Taxation and Uncertainty

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While taxes may be certain, U.S. tax policy has certainly not been. Furthermore, intrinsic economic risk makes investment decisions risky. Therefore, a serious examination of the effects of tax policy on dynamic economic behavior should consider both sources of uncertainty. This paper presents a simple theoretical and computational model that can analyze both intrinsic risk and uncertain taxation. Furthermore, it will be clear that these techniques will be useful for examining general problems of taxation and risk.

When studying the impact of past and/or proposed tax changes, one of two extreme assumptions are usually made: either agents are perfectly aware of future tax policy, a perfect foresight assumption, or they always believe that no change will ever occur, a myopic foresight assumption. These two assumptions yield substantially different views of recent tax experience, as Alan Auerbach and James Hines (1987) demonstrate in a partial-equilibrium context. Both are clearly wrong. The myopic specification assumes that individuals believe at each point in time that the current tax law will surely continue forever, even after they have been hit repeatedly with tax changes. On the other hand, it is absurd to think that in, say, 1977, a significant number of individuals perfectly knew the various tax changes that would occur during the following decade. This paper analyzes a dynamic general equilibrium model wherein taxpayers understand the uncertainty in tax policy when making their decisions.

Explicitly assuming that tax policy changes are generated by some stochastic process, instead of treating them as movements from one deterministic regime to another, will substantially alter the impact of tax changes on investment behavior. This distinction between a shock generated by a stochastic process and a change in the nature of the process is an important one, being the heart of Robert Lucas's critique of econometric practice (1976). In fact, one of the examples used by Lucas to make this point was the impact of a stochastic investment tax credit on investment in an industry facing linear costs and demand; the analysis below is general equilibrium and nonlinear.

The interaction of risk and taxation has long been a concern of tax researchers. Evsey Domar and Richard Musgrave (1944) and J. Mossin (1968) have examined the effects of income taxation on portfolio choice in single-period models. Most dynamic tax policy analysis has abstracted from all risks in technology and tastes.[1] This is clearly unrealistic and should be remedied.

This paper develops a general equilibrium approach that can solve both the existence and computational problems for a simple dynamic economy with risky technology and/or tax policy. First note that we must use an approach different from previous dynamic tax analyses that calculate the actual path of capital accumulation.[2] Calculating all possible paths in a stochastic process is

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[1]A more complete review of the relevant papers on tax incidence, dynamic general equilibrium, and real business cycles, as well as a more complete development of our equilibrium analysis can be found in our earlier papers (1988, 1989) and in Judd (1987).

[2]See, for example, William Brock and Stephen Turnovsky (1981) and Judd (1987).

obviously unwieldy. We must also take an approach different from the real business cycle literature that exploits the equivalence of competitive equilibrium and Pareto optimality, and reduces equilibrium calculations to solving a social maximization problem.[3] The optimality approach cannot work here in general, since equilibria with taxes generally solve neither the correct maximization problem nor some usable distorted problem.

We will, however, adapt the recursive equilibrium technique of Edward Prescott and Rajnish Mehra (1980) for the presence of taxation, relying on monotonicity properties of an Euler equation to demonstrate existence of equilibrium and formulate an algorithm. It will be clear that our approach to taxation problems will work for other "nonoptimal" problems, such as externalities. This implementation of recursive equilibrium is similar to the monotone operator approach taken by Lucas and Nancy Stokey (1987) in their study of a monetary economy; since monetary equilibria generally do not solve social planning problems, it is not surprising that similar techniques are useful in both monetary and taxation analyses. The result is a more realistic representation of dynamic general equilibrium, making possible general investigations of how uncertainty and taxation interact to affect capital accumulation.

Space restrictions limit the results reported here. In this paper we will focus on one application of our model, the efficiency cost of uncertainty in tax policy. Our findings are striking. We find that randomization of the capital income taxation will raise revenue, generally at a relatively low efficiency cost; this shows that naive arguments about the desirability of stable tax policies are exaggerated. On the other hand, we find that randomization of investment incentives will often be perverse, reducing both revenue and welfare.

Given the generality of the model presented below, it is clear that it could be used to study other problems; in particular, our papers (1988; 1989) and Judd (1989) address many other issues concerning fiscal policy and risk. Therefore, this paper shows how many questions concerning taxation and risk can be analyzed in a dynamic world, as well as demonstrating that conventional wisdom concerning uncertainty in tax policy is not based on general economic principles.

[3]See John Long and Charles Plosser (1983).

I. The Model

We examine a representative agent model of dynamic general equilibrium, similar to Brock and Turnovsky. Agents have a lifetime utility function $E_t\{\sum_{t=0}^{\infty}\beta^t u(c_t)\}$ where β is the discount facter, c_t is per capita consumption of the single aggregate good in period t, and $u(c)$ is the utility derived from consuming c units of the good during a period.

We model uncertainty by assuming that there are n possible states and that a Markov process describes transitions among these states. π_{ij} will denote the probability that next period's state will be j if the current state is i. Since n is arbitrary, this specification of uncertainty includes arbitrarily general stochastic processes. Tax rates depend on the state; in state i, capital income (net of depreciation) is taxed at the rate τ_i, and an investment tax credit (ITC) subsidizes gross investment at the rate θ_i. We do not explain why tax policies are uncertain. The uncertainty may reflect unpredictable changes in the balance of power among various groups of taxpayers.

We assume a constant returns to scale production function with labor and capital inputs. Output in period t and state i, gross of capital depreciation, equals $F(k_t, i)$ if k_t is the capital stock. Capital depreciates at the rate δ. To keep the analysis simple, we will consider only the case of inelastically supplied labor in this study, although this method can handle elastic labor supply. The single good is used for both consumption and investment purposes, implying an aggregate law of motion for capital stock of $k_{t+1} = k_t + f(k_t, i) - c_t$, where $f(k, i) \equiv F(k, i) - \delta k$ is net output.

The substantive restriction implied by this model is that tax rates do not depend on the endogenous variables. While we would like to relieve this restriction, a general treatment of such possibilities is beyond the scope of this study. We assume that the current tax state is known when consumption and labor supply decisions are made. Finally, since we want to study the effects of random taxation, not random government consumption, we assume that all revenues are lump sum rebated to the agents. In particular, government consumption is zero.

II. Existence and Characterization of Equilibrium

We next characterize equilibrium and demonstrate existence. The key to analyzing stochastic growth models is to examine the policy rule, that is, the function giving the actions to be taken as a function of the current state of the economy, where by *state* we mean both the current tax and productivity state and the capital stock. Such a policy rule will determine the law of motion for the economy and any other characteristic, such as tax revenues. An equation for the equilibrium policy function can be derived from consideration of basic Euler equation arguments. Suppose that $h(k,i)$ is per capita consumption when the capital stock is k and state i governs productivity and taxation. The individual must be indifferent between consuming one more unit of consumption today and investing it, consuming it and its proceeds tomorrow. This implies

$$(1)\quad u'(h(k,i)) = \sum_j \pi_{ij}\beta\big[f_k(s(k,i),j)\times(1-\tau_j)+1-\theta_j+\theta_j\delta\big]\times u'(h(s(k,i),j))\times[1-\theta_i]^{-1}$$

where $s(k,i) = k + f(k) - h(k,i)$ equals gross savings when the current period begins with a capital stock of k and the state is i.

The intertemporal condition, (1), can be understood in terms of a simple argument. Let R_j be the return, gross of taxation and depreciation, tomorrow if state j occurs. In equilibrium, an agent will be indifferent between consuming one more dollar and investing it for consumption in the next period. Because of the ITC, 1 unit of foregone consumption today if state i is in effect increases tomorrow's capital by $(1-\theta_i)^{-1}$. Tomorrow's cash flow includes the return net of taxes, $R_j(1-\tau_j)$ and depreciation allowances of $\delta\tau_j$. After depreciation, there are only $1-\delta$ extra units of capital left of the extra unit invested. To return to the originally planned level of capital, tomorrow's gross investment expenditures are reduced by $1-\delta$, increasing tomorrow's cash flow by $(1-\theta_j)(1-\delta)$. Therefore, the extra investment today will generate extra cash for consumption tomorrow equalling $[R_j(1-\tau_j)+\tau_j\delta+(1-\theta_j)(1-\delta)](1-\theta_i)^{-1}$, which, in equilibrium, equals the coefficient on tomorrow's state j marginal utility of consumption in (1).

The key step is to rewrite the equilibrium equation (1). Think of $h(k,i)$ as today's consumption policy function if state i is in force today, and let $h^+(k,j)$ be the policy function which holds tomorrow if state j occurs. In terms of h and h^+, the equilibrium arbitrage equation can be rewritten

$$(2)\quad u'(h(k,i)) = \sum_j \pi_{ij}\beta\big[f_k(s(k,i),j)\times(1-\tau_j)+1-\theta_j+\theta_j\delta\big]\times\frac{u'(h^+(s(k,i),j))}{[1-\theta_i]}$$

Suppose that $h^+(k,j)$ is continuous and increasing in k for all j. The critical fact is that h is uniquely determined by h^+ in (2). To see this, note that, for a fixed $h^+(.,.)$ function and for any k and i, (2) gives a unique solution for $h(k,i)$ since $h^+(.,j)$ is an increasing and continuous function of k for each state j, implying that the right-hand side of (2) is increasing in $h(k,i)$. Denote this relation by $h = Th^+$. Furthermore, h is also increasing and continuous in k.

If the world has a finite life, then this construction demonstrates uniqueness of equilibrium. The policy rule in the last period is $h(k,i) = k + f(k,i)$ since there is no marginal value to savings. Any equilibrium policy rule for the previous period's rule is given by (2), which has a unique, continuous, and monotonic solution. Inductively, we have the same properties for each time period's policy rule as we move back in time.

Instead of computing the sequence of consumption rules that characterize the finite-horizon equilibria, it is often preferable to compute an equilibrium to the infinite-horizon equilibrium condition, (1). The critical property of T for this construction is monotonicity; that is, if $h > g$ then $Th > Tg$. This property arises because an increase in tomorrow's consumption rule will reduce tomorrow's marginal utility of consumption, causing today's consumption to also rise to establish (2). A standard variational argument formally demonstrates the monotonicity property. Next, consider the following sequence of policy functions. Let $h^0(k,i) = k + f(k,i)$ and let $h^n = T^n h^0$. Since h^0 is the policy of eating everything and $u'(0) = \infty$, $0 < h^1 < h^0$. Monotonicity of T implies that $h^n > h^{n+1}$, for all n. Since the series of functions, h^n, is monotonically decreasing but bounded below by 0, there exists a function which is almost everywhere a pointwise limit of the h^n; call it h^∞. Under mild restrictions on tastes and technology, this limiting consumption rule will be a solution of (1), and indeed be an equilibrium rule for the infinite horizon case; the details are given in our paper (1988). Theorem 1 states the result.

THEOREM 1: *Assume that $u(c)$ and $f(k,i)$ are C^∞ in k, that the share of capital is bounded away from zero as k goes to zero, and that the elasticity of substitution in consumption is bounded away from infinity as c goes to zero. Then there exists an equilibrium h satisfying* (1), *that is also the limit of the finite-horizon model as we take the horizon off to infinity. Both h^∞, the consumption function, and $k + f - h^\infty$, the savings function, are increasing in k.*

This construction gives us not only existence of an equilibrium policy function, but also a way to compute it; this is pursued below. We will not prove any uniqueness result for the infinite-horizon case for two reasons. First, the major theoretical problem is to demonstrate the logical consistency of our description of the equilibrium, a problem solved by proving existence of an equilibrium. Second, the equilibrium we compute is the limit of finite-horizon equilibria as the horizon becomes indefinitely long. If there were multiple equilibria, the one constructed in Theorem 1 is somewhat more appealing on intuitive grounds since we would argue that there should be little difference between long finite-horizon models and infinite-horizon models.

III. Simulations

Since closed-form solutions for equilibrium are not available for this model and local approximations may not be valid for large amounts of uncertainty, we will examine numerical solutions. Since the proof of existence above is constructive, our numerical procedure is to compute the sequence constructed in the theorem, leading to an approximation of h^∞. Of course, a computer cannot store an arbitrary continuous function; we find a piecewise linear approximation of h^∞ which solves (1) exactly at the breakpoints.

In order to examine the impact of a tax policy on the performance of an economy, we develop an index. First, it must take into account any change in the expected utility of the representative agent. Second, it must also consider any change in revenues. Therefore, we will use the following approach to evaluate the impact of a tax policy change. We calculate the change in expected utility, convert it into a wealth equivalent by dividing it by the initial marginal utility of consumption, and compare it to the change in the market value of government revenue, where by government revenue we mean the receipts from the income tax minus any expenditures on the investment tax credit. The resulting ratio, denoted MEB, measures the marginal excess burden of the tax change; for example, if the ratio between the change in utility and revenue is $-.15$, then the wealth equivalent of the utility reduction is 15 cents for every extra dollar in revenue arising from the tax change.

In all of our simulations, the critical parameter is the intertemporal elasticity of consumption demand. We allow it to be constant and equal to .5, 2, and 5. While this range is at the high end of empirical estimates, it is also the range over which the distortionary effects of taxation is greatest. To economize on space, we discuss only one set of values for the other parameters; simulations show that results are not sensitive to reasonable alternatives. We assume a Cobb-Douglas production function with a .25 capital share, $\delta = .06$, and $\beta = .95$. Note that this specification of tastes and technology satisfies the conditions of Theorem 1. Furthermore, we assume that the initial tax policy has a 40 percent tax on capital income and a 5 percent ITC.

We now discuss the effects of random tax policies. The effects on investment patterns are obvious. If the income tax rate bounces between low and high rates and there is positive serial correlation in tax rates, then when the tax rate is lower than average, the economy acts as if it has a relatively large target capital stock, and when the tax rate rises to high levels, investment drops and the new target capital stock is less. Similar effects hold when the ITC is random and positively serially correlated.

Less obvious are the welfare effects of random tax policies. It is often asserted that uncertainty in taxation has a negative effect on the economy's performance. Businessmen complain that they cannot make plans if they don't have confidence in the tax structure. We can examine some of these problems in our general equilibrium model. Many elements of reality are, of course, absent in this model. However, since our model is a general equilibrium model, it is one which can test the robustness of the casual arguments often heard.

In the exercises considering uncertain taxation we will assume that the economy has experienced a deterministic tax structure, expects it to remain in effect forever, and converges to the corresponding steady state. We then change the tax policy by introducing some uncertainty but keeping the mean tax rate constant, assuming that the change is announced at the end of some initial period. First, we let τ bounce randomly (i.e., τ is serially uncorrelated) between .3 and .5. Second, we let θ bounce randomly between 0 and .1.

We find many interesting results. First, introduction of uncertainty in the income tax rate will raise nontrivial amounts of revenue, an amount around 3 percent of the initial capital stock. Furthermore, the utility loss of raising this revenue is never more and generally less than the cost of raising the same revenue through a deterministic and permanent increase in τ, the latter varying between 7 and 26 cents per dollar of revenue gain. If both capital and labor income tax rates fluctuate, the results are similar. On the other hand, the reaction of the economy to ITC randomness is very different. We find that the introduction of randomness in θ generates strong effects. In fact, both revenue and utility fall in all three cases.

The substantial differences between capital income taxation and the ITC are not surprising, since the ITC, being a more precisely targeted instrument, has much stronger incentive effects. When faced with a random ITC, firms will adjust the timing of their investment so as to use the investment subsidy more when it is high. This shifting leads to a rise in investment credits, but little change in the average level of the capital stock. Randomness in the ITC will therefore generate substantial fluctuations in investment behavior, fluctuations that are not desirable since both utility and production are concave functions. A simultaneous fall in revenue and utility are an expected consequence of a noisy ITC. On the other hand, current investment behavior does not depend on the current income tax rates but rather the income tax rates anticipated in future periods. In the case of independently, identically distributed τ shocks, the low-tax periods roughly balance the high-tax periods, indicating that variance in future income tax rates is not important for long-lived investments. In fact, to the extent that future τ uncertainty reduces utility without affecting the return to investment, it encourages investment since the negative income effect will reduce consumption, reducing the distortion caused by capital income taxation, and raising revenue. We see that, in both cases, the results on policy uncertainty are intuitive.

While these results are only suggestive, they do hold some lessons for tax analysis. In particular, we find that it is very difficult to summarize the tax code by some aggregate effective tax rate when there is policy uncertainty. In these examples, randomness in the income tax rate is not bad but randomness in the ITC generates perverse effects. This difference would likely be ignored by an effective tax rate that aggregates the components of the tax structure.

These examples are not meant to argue for the implementation of uncertain tax policies. In fact, it is unclear just how one would enact randomization in such taxes. However, this analysis does indicate that, in some cases, there is no great economic cost from policy uncertainty induced by noneconomic political forces, and no strong case for major institutional reform to insulate the tax code from those forces. Furthermore, there may be some elements of reality ignored here, such as adjustment costs, that may reverse the results. Also, the effects of uncertainty may depend on poorly estimated third-order properties of utility and production. However, the framework developed above is sufficiently flexible to incorporate more aspects of reality.

IV. Conclusion

This paper has outlined an approach to analyzing taxation and uncertainty in dynamic general equilibrium. This approach yields both a theoretical way to prove existence of an equilibrium and a numerically implementable approach to compute the equilibria that result from such policies. We find that the efficiency costs of randomness in tax policy depend strongly on which instruments are "randomized," with no general result that tax policy uncertainty is any more damaging to the economy than any other aspect of tax policy. While the applications reported here were few, analyses of many tax and macroeconomic policy questions are possible in this framework.

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