that X and Y are random variables with a joint density function $f(X, Y)$, and let Z be the expectation of Y conditional on X. We will use the following definition of conditional expectation:

Definition 2. The function $Z(X) = \mathrm{E}[Y|X]$ if and only if

$$\mathrm{E}[(Z(X) - Y)g(X)] = \int (Z(X) - Y)g(X)f(X, Y)\,\mathrm{d}X\,\mathrm{d}Y = 0 \tag{4}$$

for all continuous bounded functions, $g(X)$, of X.

Intuitively, this says that the prediction error of the conditional expectation, $\mathrm{E}[Y|X]$, is uncorrelated with any bounded, continuous function of the conditioning information, X. This definition replaces the conditional expectation with an infinite number of integration conditions. This approach can be used to approximate the conditional expectations implicit in the first-order conditions (1) and (2).[6] To check if $Z(X) = \mathrm{E}[Y|X]$, one need only verify $0 = \int (Z(X) - Y)g(X)f(X, Y)\,\mathrm{d}X\,\mathrm{d}Y$ for $g \in G$ where G is a set of functions which spans the space of continuous bounded functions of X. Of course, one cannot compute an infinite number of integrals. In practice, we approximate $Z(X)$ by finitely parameterizing $Z(X)$ and use a finite number of the unconditional expectation conditions to identify the free parameters. Below we will outline the details of applying these ideas to our specific model.

Using the definition of conditional expectation given in (4) we numerically approximate the first-order condition for the informed in (1) with the conditions

$$\mathrm{E}[\tilde{c}_{\mathrm{I}}^{\gamma}(\tilde{Z} - \hat{p}(s,x;\, a)R)H_m(s)] = 0, \quad m = 0, \dots, N_\theta. \tag{5}$$

Similarly, we approximate the first-order condition for the uninformed with the conditions

$$\mathrm{E}[\tilde{c}_{\mathrm{U}}^{\gamma}(\tilde{Z} - \hat{p}(s,x;\, a)R)H_m(\hat{p}(s,x;\, a))] = 0, \quad m = 0, \dots, N_\theta. \tag{6}$$

The projection conditions (5) constitute only a portion of the conditional expectation in Eq. (1). According to the definition (4) we would need to project $\tilde{c}_{\mathrm{I}}^{\gamma}(\tilde{Z} - \hat{p}(s,x;\, a)R)$ on all bounded, continuous functions of the conditioning information to get equivalence. The hope is that a small number of projections can yield a useful approximation. Below we will make diagnostic checks of our candidate approximations.

It is clear from the representation of the first-order condition in Eqs. (5) and (6) why we can examine models with general tastes, returns, and information structures. In the exponential-normal framework, linear closed-form solutions can be obtained because of a number of very special assumptions including (i) the joint distribution of asset payoffs, information, and liquidity trades is multivariate normal, (ii) consumption is normally distributed *conditional on observed information and prices*, and (iii) expected utility with negative exponential preferences has a closed-form representation when consumption is conditionally normal. Relaxing any of these features makes it virtually impossible to obtain a closed-form solution. The requirement that consumption be normally distributed conditional on observed information and prices rules out (i) non-linear equilibrium prices; (ii) general information structures; (iii) non-normally distributed securities such as options; and (iv) traders conditioning on other market statistics, such as trading volume, which are generally not normally distributed. Our method, however, does not make such stringent requirements.

[6] There are many obvious connections with statistical regression analysis. However, it is not advantageous to develop those analogies. It is more efficient to think of $Z(X) = \mathrm{E}[Y|X]$ as shorthand for a list of integrals, and formulate the problem in terms of integrals. Probabilistic ideas tend to lead one to think in terms of random variables and Monte Carlo methods, an approach which is substantially inferior here.

The only implicit assumptions are that the utility functions are smooth and that the random variables describing returns and information have continuous (or, better yet, C^{∞}) cumulative distribution functions. While these restrictions are mathematically substantive, they impose few economically substantive assumptions.

Equilibrium also requires market clearing. We cannot impose market clearing in each and every state and simultaneously have all traders follow rules measurable in their individual information. Therefore, we assume that deviations from market clearing are orthogonal to several of our basis functions:

$$\mathrm{E}\{[\lambda\hat{\theta}_{\mathrm{I}}(s) + (1-\lambda)\hat{\theta}_{\mathrm{U}}(\hat{p}(s,x)) + x - 1]H_j(s)H_k(x)\} = 0, \quad j,k = 0,\dots,N_p. \tag{7}$$

The disadvantage of (7) is that the market would not exactly clear in all s and x, but each trader's rule is measurable in his information. The deviations from perfect clearing can be interpreted as inventory noise on the part of market makers. Of course, it would be better to model the market maker behavior directly. This may sound like trading noise similar to the undesirable noise trader device, but we work to reduce the deviations from pure market clearing to an arbitrarily small magnitude, whereas in models with noise traders one cannot reduce the noise without also eliminating the substantive results. A simple way to check the market clearing condition is to compare the sum of the demands to the supply for randomly chosen s and x: in all the examples below we find that the deviations from market clearing are small (of the order of one-in-one-million) and can be made smaller by using more projection conditions.[7]

We now have reduced the equilibrium problem to the combined system (5)–(7) which is a collection of nonlinear equations. Hopefully this system has a solution for the unknown coefficients of $\hat{p}(s,x;\, a)$, $\hat{\theta}_{\mathrm{I}}(s;\, b^{\mathrm{I}})$, and $\hat{\theta}_{\mathrm{U}}(\hat{p}(s,x;\, a);\, b^{\mathrm{U}})$. The equilibrium relation between the proportion of informed, λ, and the cost of information, c, is determined by *fixing* λ and finding the value of c which equates the expected utilities of the informed and the uninformed. The unconditional expectations in (5)–(7) are not yet in a computable form, since they are integrals which generally have no analytic solution. To implement the system we need to replace these integrals with approximations. We use Gaussian quadrature techniques to approximate these integrals (see Judd, 1998). The details are available from the authors upon request.

[7] An alternative is to force one of the investor types to hold the remaining shares after all other investor types optimize. This will guarantee market clearing in every state of the world and reduce the number of unknowns. However, this is problematic because the residual investor's demand will not be measurable with respect to his information set.

To compute solutions to our system of non-linear equations, we use the MINPACK program HYBRD, which is an implementation of the Powell hybrid method, an adaptation of Newton's method. If it has difficulty finding a solution, HYBRD switches to a least-squares mode and returns the parametersr $(a, b^{\mathrm{I}}, b^{\mathrm{U}})$ which minimizes the sum of squared errors. Our experience is that HYBRD will find solutions in the sense that it finds values for $(a, b^{\mathrm{I}}, b^{\mathrm{U}})$ where the deviations from zero in the system (5)–(7) are all numerically indistinguishable from zero.

3.3. Results

Figs. 1–6 plot the equilibrium proportion of informed traders, λ, and the noisiness of equilibrium prices as a function of one of the exogenous parameters of the model. We provide a unit-free measure of equilibrium price noise by computing the mean absolute deviation of equilibrium prices from the full-information price divided by the equilibrium price. The results are qualitatively identical for several other reasonable measures of informational efficiency. For all of these experiments, we choose the base-case parameterization of $\gamma = -3$: $\sigma_S^2 = \sigma_\varepsilon^2 = 0.1$; $\sigma_x^2 = 0.01$; $W = 1$; $\bar{\theta} = 0.5$; $R = 1.03$; $\mu = 1$; and $c = 0.01$. These parameters yield reasonable annual, individual stock return data (6% equity risk premium and 35% annual standard deviation) and bond return data (3% real rate).

Several of our comparative statics results are consistent with those found in Grossman and Stiglitz (Theorem 4). For example, Fig. 1 demonstrates that increasing the cost of information, c, decreases information acquisition and decreases the informativeness of equilibrium prices. With fewer informed traders, equilibrium prices convey less information. Fig. 2 shows that increasing σ_S^2, keeping $\sigma_S^2 + \sigma_\varepsilon^2$ constant, first leads to an increase then a decrease in information acquisition but always increases the informativeness of equilibrium prices. The intuition for these results are discussed in Grossman and Stiglitz.

Some of our results, however, differ in important ways from those in Grossman and Stiglitz. For example, they found that increasing σ_ε^2 while holding the ratio $\sigma_S^2/\sigma_\varepsilon^2$ fixed has no effect on the informational efficiency of equilibrium prices. On one hand, for fixed λ, increasing σ_ε^2 while holding the ratio $\sigma_S^2/\sigma_\varepsilon^2$ constant decreases the informativeness of prices. The reason for this is that risk-averse informed traders act less aggressively on their private information when there is greater residual risk (after observing $\tilde{S} = s$). However, the marginal benefit of observing the signal precisely (compared to observing the price) increases which leads more traders to acquire information which in turn increases the informativeness of prices. In their model, these two effects *exactly* offset each other in equilibrium! In our model, however, the increase in price informativeness due to increased information acquisition always overwhelms the decrease in price informativeness due to the static effect of increasing σ_ε^2 in

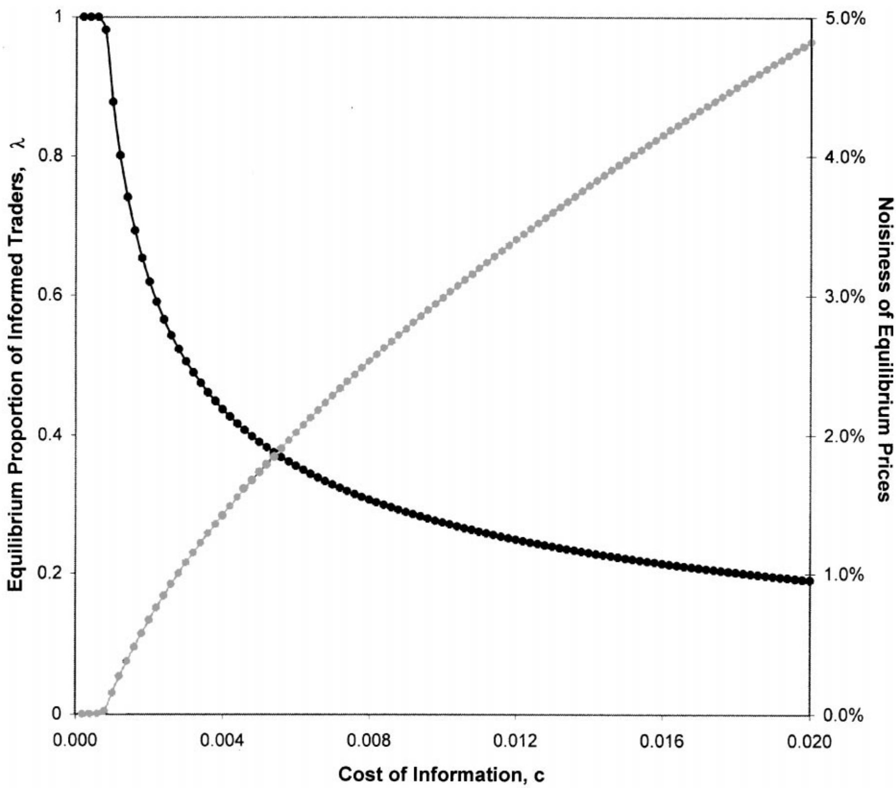

Fig. 1. The equilibrium proportion of informed traders, λ, and the noisiness of equilibrium prices as a function of the cost of information, c. The noisiness of equilibrium prices is measured by the mean absolute deviation of the equilibrium price from the full-information price divided by the equilibrium price. The equilibrium λ is plotted in black and the noisiness of equilibrium prices is plotted in gray. The base-case parameters are $\gamma = -3$; $W = 1$; $R = 1.03$; $\mu = 1$; $\sigma_S^2 = \sigma_\varepsilon^2 = 0.1$; and $\sigma_x^2 = 0.01$.

equilibrium. Fig. 3 demonstrates that as σ_ε^2 increases, holding $\sigma_S^2/\sigma_\varepsilon^2$ fixed, equilibrium prices become less informative. We verified that this comparative statics result is robust to a large space of values for the other parameters of the model.

Grossman and Stiglitz derived a similar invariance result with respect to the liquidity trade variance parameter, σ_x^2. Again, they showed that for fixed λ, an increase in σ_x^2 makes prices noisier, however, an increase in σ_x^2 also increases the marginal benefit of becoming informed thereby leading to greater information acquisition. In their model, these two effects exactly offset each other. Fig. 4 shows, however, that in our model an increase in σ_x^2 actually makes equilibrium prices more informative! The increase in information acquisition dominates the

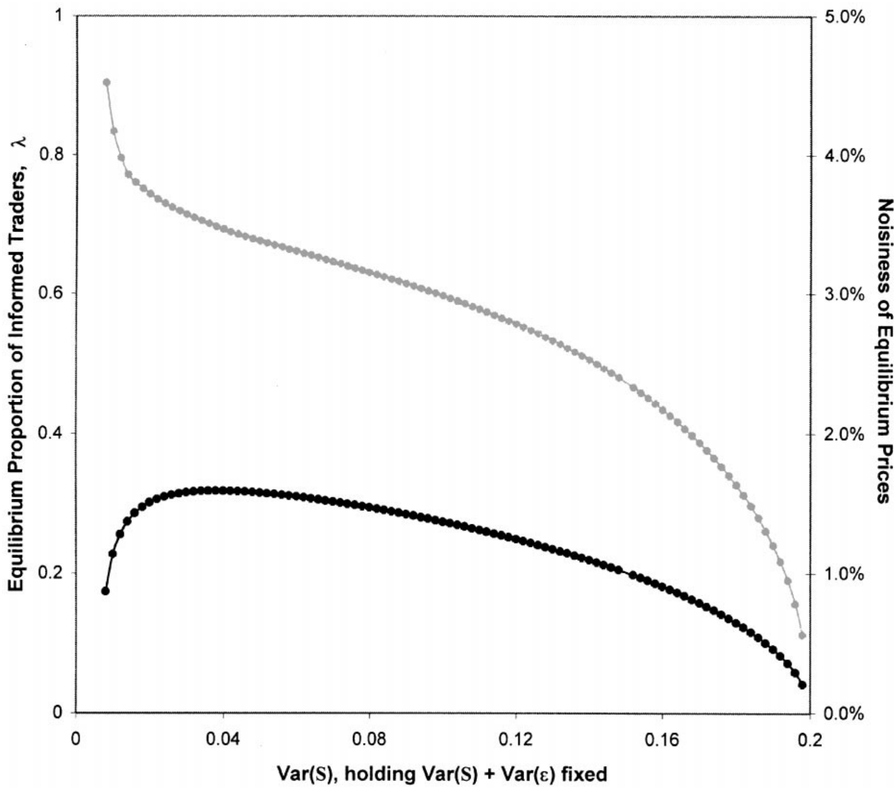

Fig. 2. The equilibrium proportion of informed traders, λ, and the noisiness of equilibrium prices as a function of σ_S^2, holding $\sigma_S^2 + \sigma_\varepsilon^2 = 0.2$ fixed. The noisiness of equilibrium prices is measured by the mean absolute deviation of the equilibrium price from the full-information price divided by the equilibrium price. The equilibrium λ is plotted in black and the noisiness of equilibrium prices is plotted in gray. The base-case parameters are $\gamma = -3$; $W = 1$; $R = 1.03$; $\mu = 1$; $\sigma_x^2 = 0.01$; and $c = 0.01$.

static effect that increasing σ_x^2 has on the informativeness of prices. Again, this comparative statics results is robust to a large space of values for the other parameters.

As in Grossman and Stiglitz, Fig. 5 demonstrates that decreasing the traders' risk aversion decreases information acquisition but also increases the informativeness of equilibrium prices. An important limitation of the Grossman and Stiglitz model is that the assumption of negative exponential utility implies no wealth effects. In our model, however, there are wealth effects because traders have CRRA preferences. In Fig. 6 we show that increasing the wealth of the potentially informed traders decreases equilibrium information acquisition but increases the informativeness of equilibrium prices. At first glance, this seems

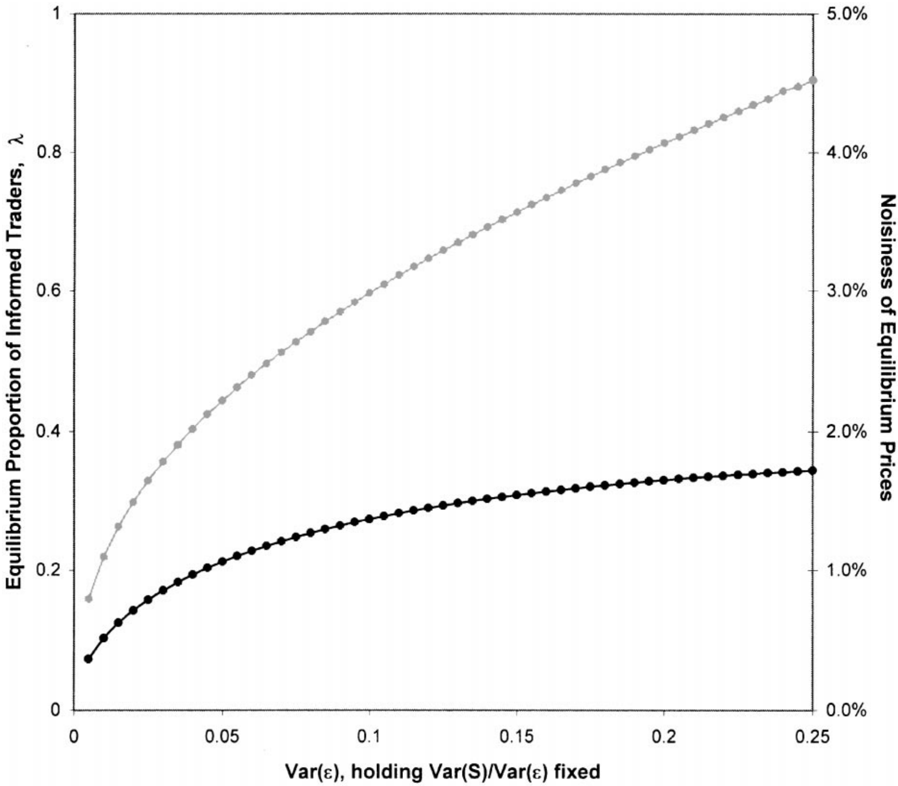

Fig. 3. The equilibrium proportion of informed traders, λ, and the noisiness of equilibrium prices as a function of σ_ε^2, holding $\sigma_S^2/\sigma_\varepsilon^2 = 1$. The noisiness of equilibrium prices is measured by the mean absolute deviation of the equilibrium price from the full-information price divided by the equilibrium price. The equilibrium λ is plotted in black and the noisiness of equilibrium prices is plotted in gray. The base-case parameters are $\gamma = -3$; $W = 1$; $R = 1.03$; $\mu = 1$; $\sigma_x^2 = 0.01$; and $c = 0.01$.

surprising. One would expect that if information acquisition is reduced so would the informativeness of equilibrium prices. However, wealthier traders in our model act more aggressively on their private information because they have decreasing absolute risk aversion. While there are fewer informed traders, their aggressive trading behavior impounds more information into prices, so much so that it dominates the effect that reduced information acquisition has on price informativeness. This intuition applies to the results in Fig. 5: although reducing risk aversion reduces information acquisition, it increases the informativeness of equilibrium prices via the more aggressive behavior of the informed traders.

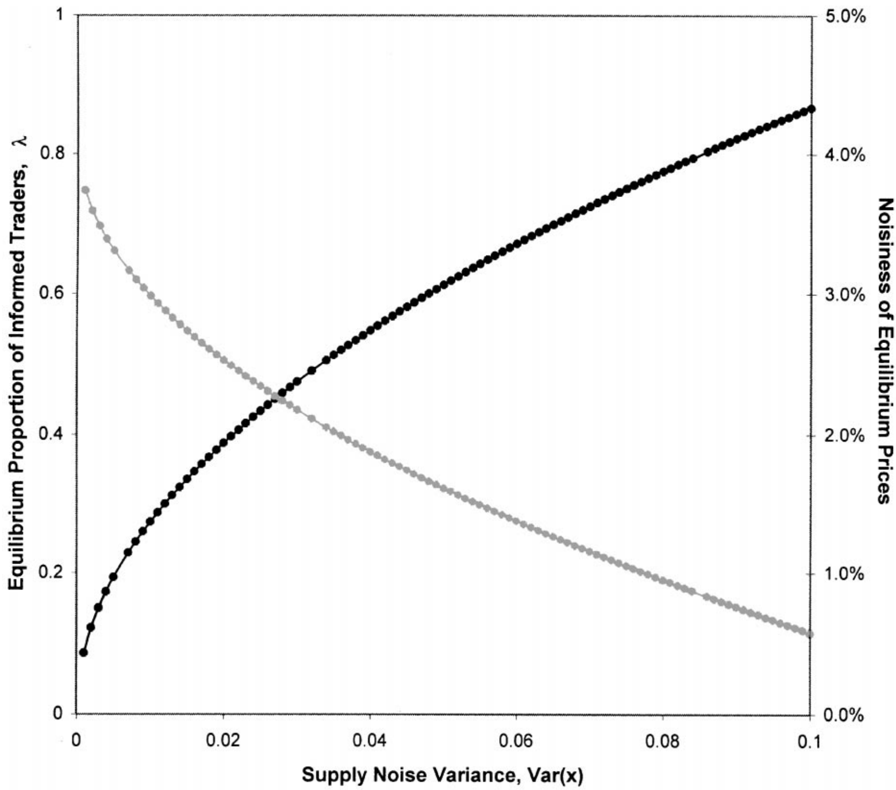

Fig. 4. The equilibrium proportion of informed traders, λ, and the noisiness of equilibrium prices as a function of the supply noise variance, σ_x^2. The noisiness of equilibrium prices is measured by the mean absolute deviation of the equilibrium price from the full-information price divided by the equilibrium price. The equilibrium λ is plotted in black and the noisiness of equilibrium prices is plotted in gray. The base-case parameters are $\gamma = -3$; $W = 1$; $R = 1.03$; $\mu = 1$; $\sigma_S^2 = \sigma_\varepsilon^2 = 0.1$; and $c = 0.01$.

Figs. 7A–D examine the effects of the underlying parameters of the model on unconditional expected stock returns. This type of analysis is difficult in the standard exponential-normal models because normally distributed payoffs allow negative prices in some states in which case expected returns are not well defined. Several papers have tried to apply the exponential-normal model to return data with considerable difficulty (e.g., Campbell and Kyle, 1993; Spiegel, 1998). In contrast, computing expected stock returns is a straightforward exercise with our distributional assumptions. For the results in Fig. 7, we choose a different set of base-case parameters than in Figs. 1–6 to reflect a stock market index rather than individual stocks. The unconditional standard deviation of stock returns in this calibration mimic the historical U.S. market average of

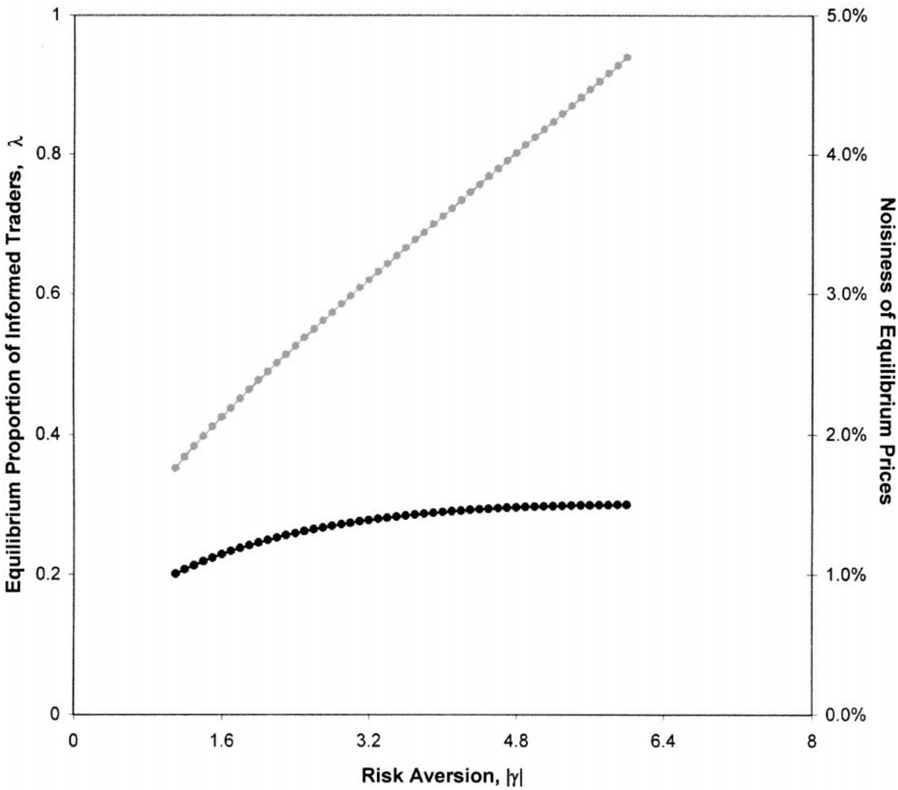

Fig. 5. The equilibrium proportion of informed traders, λ, and the noisiness of equilibrium prices as a function of the risk aversion parameter, $|\gamma|$. The noisiness of equilibrium prices is measured by the mean absolute deviation of the equilibrium price from the full-information price divided by the equilibrium price. The equilibrium λ is plotted in black and the noisiness of equilibrium prices is plotted in gray. The base-case parameters are $W = 1$; $R = 1.03$; $\mu = 1$; $\sigma_S^2 = \sigma_\varepsilon^2 = 0.1$; $\sigma_x^2 = 0.01$; and $c = 0.01$.

20%/year. Furthermore, it is assumed that investors have a relative risk aversion parameter of $\gamma = -1.5$ and non-stock market wealth roughly twice their stock market wealth.

Several interesting results emerge from our analysis. Our asymmetric information model predicts that the unconditional equity risk premium is increasing in risk aversion, decreasing in wealth, increasing in the residual payoff variance, and largely invariant to the liquidity trade variance. An increase in risk aversion (and a decrease in wealth) reduces the aggregate risk tolerance in the economy thereby increasing the equity risk premium. An increase in the residual payoff variance increases the supply of risk in the economy thereby increasing

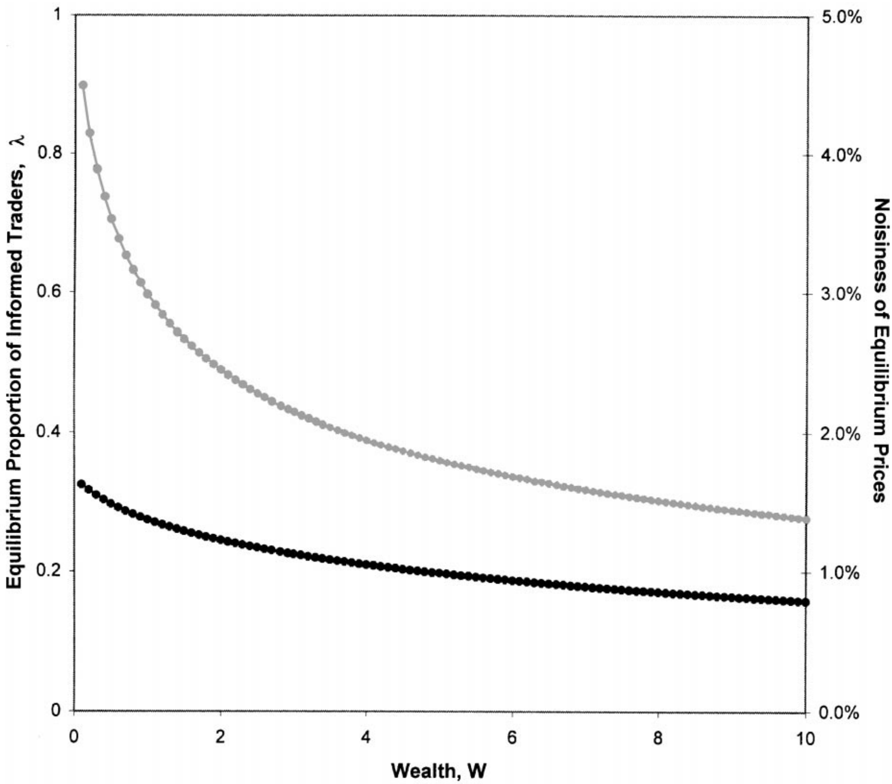

Fig. 6. The equilibrium proportion of informed traders, λ, and the noisiness of equilibrium prices as a function of wealth, W. The noisiness of equilibrium prices is measured by the mean absolute deviation of the equilibrium price from the full-information price divided by the equilibrium price. The equilibrium λ is plotted in black and the noisiness of equilibrium prices is plotted in gray. The base-case parameters are $\gamma = -3$; $R = 1.03$; $\mu = 1$; $\sigma_S^2 = \sigma_\varepsilon^2 = 0.1$; $\sigma_x^2 = 0.01$; and $c = 0.01$.

the equity risk premium (due to concave preferences). Liquidity trade variance surprisingly has no meaningful impact on the unconditional equity risk premium. For a fixed level of information acquisition, an increase in liquidity trade variance should increase the equity risk premium because it increases the supply of risk. However, we know from Fig. 4 that an increase in σ_x^2 leads to an increase in information acquisition (improving the equilibrium informativeness of stock prices) which reduces total risk. In our analysis, these effects largely offset each other. Finally, our model predicts reasonable values for the unconditional equity risk premium (roughly 6%) with risk aversion parameters of the order $\gamma = -5.5$ in contrast to the much higher risk aversion parameters required in symmetric information models.

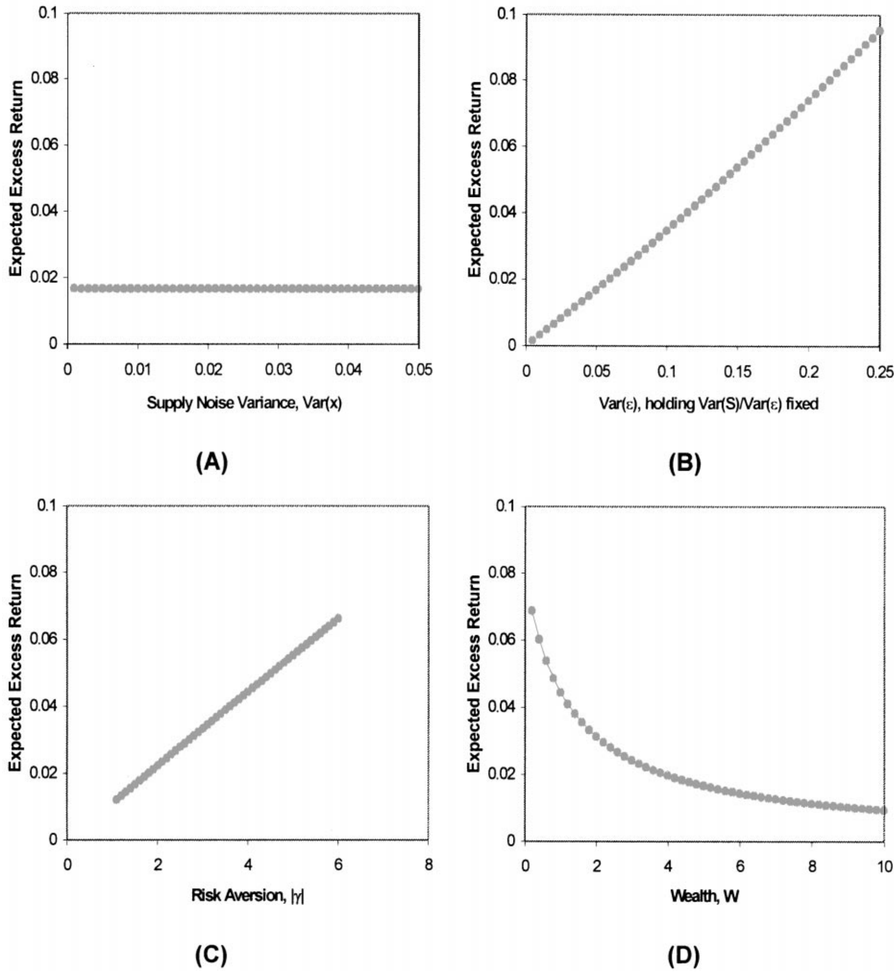

Fig. 7. (A–D). The unconditional expected return on the stock as a function of (A) supply noise variance, σ_x^2; (B) residual payoff variance, σ_ε^2, holding $\sigma_S^2/\sigma_\varepsilon^2 = 1$ fixed; (C) risk aversion, $|\gamma|$; and (D) wealth, W. The base-case parameters are $\gamma = -1.5$; $W = 5$; $R = 1.03$; $\mu = 1$; $\sigma_S^2 = \sigma_\varepsilon^2 = 0.1$; $\sigma_x^2 = 0.01$; and $c = 0.01$.

4. Applications to soluble models

We now evaluate the accuracy of our methodology. Fortunately, there are some numerically non-trivial cases where we know the solution. In this section, we follow standard practice in the numerical literature and test our method by applying it to problems where we know the true solution. This will give us some idea as to the accuracy of the procedure and its performance. We then develop a methodology for checking the quality of approximate solutions in cases where we do not know the true solution.

4.1. No-trade examples

If all traders have the same utility function and endowment then there will be no trade (each trader will hold their endowment) in a rational expectations equilibrium, regardless of the distribution of private information. Furthermore, the equilibrium price will be the full-information price (see Milgrom and Stokey, 1982). Numerical calculation of the full information prices is a straightforward calculation of marginal rates of substitution, involving only numerical integration which in this case, because of the smooth functions involved, will be very accurate. While we may know these facts, the algorithm does not 'know' these facts and instead approaches the problem in the general way. Therefore, we can check our algorithm on these cases.

We considered the following model covered by the Milgrom–Stokey theorem. We assume that there is a group of traders endowed with private information $\tilde{Y} = \tilde{Z} + \tilde{\varepsilon}$ where $\tilde{Z}$ is the random stock payoff and ε is signal noise which is normally distributed with mean 0 and variance 0.1, and three groups of traders who have no private information but can observe the equilibrium price. We applied our algorithm to compute equilibrium for twenty cases covered by the Milgrom–Stokey theorem with a wide variety of utility functions and returns. By comparing it with the full information calculations, we found that this method generated the correct prices and holding strategies to within at least three and often four or five significant digits when we used degree three polynomials for the pricing and demand functions. We also found the correct solution even if the initial guesses were poor, indicating the stability of the method.

Table 1 displays a typical example of the no-trade result. In this example, all traders have constant relative risk aversion (CRRA) preferences with parameter $\gamma = -3$ and the stock payoff is lognormally distributed, i.e. $\log \tilde{Z}$ is distributed normal with mean 0.25 and variance 0.1. All traders are endowed with cash wealth of 1.0 and $\frac{1}{4}$ shares of the stock. Even though some traders are informed and others not, there should be no trading under any conditions (i.e. they should always hold their endowment) and equilibrium prices should equal the full information price in each state.

The column in Table 1 labelled y denotes the realization of the signal Y measured in terms of the standard deviation. Thus, the value $y = -1$ implies that the informed traders observe a signal which is one standard deviation below the mean. The Computed price column denotes the equilibrium price in that state, the Full-info price column denotes the equilibrium price in a hypothetical economy in which all traders observed all the information directly, and the

Table 1
No-trade example[a]

y	Computed price	Full-info price	θ_{I}	θ_{U}
− 2	0.82022	0.82024	0.24995	0.25007
− 1	1.02078	1.02079	0.25011	0.24995
0	1.26937	1.26936	0.24995	0.25000
1	1.57751	1.57751	0.24998	0.25004
2	1.95947	1.95947	0.25014	0.24988

[a]Table 1 demonstrates a typical example in which the Milgrom–Stokey (1982) no-trade theorem holds. Prices and shareholdings are correct to roughly four significant digits with cubic polynomial approximation and 7-point quadrature rules.

θ_{I} and θ_{U} columns denote the shareholdings after trading for the informed group and each of the three uninformed groups, respectively. Using complete, cubic polynomial approximations for the price and demand functions and 7 quadrature nodes for computing integrals we find that the projection method yields a very accurate approximation of the true equilibrium. The computed price and demand functions are correct to about 4 decimal places. This example is typical over a wide range of tastes and information specifications. Since the combination of cubic polynomial approximations and 7-point product integration formulas did so well here, we use them below in later examples.

4.2. Known asset demand example

DeMarzo and Skiadas (1998) derived closed-form expressions for asset demands in competitive models with asymmetric information when traders have identical, linear risk tolerance. These expressions hold regardless of the specific distributional assumptions on returns and information, and for any rational expectations prices, fully informative or not. In particular, they showed that if traders have CRRA preferences with identical parameters of relative risk aversion then (stochastic) asset demands are given by

$$\theta_i^* = \left(\frac{W_i + \bar{\theta}_i p}{W + \bar{\theta} p}\right)\bar{\theta},$$

where p is the equilibrium price, W_i is trader i's cash wealth, $\bar{\theta}_i$ is trader i's share endowment, $W \equiv \sum_i W_i$, and $\bar{\theta} \equiv \sum_i \bar{\theta}_i = 1$.

This expression for asset demands is independent of private information but is nonlinear in p and presents a useful benchmark for testing the quality of the

Table 2
Known asset demand example[b]

y	Computed price	$\theta_{\mathrm{I}} - \theta_{\mathrm{I}}^*$	$\theta_{\mathrm{U_1}} - \theta_{\mathrm{U_1}}^*$	$\theta_{\mathrm{U_2}} - \theta_{\mathrm{U_2}}^*$
− 2	0.78945	− 0.00001	0.00002	0.00000
− 1	0.97742	0.00001	− 0.00004	0.00002
0	1.20914	0.00000	0.00002	− 0.00002
1	1.49507	0.00000	0.00002	− 0.00001
2	1.84811	0.00001	− 0.00017	0.00014

[b]Table 2 demonstrates an example in which asset demands are known exactly. The numerical approximation is correct to four and five significant digits.

approximate demand functions computed using our method. Table 2 demonstrates the results of a model in which all traders have identical CRRA preferences with risk aversion parameter -4.5 but differ in their information and their cash and share endowments. As in our Milgrom–Stokey example above, the log of the stock payoff, $\log Z$, is distributed normal with mean 0.25 and variance 0.1. There is one informed group of traders who observe $Y = Z + \varepsilon$ with ε distributed normal with mean 0 and variance 0.1. There are two groups of uninformed traders who only learn about Z via the equilibrium price. The informed group is endowed with cash wealth of 1 and 0.4 shares of the stock, the first uninformed group is endowed with cash wealth of 1 and 0.4 shares of the stock, and the second uninformed group is endowed with zero cash wealth and 0.2 shares of stock. In the last three columns of Table 2 we compare the demands in various states using cubic polynomial approximation, θ_i, to the closed-form expression derived in DeMarzo and Skiadis, θ_i^*, and find that our approximate demand functions are correct to four and five significant digits when computed at numerous states.

4.3. Accuracy measures with bounded rationality interpretations

Even after we have computed a candidate solution to the system (5)–(7) we cannot uncritically accept it. We saw above that the algorithm did well in some cases where we knew the true solution. However, we want to be able to evaluate the candidate solution in all cases, not just in those special cases where we know the answer. Just because we have a solution to the system (5)–(7) does not mean that it is a good approximation to the solution of the real problem, (1)–(3). For example, it could be that we chose approximations of insufficient flexibility, or that the integration formulas are too imprecise; in either case it is unlikely that our candidate is a good approximation to the true solution. We need tests to indicate whether we can accept the solution to (5)–(7) or if we need to go back

Table 3
$\log_{10}$ projection errors

Approx.:	2			3			3		
Quad. rule:	5			5			7		
Projection:	5	6	7	5	6	7	5	6	7
$\gamma = -1.5$	− 6.19	− 6.35	− 6.91	− 6.41	− 7.12	− 7.04	− 6.53	− 7.47	− 7.15
$\gamma = -2.5$	− 5.81	− 6.51	− 6.38	− 5.86	− 6.16	− 6.25	− 6.08	− 6.49	− 6.52
$\gamma = -3.5$	− 5.23	− 5.71	− 5.80	− 5.36	− 5.72	− 5.89	− 5.52	− 5.88	− 6.01[c]

[c]Table 3 reports ($\log_{10}$) consumption errors, as a proportion of wealth, when projecting on higher-order basis functions. Thus, an entry of -6 implies consumption errors of one-millionth part of wealth.

and make different choices for the quadrature formula and/or the approximation form.

We attack this problem by computing a measure of inaccuracy. Once we have a candidate solution for the price and trading rules, we can ask how much better could a trader do if he used more information than implicit in the equilibrium conditions. Note that the equilibrium conditions force him to choose decision rules which yield Euler equation residuals which are orthogonal to a restricted set of test functions. If we think of the trader as being an econometrician, this essentially allows him to do only a limited regression analysis of the data. Such a trader could look at the data and use more projections than we use in (5)–(7).

To check the quality of our candidate equilibrium, we compute the wealth equivalent of the Euler equation residual when projected in other directions. This is the *consumption error*, that is, the difference, in consumption units, between following the candidate equilibrium rule versus following a rule which uses more information in making inferences from the price. We operationalize this by taking the equilibrium law and subjecting it to a more refined regression analysis and asking how much a trader will gain if he is allowed to use the better inference rule. As Table 3 suggests, the Euler equation residual errors are very small, approximately one in ten-thousand to one in one-hundred-thousand parts of wealth, when projected in directions not used to approximate the equilibrium. Moreover, the Euler equation errors give us clues about the optimal number of basis functions and quadrature nodes to use in the approximation. We find that cubic approximation works extremely well, with little to be gained by moving to quartic approximation. We also find that using twice as many quadrature nodes in each dimension as the degree of approximation works very well. Thus, for cubic approximation one should use 6 or 7 quadrature nodes for each dimension of integration.

In this table we report $\log_{10}$ consumption errors. Thus, a consumption error of -6 implies that Euler equation residuals, when projected on polynomials not used in determining the equilibrium, are approximately one in one-millionth part of wealth. We report consumption errors for several risk aversion parameters, degrees of polynomial approximation, and quadrature rules.

This procedure reflects a bounded rationality attitude towards our calculations. If these computational methods produce a policy function with small optimization errors, then that approximate policy function is as compelling a description of behavior as the equilibrium policy function since it is unclear why individuals would bother making the nontrivial effort to find the 'true' policy function if the gain is small. From this perspective, the challenge of numerical economic modeling is not in finding the perfectly accurate description of the mathematical equilibrium, but in finding the collection of behaviors which are approximately rational.

The size of the errors in Table 3 should also be compared with the existing standards of the published literature. Any numerical approximation involves error, and we must make judgments about what constitutes an acceptably small error. The errors in Table 3 are far smaller than the standards used currently. For example, Heaton and Lucas (1996) develop a numerical method to compute equilibria in dynamic symmetric information models and accept equilibria where the demand and supply price differ by up to one percent in each period. The Euler equation errors reported in Table 3 are orders of magnitude smaller. This analysis also shows that projection methods applied to asset pricing models can produce approximations of high quality relative to other methods in use.

5. Conclusions

In this paper we have shown how to compute approximate solutions to asset market equilibrium with asymmetric information. We developed a numerical methodology which can handle arbitrary tastes, return distributions, and asset structures. We demonstrate the effectiveness of these methods by analyzing a version of Grossman and Stiglitz (1980) in which traders have constant relative risk aversion (CRRA) preferences and stock returns are lognormally distributed. This combination of preferences and returns precludes closed-form solutions. Our results demonstrates that the conclusions from the Grossman and Stiglitz model are not robust to changes in the parametric assumptions.

The exercises in this paper are just a basic application of the method. The speed and accuracy of these methods when applied to these simple problems indicate that more complex problems are tractable. From our experience, it is clear that this approach can produce robust analyses of many theoretical and practical issues. In particular, these methods could be used to model the informational role of other endogenous statistics, such as trading volume (Bernardo and Judd, 1998). Since trading volume is not normally distributed, it is difficult to find closed-form solutions to equilibrium because of the complex distribution of payoff-relevant variables conditional on observed trading volume. Our methodology is ideally suited to examine the informational role of trading volume because it does not require knowledge of the joint conditional distribution but only requires the joint unconditional distribution of all random variables.

References

Admati, A., 1989. Information in financial markets, In: Bhattacharya, S., Constantinides, G. (Eds.), Financial Markets and Incomplete Information. Rowman and Littlefield, Savage, MD, pp. 139–152.

Allen, B., 1985a. The existence of fully rational expectations approximate equilibria with noisy price observations. Journal of Economic Theory 37, 213–253.

Allen, B., 1985b. The existence of rational expectations equilibria in a large economy with noisy price observations. Journal of Mathematical Economics 14, 67–103.

Anderson, R.M., Sonnenschein, H., 1982. On the existence of rational expectations equilibrium. Journal of Economic Theory 26, 261–278.

Ausubel, L.M., 1990a. Partially-revealing rational expectations equilibrium in a competitive economy. Journal of Economic Theory 50, 93–126.

Ausubel, L.M., 1990b. Insider trading in a rational expectations economy. American Economic Review 80, 1022–1041.

Bernardo, A.E., 1999. Contractual restrictions on insider trading: a welfare analysis. Unpublished U.C.L.A. Working Paper.

Bernardo, A.E., Judd, K.L., 1998. Volume and price formation in an asset trading model with asymmetric information. Unpublished U.C.L.A. Working Paper.

Campbell, J.Y., Kyle, A., 1993. Smart money, noise trading and stock price behaviour. Review of Economic Studies 60, 1–34.

De Long, J.B., Shleifer, A., Summers, L., Waldmann, R., 1990. Noise trader risk in financial markets. Journal of Political Economy 98, 703–738.

De Long, J.B., Shleifer, A., Summers, L., Waldmann, R., 1991. The survival of noise traders in financial markets. Journal of Business 64, 1–19.

DeMarzo, P., Skiadas, C., 1998. Aggregation, determinacy, and informational efficiency for a class of economies with asymmetric information. Journal of Economic Theory, forthcoming.

Grossman, S.J., 1976. On the efficiency of competitive stock markets where agents have diverse information. Journal of Finance 18, 81–101.

Grossman, S.J., Stiglitz, J.E., 1980. On the impossibility of informationally efficient markets. American Economic Review 70, 393–408.

Heaton, J., Lucas, D., 1996. Evaluating the effects of incomplete markets on risk sharing and asset pricing. Journal of Political Economy 104, 443–487.

Jordan, J., 1982. The generic existence of rational expectations equilibrium in the higher dimensional case. Journal of Economic Theory 26, 224–243.

Jordan, J., 1983. On the efficient markets hypothesis. Econometrica 51, 1325–1344.

Judd, K.L., 1992. Projection methods for solving aggregate growth models. Journal of Economic Theory 58, 410–452.

Judd, K.L., 1998. Numerical Methods in Economics. MIT Press, Cambridge, MA.

Kreps, D., 1977. A note on fulfilled expectations equilibria. Journal of Economic Theory 44, 32–43.

Milgrom, P., Stokey, N., 1982. Information, trade, and common knowledge. Journal of Economic Theory 26, 17–27.
Radner, R., 1979. Rational expectations equilibrium: generic existence and the information revealed by prices. Econometrica 47, 655–678.
Spiegel, M., 1998. Stock price volatility in a multiple security overlapping generations model. Review of Financial Studies 11, 419–447.
Wang, J., 1994. A model of competitive stock trading volume. Journal of Political Economy 102, 127–168.