

Capital Market Imperfections and Tax Policy Analysis in the Life Cycle Model

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ABSTRACT. – One issue often overlooked in dynamic fiscal policy analysis is the importance of assumptions made regarding individuals' abilities to use capital markets to transfer income across time. We focus on the impact of borrowing restrictions on consumption functions within the life cycle framework, and address a set of fiscal policy issues. Our principal concerns are two. First, taking into account the effect of borrowing constraints affects the calculation of the efficiency costs of taxation substantially. Second, we address within the life cycle model the importance of precautionary saving in response to capital market imperfections and consequent implications for effects of social insurance programs.

Imperfection du marché du capital et analyse de l'incidence fiscale dans le modèle de cycle de vie

RÉSUMÉ. – Les analyses de politique fiscale dynamique négligent souvent les hypothèses faites sur la capacité des individus à recourir au marché du capital pour transférer des ressources dans le temps. On s'intéresse principalement à l'impact des contraintes d'endettement sur les fonctions de consommation dans le cadre du modèle de cycle de vie et on étudie un certain nombre de questions fiscales. D'une part, on montre que la prise en compte des contraintes d'endettement exerce une forte influence sur le calcul du coût d'efficacité de la fiscalité. D'autre part, on analyse dans le cadre du modèle de cycle de vie l'épargne de précaution résultant des imperfections du marché du capital. Cette épargne modifie l'étude de l'incidence des dépenses sociales sur l'accumulation.

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1 Introduction

Central to modeling the economic impacts of tax policy and tax reform is an analysis of the relationship between taxation and taxpayers' decisions about consumption, saving, and work effort. One issue often overlooked in policy applications is that such analysis is especially sensitive to assumptions made regarding individuals' abilities to use capital markets to transfer income across time. We focus on the impact of borrowing restrictions on consumption functions within the life cycle framework, and address a set of fiscal policy issues.

One general issue lies within concerns of optimal taxation. Recent applications of life cycle models have facilitated comparisons of the effects of alternative policies. These models generally assume that individuals or households maximize well-being over their lifetime, subject to the restrictions that the present value of consumption is no greater than the present value of income. The models then assess how tax policies can alter the rewards to saving and working, thereby distorting intertemporal choices about consumption and work effort. However, models with an overall lifetime budget constraint ignore the many restrictions on individuals' ability to shift income in a world of capital market imperfections, restrictions that substantially affect those intertemporal choices. Taking into account the effect of borrowing constraints affects the calculation of the efficiency costs of taxation substantially; prevailing arguments, based on "perfect market" models, against capital taxation or progressive income taxation and in favor of wage and consumption taxation must be substantially muted and often reversed.

Second, we address within the life cycle framework the importance of precautionary saving in response to capital market imperfections and consequent implications for effects of social insurance programs. In particular, we focus on precautionary saving against uncertainty over longevity in the presence of imperfections in private annuity markets. The introduction of social security annuities can be efficiency-improving in such a world, but gains in efficiency depend critically on the significance of borrowing restrictions (because of the importance of being able to borrow against the gains from access to social security annuities and because of the proportional payroll tax financing of social security).

The paper is organized as follows. Section 2 discusses the general importance of capital market imperfections (in particular, restrictions on borrowing) for analyzing consumption decisions in the life cycle model. The development of an extended life cycle model for tax policy analysis is accomplished in section 3. Simulation exercises related to tax reforms are presented in section 4. We consider lifetime uncertainty and the impact of social security on saving and welfare under various assumptions about capital markets in section 5. Some conclusions and implications are presented in section 6.

2 Capital Market Imperfections and Consumption

2.1. Analyzing Borrowing Constraints

The effects of various types of borrowing constraints on consumption have been considered in many empirical studies. One approach builds on HALL's [1978] "Euler equation" method for testing the sensitivity of consumption to changes in current income. In Hall's model, to be consistent with the permanent-income hypothesis under rational expectations (with no borrowing restrictions), conditional on lagged consumption, expected consumption should be independent of other lagged information. Other empirical studies find consumer spending to be sensitive to income changes (see FLAVIN [1981]; HAYASHI [1982]; HALL and MISHKIN [1982]).

Our analysis is predicated on the idea that this excess sensitivity can be traced to the operation of borrowing constraints, the aggregate importance of which for the United States is amply illustrated by historical evidence (see for example DE LONG and SUMMERS [1986]). HAYASHI [1982] finds that liquidity-constrained consumers accounted for approximately 20% of all consumption in post-World War II United States. In a separate effort using microeconomic data, HAYASHI [1985] also notes that the relationship between consumption and income movements differs significantly for "high saving" and "low saving" families (see also RUNKLE [1983] and ZELDES [1985]). BERNANKE [1984] finds no evidence that the permanent-income hypothesis needs to be amended for liquidity constraints in his examination of individual expenditures on automobiles. Automobile loans, however, are self-collateralized, and our focus is on the relative unavailability of noncollateralized consumption loans. FLAVIN [1984] finds that the estimate of the marginal propensity to consume is affected dramatically by the inclusion of proxies for liquidity constraints. In her econometric work, Flavin uses the aggregate unemployment rate as a proxy for borrowing constraints and tests "myopia" and borrowing-constraint explanations of the excess sensitivity findings. She reports that the estimated marginal propensity to consume out of transitory income is explained almost entirely by proxies for liquidity constraints.

We analyze borrowing restrictions arising from a nonnegativity constraint on net worth. That is, consumers are not permitted to borrow against income to be received in the future; current consumption is limited by current resources. For consumers who cannot finance their desired level of consumption with current wealth, consumption is responsive to changes in

disposable income (even those changes which are anticipated). The imposition of a borrowing restriction of this form requires motivation.¹ Here we rely on observed collateral requirements for borrowing in US capital markets, *i. e.*, restricted access to consumption loans.² Institutional motivations for the constraint include legal restrictions prohibiting the inclusion of human capital as an asset in bankruptcy proceedings or, for our purposes later, the assignment of future social security benefits.³ Allowing individuals to borrow some fixed “small” amount would increase significantly the numerical complexity of the problem, without qualitatively altering the results.⁴

In the absence of borrowing restrictions, life cycle consumption typically exceeds earnings in youth, while earnings exceed consumption in middle age and then decline in retirement. Given a nonnegativity constraint on net worth, however, consumption can never exceed current resources. Given an upward sloping earnings profile, consumption follows earnings during youth, when the constraint is binding, then increases relative to the “perfect capital markets” profile thereafter. Lifetime utility from consumption is reduced by the constraint, with the magnitude of the reduction increasing the flatter is the desired age-consumption profile. Fiscal policies that depress consumers’ net earnings during constrained periods will reduce consumption dollar for dollar.

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1. The source of the borrowing restrictions is important. HAYASHI [1985] and YOTSUZUKA [1986] have argued that liquidity constraints adjust in response to changes in tax policy, so that Ricardian equivalence holds. BERNHEIM [1987] shows that the Hayashi-Yotsuzuka results are sensitive to assumptions about the distribution of future tax liabilities. Hayashi and Yotsuzuka assume that taxes are fixed and lump-sum, while Bernheim considers (and we consider) the case in which taxes paid depend positively on earnings.
 2. For a discussion of potential information problems in credit markets and restrictions on borrowing in the absence of collateral, see STIGLITZ and WEISS [1981] and CALOMIRIS and HUBBARD [1985, 1987].
 3. We do not consider the additional source of borrowing restrictions arising from uncertain earnings fluctuations around an expected profile (see for example BARKSY, MANKIW, and ZELDES [1986]; HUBBARD and JUDD [1986]; and ZELDES [1986]). Adding this complexity would not alter our qualitative result; indeed it would strengthen the importance of liquidity constraints in accounting for national saving. Our characterization of borrowing restrictions probably understates their importance in the real world. We have disallowed only net consumption loans; real-world restrictions in addition to collateral requirements (e. g., minimum-income requirements for debt service) would strengthen our findings. Moreover, our constraint does not rule collateralized loans for such durables as homes or cars.
 4. See for example the simulations in ZELDES [1986]. CALOMIRIS and HUBBARD [1987] develop a model in which changes in the relative importance of certain and uncertain resources for borrowers is important for participation in credit markets.

3 Life Cycle Models and Tax Policy Analysis

3.1. Issues

Most questions of fiscal policy or tax reform are best suited to multiperiod models that consider long-run effects, either for individuals or from the economy. It is not surprising that the life cycle model has been the principal focus of analyses of the effects of tax changes on consumption and welfare.⁵ The model provides a realistic number of periods in an individual's life to permit consideration of the effect of even temporary policy changes on lifetime consumption and welfare. It is extremely valuable as an analytical tool because it does not rely on ad hoc rules of behavior and is consistent with basic demand and supply theory and rational expectations methodology. The life cycle framework has been used in the construction of dynamic general equilibrium models that permit comparison of steady states of different policy regimes (SUMMERS [1981]; AUERBACH, KOTLIKOFF, and SKINNER [1983]; HUBBARD and JUDD [1986]).

Most empirical questions about the validity of the life cycle model do not necessarily refute the basic insight of the theory—that consumers are forward looking in their behavior and optimize over a long (lifetime) horizon.⁶ Here we maintain that insight and focus on the implications of borrowing restrictions for analyses of the impact of tax changes or tax

5. For a description of the model, see MODIGLIANI and BRUMBERG [1954] and ANDO and MODIGLIANI [1963]. For a survey of applications to fiscal policy issues, see KOTLIKOFF [1984].

6. While the life cycle model provides a theoretically consistent and logical framework for an examination of policy-induced changes in consumption, it is not without its problems. At the theoretical level, it has been argued that the model requires consumers to be both “not forward looking enough” and “too forward looking.” In one case, critics assert that consumers are much more myopic than the life cycle model would allow, that they might be guided, for example, by rule-of-thumb saving behavior (THALER and SHEFFIN [1981]). It is not obvious what effect tax policy would have in such a world. In the alternative case, most closely associated with BARRO [1974], critics argue that if consumers consider their heirs when making their own consumption decisions, the relevant optimizing horizon may be infinite, and the capital stock would not arise out of a mismatching of earnings and desired consumption over individuals' lives. The life cycle model's predictions have also been criticized in empirical studies. For example, some studies of age-wealth profiles find a significant number of individuals or households with “too little wealth” (see KING and DICKS-MIREAUX [1982]; and HUBBARD [1986]). In addition, econometric studies typically find lower dissaving among the retired elderly than the theory would predict (MIRER [1979]), though DAVIES [1981] and HUBBARD [1987] have offered uncertainty about life expectancy and the fear of living too long as plausible explanations within the life-cycle framework; we consider this issue in more detail later. Perhaps most challenging to applications of the basic model is the claim by KOTLIKOFF and SUMMERS [1981] that life-cycle saving as they define it can explain only a

reform on national saving and individual welfare. The specification of a lifetime budget constraint in standard uses of the life cycle model is not an accurate representation of restrictions on consumption smoothing when capital markets are characterized by collateral restrictions, differences in borrowing and lending rates, and credit rationing. The central issue within the life cycle framework is as follows. Hump-shaped lifetime earnings profiles rising toward middle age then leveling off and declining in old age imply that individuals will want to consume more than their current resources allow when young. They cannot do so if borrowing constraints are binding.

3.2. Simulating the Importance of Liquidity Constraints

To simulate the effects of alternative tax and fiscal policies on consumption, we make use of a life cycle model in which no bequests are desired and individuals consume so as to maximize an intertemporal utility function subject to a lifetime budget constraint. Individuals live for T years, working only for the first R years; the retirement age of R is taken as exogenous, and, for the moment, we assume that labor is supplied inelastically. (We use a model with elastic labor supply in our simulations; see the discussion of the full model in the appendix). Model simulations begin at the commencement of individuals' working lives, assumed to be age twenty. Retirement occurs after forty-five years of work; death occurs ten years later.

The addition of substantial bequests would complicate the model.⁷ Bequests can be either planned or "accidental." The latter occur when individuals who have saved to insure against long life die prematurely and with positive wealth. In the context of accidental bequests generated by precautionary saving, the effects of liquidity constraints remain important. In fact, because many liquidity-constrained individuals are effectively constrained only early in life, to the extent that accidental bequests are received relatively late in life, our conclusions about the importance of constraints are strengthened, since desired consumption in youth would be further increased relative to current resources.

Nor would planned bequests seriously weaken this analysis. If an individual plans to leave a bequest to his child, his desired lifetime consumption is reduced. However, if he himself receives a bequest from a parent, that bequest could be used to finance his bequest to his child. As long as the bequest he receives from his parent arrives after his constrained periods

small portion of the capital stock in the United States, with the clear implication that models emphasizing intergenerational transfers as the dominant factor in saving decisions deserve more attention.

7. HURD [1986] finds essentially no bequest motive evident in the saving behavior of households in the Retirement History Survey, concluding that bequests are much more likely to be of the "accidental" variety attributable to lifetime uncertainty.

and he plans on leaving a comparably sized bequest to his child—a natural steady-state assumption—his consumption path is financed by his earnings, and bequests received and given are not important for consumption decisions. Therefore, ignoring bequests does not seriously reduce the plausibility of our analysis for the issues we examine below; we reconsider the role of bequests in our analysis in section 5 of precautionary saving against lifetime uncertainty.

Assuming that utility is additively separable across periods, individuals maximize lifetime utility

$$(1) \quad \int_0^T U(c) e^{-\rho t} dt,$$

subject to

$$(2) \quad \dot{A} = (1 - \tau_L) E + (1 - \tau_K) r A - c, \quad A(0) = A(T) = 0,$$

where c , ρ , and r represent consumption, the subjective discount rate, and the interest rate, respectively. The coefficients τ_L and τ_K denote tax rates on labor and capital income, respectively; A represents the stock of accumulated assets. A dot over a variable denotes a time rate of change. The income stream E represents labor earnings.

For simplicity, we assume that the utility function is of the isoelastic form, and rewrite equation (1) as

$$(3) \quad \max \int_0^T \frac{1}{1 - \beta^{-1}} c^{1 - \beta^{-1}} e^{-\rho t} dt,$$

where β measures the intertemporal elasticity of substitution in consumption.

If λ denotes the marginal utility of consumption and $c(\lambda)$ is defined as consumption corresponding to λ , the differential equations describing the time paths of consumption and wealth accumulation are given by

$$(4a) \quad \dot{\lambda} = [\rho - (1 - \tau_K) r] \lambda,$$

and

$$(4b) \quad \dot{A} = (1 - \tau_K) r A + (1 - \tau_L) E - c(\lambda),$$

together with the boundary conditions $A(0) = A(T) = 0$.

In the absence of borrowing restrictions, the capital stock will be a function of underlying parameter values, the age distribution of the population, and the shapes of individual earnings profiles. In the basic life cycle model, individual savings profiles are expected to be hump-shaped, with borrowing by young people, substantial asset accumulation by those in middle age, and dissaving by the elderly. Within this framework, tax reforms—either changes in tax rates or shifts in relative reliance on different tax bases—or social insurance programs have real effects only to the extent that they alter steady-state factor prices.

These results will be qualified in the presence of borrowing restrictions. It is here that the choice of pretax earnings profiles for simulation exercises

is particularly relevant. We chose the profile used by DAVIES [1981].⁸ By using an average earnings profile, we are ignoring individual-specific fluctuations in earnings. Since the marginal loss due to a tighter liquidity constraint is negligible if the constraint is slack and greater when the constraint is binding, losses due to the liquidity constraint are convex in the tightness of the constraint. Using the average earnings pattern means that losses are underestimated, since the distribution of earnings patterns would include some with much tighter constraints as well as some with looser constraints.

When we impose the constraint that net worth must be nonnegative at all times, we substantially change the nature of the consumer's optimization problem. The budget constraint in equation (2) becomes

$$(5) \quad \dot{A} = (1 - \tau_L) E + (1 - \tau_K) r A - c, \quad A(T) \geq 0.$$

The first-order conditions must be altered to take into account this state constraint.⁹ The new arbitrage conditions become

$$(4a') \quad \begin{aligned} \dot{\lambda} &= [\rho - (1 - \tau_K) r] \lambda, & \text{if } A > 0 \text{ or } \lambda > U'[(1 - \tau_L) E], \\ \lambda &= U'[(1 - \tau_L) E], & \text{if } A = 0 \text{ and } \lambda \leq U'[(1 - \tau_L) E], \end{aligned}$$

where λ is continuous. Assets obey

$$(4b') \quad \begin{aligned} \dot{A} &= 0, & \text{if } A = 0 \text{ and } \lambda \leq U'[(1 - \tau_L) E], \\ \dot{A} &= (1 - \tau_K) r A + (1 - \tau_L) E - c(\lambda), & \text{otherwise.} \end{aligned}$$

If assets are positive or if earnings exceed consumption, then equation (2) still holds. Otherwise, consumption is limited to current earnings, and the consumer's optimization problem is divided into constrained and unconstrained intervals; equation (4b') governs how these intervals meet.

Aggregate consumption is determined in the model by summing over the individual consumption of those alive at a given time. That is, consumption of individuals at each age is determined, with the relative number of individuals at each age depending on the rate of growth of population. Aggregate asset stocks are constructed similarly.

For borrowing constraints to be important for aggregate fiscal policy analysis, a nontrivial fraction of the population must be constrained, with subsequent effects on saving. To evaluate the importance of the constraints for steady-state measures of the aggregate capital stock, we allow for variable factor prices. We assume that output is produced according to a Cobb-Douglas production function in capital and labor, with a capital share equal to α . Factor markets are assumed to be competitive, so that capital and labor are paid their marginal products. That is, the gross interest rate,

8. That is, as in DAVIES [1981], p. 572, the lifetime path of mean noninvestment income E is approximated by a fourth-order polynomial:

$$E(t) = -36,999.4 + 3520.22 t - 101.878 t^2 + 1.34816 t^3 - 0.00706233 t^4.$$

9. For a discussion of such problems, see KAMIEN and SCHWARTZ [1982].

r , and wage rate, w , satisfy.

$$(6) \quad r = \alpha k^{\alpha-1},$$

and

$$(7) \quad w = (1 - \alpha) k^{\alpha},$$

where k represents the capital-labor ratio.

Within this framework, the steady state can be solved as follows. A guess is made for k . Solutions for r and w are then generated from the marginal productivity conditions to produce individual consumption and asset accumulation profiles. The resulting aggregate consumption and capital stock are compared with the initial guess, and iteration proceeds until convergence is reached.

TABLE 1

*Borrowing Constraints and Steady-State Capital Accumulation**

Elasticity of substitution in consumption (β)	Capital-income ratio		Number of periods constrained	Percent of population constrained	Percent of income held by constrained consumers
	Unconstrained	Constrained			
1.10	4.40	4.43	4	7.3	2.7
0.50	2.79	3.26	6	12.0	5.5
0.20	1.08	1.66	9	19.0	10.8
0.10	0.68	1.27	11	23.5	14.9

* The model assumes a general proportional tax on both capital (τ_k) and labor (τ_l) income of 0.30.

In Table 1, we present findings on the effects of borrowing restrictions on the capital stock and individual welfare when $\alpha=0.3$. The discount rate was chosen to be 1.5%, and population growth was assumed to be 1% per year; the interest rate is determined endogenously. Results were robust to minor variations in the rate of time preference. We felt that $\rho=0.015$ was a reasonable choice, and the selection of a low discount rate furnishes a lower bound of the effects of borrowing constraints on consumption and welfare, since a higher rate would only have increased desired consumption relative to current resources.

There is some evidence to the literature on the value of β (see for example FRIEND and BLUME [1975]; FARBER [1978]; HANSEN and SINGLETON [1983]; SUMMERS [1982]; and HALL [1985]). We chose a set of values of β to span these estimates. The model was simulated for the case of a general proportional income tax of 30% for four values of β —1.10, 0.50, 0.20, and 0.10. Results are presented for the capital-income ratio in the absence and presence of the wealth-nonnegativity constraint, the number of periods for which the consumption is constrained (determined endogenously by the model), the percentage of the population constrained, and the percentage

of income earned by individuals whose consumption is limited by current resources.

As expected, as the intertemporal elasticity of substitution in consumption declines, reflected in lower values of β , the capital-income ratio rises in the constrained case relative to the unconstrained case. This effect of forced saving through liquidity constraints is also evident in the corresponding increase in the number of periods constrained, the percentage of the population constrained, and the fraction of total income earned by constrained consumers.¹⁰ Note, for example, that in the case wherein $\beta=0.20$, the constrained capital stock is 54% larger than the unconstrained capital stock, with borrowing constraints binding on 19% of the population (receiving 11% of disposable income). These results suggest that tax policies that exacerbate the severity or duration of liquidity constraints are likely to lead to substantial efficiency costs, especially when the intertemporal elasticity of substitution in consumption is small.

In summary, the simulations show that a nonnegativity constraint for personal wealth has a substantial effect on the long-run supply of capital. The potential efficiency costs associated with the effects of liquidity constraints on consumption do not, of course, imply that society should make large transfers to young, constrained workers. Constraints with respect to incentive effects and social convention abound. The marginal contribution of taxation to those welfare costs is, however, a legitimate concern of public policy within the context of optimal taxation.

4 Liquidity Constraints and Tax Policy Evaluation

4.1. Issues

Life cycle simulation models have been used extensively in the area of tax reform—in particular, to measure the welfare effects of switching from a general income tax on both capital and labor income to a tax on labor income alone and from a progressive income tax to a proportional income tax.¹¹ According to the findings of recent studies, the efficiency effects of either shift can be quite large.

10. Our predictions for the fraction of aggregate disposable income earned by constrained individuals are lower than HAYASHI's [1982] point estimate of 17.1%, but our description of borrowing restrictions is most likely an underestimate of their severity in the real world.

11. See for example SUMMERS [1981], AUERBACH, KOTLIKOFF, and SKINNER [1983], EVANS [1983], SEIDMAN [1984].

Many recent studies have focused on intertemporal distortion arising from the taxation of capital income. Since capital taxes reduce the net rate of interest, they discourage saving and discriminate in favor of present consumption at the expense of future consumption. Because substitution effects diminish for consumers for whom liquidity constraints are binding or expected to bind, the potential efficiency gains from policy reform are reduced according to the importance of liquidity-constrained consumers in the population.

SUMMERS [1981] shows that efficiency costs of capital income taxation analyzed in a realistic life cycle model are likely to be large. The channel for this effect is intuitive. A cut in the capital income tax rate raises the net return to saving, and the substitution effect discourages consumption; for individuals in debt, the income effect further depresses consumption. The resulting decline in consumption can lead to a substantial increase in lifetime saving.

Similarly, arguments against progressive income taxation focus on distortions in intertemporal consumption decisions and in labor-supply decisions. Using a dynamic life cycle simulation model with a labor-leisure choice in addition to intertemporal consumption decisions, AUERBACH, KOTLIKOFF, and SKINNER [1983] find substantial efficiency costs to using a progressive income tax rather than a proportional income tax in financing a given level of government spending.

The effect of tax reform on the welfare of liquidity-constrained consumers will be important for assessing the aggregate welfare effects of such policy changes. Any switch away from capital income taxation toward increased taxation of wage income will entail, *ceteris paribus*, an efficiency loss from a decline in the consumption of constrained consumers, a loss that must be weighed against the efficiency gain from lessening tax-induced distortions in intertemporal consumption decisions. Similarly, to the extent that earnings rise with age over most of the working life, progressive taxation shifts the individual lifetime burden of taxation away from constrained periods, permitting gains in lifetime consumption and welfare. Earlier studies focus only on the efficiency costs of distortions in labor supply under progressive taxation.¹²

4.2. Efficiency Costs of Capital and Labor Income Taxation

As noted by SUMMERS [1981], capital taxation in a life cycle growth model can lead to substantial reductions in individual welfare when individuals supply labor inelastically throughout their working lives. Since reductions in the level of capital taxation will in general be financed by higher taxes on labor income, to the extent that labor is elastically supplied, the efficiency

12. These distortions can, of course, be large; see for example the estimates in HAUSMAN [1981].

effects of policy reform are no longer obvious. We show that intertemporal distortions induced by capital income taxation dominate the contemporaneous distortions due to labor income taxation when capital markets are assumed perfect. Our concern is with the complications that borrowing restrictions introduce.

Our first experiment eliminates the capital income tax and finance the reform by higher taxes on labor income. We take as our base case a proportional tax on capital and labor income such that ¹³ $\tau_K = \tau_L = 0.30$. As τ_K is set equal to zero in this experiment we solve for the labor tax rate τ_L that raises the same revenue as the proportional general income tax. We assume elastic labor supply, making the individual's lifetime utility function described in equation (1) additively separable in consumption and leisure (see the detailed discussion in the appendix). The exercise is conducted under an open-economy assumption, where the interest rate remains at the same level as that prevailing prior to the policy experiment.

To allow for a reasonable range of the underlying parameter values, we perform the experiment for two values of the interest rate, ¹⁴ 0.04 and 0.08. We also examine two values of the compensated wage elasticity of labor supply, 0.1 and 0.5 (see the discussion in the appendix), and four values of $\beta - 1.1, 0.5, 0.2,$ and 0.1. Results for changes in individual welfare in unconstrained and constrained cases are reported in Table 2. In all cases, efficiency gains are expressed as a percentage of lifetime taxes paid. ¹⁵

As expected, in the unconstrained case, substantial efficiency gains are achieved by moving from capital to labor income taxation. Gains are largest for high values of the intertemporal elasticity of substitution in consumption, where the sensitivity of saving to changes in the net return is substantial. These gains are mitigated significantly in the constrained case. As the intertemporal elasticity of substitution in consumption takes on smaller values in the constrained case, the effect of the borrowing restrictions becomes more significant, and the switch from capital income to labor income taxation leads to much smaller gains than in the unconstrained case and, in some cases, even to welfare *costs*. For example, from Table 2, when $r = 0.04$, $\beta = 0.2$, and $\eta = 0.1$, a 0.6% gain from eliminating capital taxation when capital markets are perfect becomes a 2.0% *loss*

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13. SUMMERS [1981] and FELDSTEIN and SUMMERS [1979] describe the US tax system as being characterized by $\tau_K = 0.5$ and $\tau_L = 0.2$. That description is probably unrealistic in the current environment of declining capital taxation. In any event, starting with a higher tax on interest income would accentuate our findings in the constrained regime.
 14. While an 8% interest rate is unrealistically high as a representation of any available riskless return, we include it here since risky assets have expected returns in that range. Since it is not known how to account properly for the riskiness of real-world investments in our deterministic model, we choose to examine these alternative values.
 15. At first blush, the pattern of results in Table 2 by labor supply elasticity may appear counterintuitive. One might suspect that the efficiency gain from switching from capital taxation to labor taxation should be less the more elastic is labor supply with respect to changes in the net wage. Here the intertemporal distortion from interest taxation is more important than the static consumption-leisure distortion. That is, reducing the intertemporal distortion dominates the fact that the static distortion becomes greater.

TABLE 2

*Efficiency Gains, as Percent of Lifetime Taxes Paid, from Switching from Capital Taxation to Labor Taxation**

Real Interest rate (r)	Elasticity of substitution in consumption (β)	Unconstrained case		Constrained case	
		Labor supply elasticity (η)		Labor supply elasticity (η)	
		0.1	0.5	0.1	0.5
0.04	1.1	4.2	5.7	2.8	3.9
0.04	0.5	1.6	2.0	0.0	0.5
0.04	0.2	0.6	0.6	-2.0	-0.5
0.04	0.1	0.2	0.3	-2.3	-0.5
0.08	1.1	15.4	22.9	15.0	21.1
0.08	0.5	4.5	5.5	2.1	3.3
0.08	0.2	1.3	1.5	-2.4	-0.3
0.08	0.1	0.0	0.7	-2.5	-0.5

* The initial tax system is characterized by a tax on both capital (τ_k) and labor (τ_l) income of 0.30.

with binding liquidity constraints. The addition of borrowing constraints substantially affects and often reverses the efficiency gains that would otherwise arise.

4.3. Reconsidering the Costs of Progressive Income Taxation

Debates over the effects of progressive income taxation on work effort and saving have figured prominently in policy discussions. In particular, estimates of significant effects of the tax system on labor supply have raised the specter of large deadweight losses on the margin from progressive taxation (e. g., HAUSMAN [1981]). Proposals to modify the general income tax toward a proportional tax system cite the potential efficiency gains from mitigating these distortions (see for example HALL and RABUSHKA [1985]).

Our findings for the case of capital taxation suggest that, in the presence of significant restrictions on borrowing, delays in tax collection over an individual's lifetime are important in assessing the efficiency effects of tax reform. For many plausible underlying parameter values, capital income taxation is efficiency-improving because it effectively delays the collection of tax payments over an individual's life cycle. A switch from progressive to proportional income taxation would speed up tax collection, raising tax rates on low-income consumers and reducing their consumption substantially when borrowing constraints are important.

Proponents of proportional taxation in the United States have suggested that increased exemptions could maintain the equity or political acceptability of the tax code. When borrowing restrictions are important, exemptions can serve *efficiency* functions as well. Below, we examine the effects of going from a strictly proportional tax on all income to a "progressive" tax

with an intercept—exemption—and a single marginal tax rate that raises the same revenue. Including a sequence of tax brackets applying to different income levels would raise the efficiency costs of progressive taxation. Our aim here is to call attention to the role of exemptions, to introduce the possibility that progressive taxation per se need not carry a deadweight loss. The implicit tradeoff is between the saving and labor supply distortions from higher marginal taxes on capital and labor income under progressive taxation on the one hand and the efficiency gain from relaxing borrowing constraints on low-income individuals on the other. For low values of the intertemporal elasticity of substitution in consumption, the latter effect is likely to dominate; for high values, the former.

Using an exemption, we calculate the constant marginal tax rate required to raise the same revenue as a proportional income tax at a rate of 30%. The exemption is calibrated to be two-thirds of first-year earnings (about 15% of maximum earnings). Simulations are performed using interest rate values of 0.04 and 0.08, labor supply elasticity values of 0.1 and 0.5, and β values of 1.1, 0.5, 0.2, and 0.1. Dollar-equivalent efficiency gains and losses are calculated relative to taxes shifted. That is, for each individual, the switch to progressive taxation grants an increment to income equal to the present value of the new marginal tax rate times the exemption level, at a cost of higher taxes paid on the margin at the new marginal tax rate. Our percentage efficiency change is calculated as the quotient of the dollar-equivalent efficiency gain or loss and this effective income change from the imposition of the exemption.

Results are reported in Table 3.¹⁶ The change to progressive taxation leads to substantial efficiency losses under perfect capital markets; the losses increase with the magnitude of the elasticity of labor supply, as expected. Even with the significant increase in marginal tax rates (and hence distortions, which initially rise with the square of the tax rate) required to raise the same amount of revenue over the individual's lifetime, the inclusion of borrowing restrictions in the analysis substantially mitigates losses in all cases. For example, in the case with $r=0.04$, $\beta=0.5$, and $\eta=0.1$, the efficiency loss of 10.3% (relative to taxes shifted) under perfect capital markets becomes a 2.3% gain in the liquidity-constrained regime. Even more striking is the finding that for a relatively inelastic labor supply and a low intertemporal elasticity of substitution in consumption, *efficiency gains* accompany progressive taxation. That is, the exemption here serves an efficiency role in the presence of liquidity constraints.¹⁷

16. While an analysis of the transition from the steady state of the economy under proportional taxation to one with the progressive tax would be desirable, there are several problems. Most worrisome is the fact that overlapping generations models such as these can have a continuum of such transition paths. See the discussion in HUBBARD and JUDD [1986].

17. This is in contrast to simulations ignoring the potential effects of liquidity constraints. For example, HAUSMAN [1981], p. 64, Table 1, finds that the welfare cost of progressive income taxation described in this way increases with the amount of the exemption.

TABLE 3

*Efficiency Gains, as Percent of Taxes Shifted, from Switching from Proportional Income to Progressive Income Taxation**

Real Interest rate (r)	Elasticity of substitution in consumption (β)	Unconstrained case		Constrained case	
		Labor supply elasticity (η)		Labor supply elasticity (η)	
		0.1	0.5	0.1	0.5
0.04	1.1	-18.8	-42.9	-6.5	-28.2
0.04	0.5	-10.3	-23.6	2.3	-3.1
0.04	0.2	-5.6	-10.6	4.1	1.1
0.04	0.1	-4.6	-8.2	4.5	1.3
0.08	1.1	-65.3	-152.6	-54.3	-114.5
0.08	0.5	-28.8	-54.9	-4.4	-18.3
0.08	0.2	-12.5	-23.3	6.3	-0.3
0.08	0.1	-9.8	-18.5	6.8	0.6

* The proportional tax is characterized by a tax on both capital (τ_K) and labor (τ_L) income of 0.30. The exemption under the progressive tax is described in the text.

TABLE 4

*Efficiency Gains, as Percent of Taxes Shifted, from Switching from Proportional Income to Progressive Labor Taxation**

Real Interest rate (r)	Elasticity of substitution in consumption (β)	Unconstrained case		Constrained case	
		Labor supply elasticity (η)		Labor supply elasticity (η)	
		0.1	0.5	0.1	0.5
0.04	1.1	-8.3	-33.4	-1.1	-18.1
0.04	0.5	-8.0	-28.3	3.4	-1.1
0.04	0.2	-5.8	-16.9	5.2	1.5
0.04	0.1	-5.2	-10.2	5.1	1.4
0.08	1.1	-13.1	-34.2	-9.1	-41.1
0.08	0.5	-7.9	-39.7	2.1	-6.0
0.08	0.2	-8.2	-21.9	7.2	2.4
0.08	0.1	-13.7	-6.7	7.3	2.8

* The proportional tax is characterized by a tax on both capital (τ_K) and labor (τ_L) income of 0.30. The exemption under the progressive tax is described in the text.

Our findings in the experiments with capital and labor taxation and with progressive and proportional taxation suggest potential gains from a progressive tax on labor income only, with no taxation of interest income. Such a reform would mitigate the impact of taxation on the consumption of constrained individuals without introducing the intertemporal distortions inherent in capital taxation. To examine this hypothesis, we simulate a shift away from a proportional general income tax of 30%. The new system sets $\tau_K=0$, imposing all taxation on labor income and granting the same exemption as before. The constant marginal tax rate on labor income above the exemption is raised to ensure revenue

neutrality. Results presented in Table 4 for the same parameters as before show that exemptions are generally less costly and more often beneficial when imposed on top of a labor income tax than when imposed on both capital and labor taxes. This finding validates our intuition that the exemption attacks the liquidity-constraint problem and works better when it does not aggravate intertemporal distortions, even though it must aggravate labor supply distortions even more.

Comparing the findings of Tables 2 and 3 reveals the significance of liquidity constraints for considering alternative tax changes. The efficiency changes in Table 2 can be expressed in units comparable to those in Tables 3 and 4, given information on taxes shifted relative to lifetime taxes paid. As a benchmark, the present value of the exemption is about one-fifth of lifetime taxes when intertemporal substitution is high, and about one-third of lifetime taxes when intertemporal substitution is low. The efficiency changes of all three policies considered are thus of comparable magnitude.

5 Borrowing Constraints and Social Security in the Life Cycle Framework

5.1. Issues

Since the pioneering paper by FELDSTEIN [1974], studies of the effects of social security on saving have considered impacts over the whole life cycle and not just in old age. Empirical work within the life cycle framework has tested the impact on consumption of intergenerational wealth transfers accompanying the introduction of a “pay as you go” social security system. Our concern is with the influence of realistic capital-market imperfections on the effect of social security annuities on national saving and individual welfare. In particular, we consider the relationship between two—(i) market failure in the private provision of annuities, and (ii) restrictions on borrowing against anticipated future gains from participating in social security.

The first has been examined in isolation. ABEL [1985] and HUBBARD [1987] have shown in the context of lifetime uncertainty that even an actuarially fair, fully funded social security system can generate increases in individual lifetime welfare. However, as we have emphasized, the specification of a *lifetime* budget constraint may be too narrow a description of restrictions on individuals’ optimizing behavior in the presence of capital market imperfections. The presence of borrowing constraints can reduce substantially the efficiency gains from introducing social security annuities under payroll tax finance. The extent to which agents can spread the benefits from participation in social security annuities over their lifetimes depends on the degree to which capital markets permit consumption to be smoothed when current resources are insufficient. More important, since

social security is financed through a proportional payroll tax on current earnings, payroll taxes depress consumption dollar for dollar when borrowing constraints are binding. Including realistic limitations on borrowing introduces the possibility that increasing the provision of social security coverage (financed by the payroll tax) may leave individuals worse off in terms of utility gains from lifetime consumption, while at the same time increasing *potential* lifetime consumption (that is, if constraints were not binding). In general, one expects that an optimal tax structure should reallocate this burden over an individual's lifetime.

5.2. Lifetime Uncertainty, Borrowing Restrictions, and Saving Behavior

Precautionary saving against lifetime uncertainty¹⁸ will arise in the life cycle model to the extent that there are imperfections in the provision of old-age annuities.¹⁹ To the extent that such precautionary saving is quantitatively important, modifying the basic life cycle model to include uncertainty may account for much of the failure of the model to explain actual wealth-age profiles (particularly among the elderly).

Following ABEL [1985] and HUBBARD [1987], we do not explicitly model the reason for the absence of annuity markets. We assume that they simply exist neither prior to nor after the introduction of the social security system. This is an appropriate assumption given our focus on the importance of the *interaction* of liquidity constraints and the social security system. By ruling out all annuities, we make the impact unrealistically large and bias the results in favor of social security. If we were to add reasonable features such as overpriced annuities, manipulative bequest motives (BERNHEIM, SHLEIFER, and SUMMERS [1985]), or altruistic bequest motives, the value of social security annuities would decline. The importance of borrowing constraints would not be affected, however, implying that our negative results would be more likely to hold.

Our life cycle model outlined in section 4 is modified as follows. Individuals live in a maximum of T years, working only for the first R ; the retirement age of R is taken as exogenous, and labor is supplied inelastically. The probability of surviving through period t is $p(t)$ for each t . As before, our simulations begin at the beginning of individuals' working lives (assumed to be age twenty). Retirement occurs at model age forty-five. The maximum model age to which one can survive is ninety.

18. YAARI's [1965] seminal paper showed that with an uncertain lifetime, intertemporal utility maximization can dictate saving against the possibility of living longer than expected.

19. This market failure is likely because of asymmetries of information between individuals and insurers, the classic adverse selection problem discussed by ROTHSCILD and STIGLITZ [1976] and elaborated in the context of social security by ECKSTEIN, EICHENBAUM, and PELED [1985]. FRIEDMAN and WARSHAWSKY [1985] show that under plausible assumptions about risk aversion, the returns on life annuity contracts actually offered in the market would not be purchased by optimizing individuals.

Following YAARI [1965] and BARRO and FRIEDMAN [1977], we let utility be additively separable across periods, and let utility from consumption $U(c)$ be evaluated contingent on being alive. That is, with no restrictions on borrowing, individuals choose c so as to maximize

$$(8) \quad \int_0^T p U(c) e^{-\rho t} dt,$$

subject to

$$(9) \quad \dot{A} = E + rA + B - c, \quad A(0) = 0, \quad A(T) \geq 0,$$

where B represents resources from unplanned bequests from the previous generation.

We assume again that the utility function is isoelastic. If $h(t) dt$ is the probability of death during $(t, t + dt)$ conditional on being alive at t —i. e., the hazard rate—the problem in (8) then becomes

$$(10) \quad \max \frac{1}{1 - \beta^{-1}} \int_0^T e^{-\int_0^t (\rho + h(s)) ds} c^{1 - \beta^{-1}} dt.$$

subject to (9), where β is the intertemporal elasticity of substitution in consumption. Denoting the marginal utility of consumption conditional on being alive by λ and the sum of the rate of time preference and the hazard rate reflecting lifetime uncertainty by $\tilde{\rho}$, the differential equations describing the time paths of consumption and asset accumulation (in the absence of borrowing restrictions) are now given by

$$(11) \quad \dot{\lambda} = (\tilde{\rho} - r)\lambda,$$

and

$$(12) \quad \dot{A} = rA + E + B - c(\lambda),$$

together with the boundary conditions $A(0) = A(T) = 0$. Data on average survival probabilities in the United States are taken from FABER [1982].

Unplanned bequests are incorporated as follows. Using information on p over the life cycle, a distribution of initial bequests can be generated. We consider intergenerational transfers from a generation of single “parents” to the next generation. Iteration proceeds for a given set of parameter values until an individual would transmit (in expected present value) the same bequest he receives. As our principal concern is with the first-order effects of lifetime uncertainty (and later social security) on the aggregate capital stock (and output and consumption), we do not discuss the impact of lifetime uncertainty or social security on the steady-state *distribution* of bequests (see for example ABEL [1985]). Given our emphasis on life cycle patterns of consumption and savings, it is important that we do not aggregate intertemporally. We must allow the individual to live for several periods if we are to get a quantitative idea of the importance of borrowing constraints. However, given this fineness in our intertemporal consumption patterns, it would be numerically intractable to calculate a steady-state rational expectations distribution of bequests. Therefore, all individuals

are assumed to receive the weighted average bequest regardless of their particular family mortality history. The implicit assumption is that individual bequests are taxed away by the government and redistributed lump sum to individuals. To model the observation that these bequests are most likely to occur when the recipient is in early middle age, such receipts are assumed to be obtained after twenty periods.

As before, when we impose the constraint that net worth must be nonnegative at all times, we change the consumer's optimization problems. The arbitrage equations are modified as before.

HUBBARD [1987] and HUBBARD and JUDD [1987] show that precautionary saving in such a model is substantial. In the absence of borrowing restrictions, unplanned bequests accompanying lifetime uncertainty are significant relative to lifetime earnings. Aggregate capital stocks implied by the life cycle model when lifetime is certain are small relative to those implied by the uncertain-lifetime case.²⁰ The results in HUBBARD and JUDD [1987] also illustrate the additional relevance of liquidity constraints for the size of the capital stock in the uncertain-lifetime case. The constraint is binding for a significant interval; the aggregate capital stock is higher in the liquidity-constrained regime.²¹ We now take up the effects of social security on precautionary saving in this case.

5.3. Social Security, Precautionary Saving, and Life Cycle Saving

In previous papers, ABEL [1985] and HUBBARD [1987] have shown that public provision of annuities through social security leads to partial equilibrium increases in individual welfare and decreases in national saving. In the context of our multiperiod life cycle model, a brief outline of such a social security system follows. Individuals are compelled to pay a payroll tax τ_s on gross wages, from which the social security system is funded. During retirement they receive an annuity benefit S in each period until death. The asset accumulation constraint (in the absence of restrictions) becomes.

$$(13) \quad \dot{A} = rA + (1 - \tau_s)E + B + S - C, \quad A(T) \geq 0.$$

-
20. The unrealistically low capital stocks in the certain-lifetime case lend further support to the finding in KOTLIKOFF and SUMMERS [1981] that the basic life cycle model can explain only a small portion of the aggregate capital stock. The importance of precautionary saving could help to explain this discrepancy without relying on intentional bequest motives.
21. The potential welfare effects of liquidity constraints in this framework stem from the excess of desired consumption over earnings when young. These effects could be mitigated, of course, to the extent that individuals have "humped" pattern of desired consumption as well—say to finance the expenses of rearing and educating children in middle age (see for example GHEZ and BECKER [1975]). In HUBBARD and JUDD [1986], we show that the general results of the impact of liquidity constraints on saving and welfare are robust to the consideration of changes in family size over the life cycle (using the family weights presented in DOLDE [1978]).

The uniform economy-wide actuarially fair social security benefit S satisfies the condition that

$$(14) \quad S \int_R^T p e^{-rt} dt = \tau_s \int_0^R p w e^{-rt} dt.$$

In the absence of borrowing restrictions, the system generates an increase in the propensity to consume out of lifetime resources. This increase in lifetime consumption occurs even in a system which is actuarially fair and fully funded.

With respect to our focus on borrowing constraints, potential efficiency gains from the introduction of social security comes about because of increases in *pre-retirement* consumption made possible by the annuity provisions. With a nonnegativity constraint on net worth, however, the social security payroll tax depresses pre-retirement consumption as long as the constraint binds, and increases consumption after the constraint ceases to bind. Hence the effect of an actuarially fair social security system is to increase desired consumption of the young, while decreasing actual consumption due to the interaction of the payroll tax and restrictions on borrowing.

In Table 5, we simulate the partial equilibrium impact on initial participants of the imposition of an actuarially fair social security system financed by a proportional payroll tax of 6%.²² Percentage changes in the capital-output ratio and average bequests as a fraction of lifetime earnings are reported, as is the change in lifetime welfare (expressed in terms of a percentage change in initial lifetime earnings). As before, simulation results are conducted over four values of β ; we also consider three values of (0.02, 0.04, and 0.06). Results both in the absence of an in the presence of liquidity constraints are reported; numbers referring to the constrained case are in parentheses. For the constrained case, the number of periods for which the constraint is binding is also reported.

In the absence of borrowing restrictions, the system generates very large initial declines in the capital stock. Given the dramatic reduction in the size of accidental bequests shown in the initial participants from introducing an actuarially fair social security system are significant for plausible parameter values.²³

Realistic borrowing restrictions mitigate these large welfare gains. Because the system is financed by a proportional payroll tax on earnings, forced saving occurs in youth. The last set of columns in Table 5 illustrates the importance of the restriction on borrowing against future net earnings even under a moderate social security tax—consumption is constrained for

22. We chose a moderate payroll tax rate between the rate assessed at the beginning of the US system and the much higher rate in place now. The results are not qualitatively sensitive to the choice of the payroll tax rate. When the wealth nonnegativity constraint is introduced, welfare losses from the interaction of the borrowing constraint and the payroll tax increase with the tax rate in a nonlinear fashion.

23. For example, KOTLIKOFF and SPIVAK [1981] report very large welfare gains from the introduction of a perfect annuity market.

TABLE 5

Social Security, the Capital Stock, and Individual Welfare

	Percentage Change in Bequests as a Fraction of Lifetime Earnings										Δ		Periods Constrained		
	β					β					β		β		
	0.50	0.25	0.20	1.10	0.50	0.25	0.20	1.10	0.50	0.25	0.20	1.10	0.50	0.25	0.20
	48.2%	-58.1%	-58.9%	-57.9%	-88.0%	-83.5%	-79.3%	4.4%	8.2%	11.1%	12.6%	13	12	11	10
	(-28.4)	(-39.2)	(-42.4)	(-42.7)	(-82.1)	(-77.1)	(-72.9)	(3.5)	(4.5)	(0.1)	(-4.3)				
	-27.7	-50.4	-65.2	-70.1	-80.1	-94.2	-98.3	5.4	6.6	7.7	8.1	5	9	12	13
	(-24.5)	(-39.0)	(-47.8)	(-51.4)	(-78.6)	(-89.5)	(-93.3)	(5.0)	(3.9)	(-1.8)	(-5.2)				
	-17.0	-32.0	-36.5	-39.3	-73.8	-95.5	-97.5	6.1	3.8	1.9	1.3	2	7	11	12
	(-16.7)	(-29.1)	(-31.4)	(-32.8)	(-73.6)	(-93.6)	(-95.8)	(5.9)	(2.4)	(-3.5)	(-5.6)				

Notes: Numbers in parentheses represent the alternative result in the presence of liquidity constraints. Δ is expressed as percentage of lifetime earnings. Δ and the percentage change in K/Y are measured with respect to the corresponding no-social-security cases under lifetime uncertainty. In all cases $t_s=0.06$.

at least 10 periods at an interest rate of 0.02. For moderate measures of the intertemporal elasticity of substitution in consumption ($\beta=0.50$), the gains in lifetime welfare reported are substantially smaller than the potential gains in the absence of borrowing restrictions. When $\beta=0.20$, the gains are trivial when the interest rate is 0.02, and at interest rates of 0.04 or 0.06, the operation of the social security system actually leads to an efficiency loss, as it does for all cases when $\beta=0.10$. These results suggest the importance of both uncertainty and the method used to finance the system in evaluating the impact of social security on individual saving behavior.

5.4. Progressive Taxation and Welfare Gains from Social Security

To examine the influence of the method of financing social security on its impact on the capital stock and individual welfare, we now remove the assumption of proportional payroll tax finance. In its place we institute a progressive social security tax in which the first 15 working periods are exempt from payroll taxation. To preserve comparability with our previous results, the retirement benefit is kept the same as under the proportional tax case where $\tau_s=0.06$. A new, higher flat tax rate is instituted in the sixteenth period to maintain the average actuarial fairness of the system. The use of such an exemption alleviates the added contribution to the social security payroll tax to liquidity constraints on consumption. By shifting the burden of the tax intertemporally through higher taxes later in life, the same present value of contribution can be collected with an increase in individual lifetime utility from consumption. Cross-sectionally, the use of exemption corresponds to progressive taxation.

Partial equilibrium results for the impact of social security on lifetime welfare (comparable to Table 5) are presented in Table 6. Simulations are performed for four values of β and three values of r as before. The top entry in each cell represents the gain in lifetime welfare (expressed as an equivalent percentage of lifetime earnings) from participating in the "progressive-tax-financed" social security system. The numbers in parentheses below are the corresponding gains from the "proportional-tax-financed" system in Table 5. This change in the structure of payroll tax finance has important implications for the magnitude of the welfare gains made possible by social security. Gains are positive in all cases in Table 6, and for many parameterizations are substantially higher than in the proportional-tax-financed case. As with the case of the tax reforms discussed before, this experiment puts in a new light claims based on "perfect markets" models that large efficiency costs necessarily accompany progressive taxation.

TABLE 6

“Progressive” Taxation and Social Security: Reassessing the Impact on Individual Welfare

	Δ			
	$\beta=1.10$	$\beta=0.50$	$\beta=0.25$	$\beta=0.20$
$r = .02$	4.2% (3.5)	6.3% (4.5)	4.3% (0.1)	1.4% (-4.3)
$r = .04$	5.1 (5.0)	5.3 (3.9)	2.7 (-1.8)	0.6 (-5.2)
$r = .06$	6.0 (5.9)	4.0 (2.4)	0.8 (-3.5)	0.2 (-5.6)

Notes: Δ is measured with respect to the corresponding no-social-security cases under lifetime uncertainty, with borrowing restrictions imposed. Individuals are exempt from social security taxation for the first fifteen working periods. Numbers in parentheses are the welfare gains from imposing a social security system of identical size financed by a proportional payroll tax.

6 Conclusions and Implications

The existence of borrowing constraints as a preexisting distortion is important in the determination of consumption behavior in a life cycle setting. We show here that consideration of such constraints will have a significant impact on how one views various tax changes. Many of the principal findings about the saving and efficiency effects of tax reforms drawn from simulation models rely heavily on the assumptions that capital markets are perfect and that individuals can borrow and lend freely to smooth consumption in response to policy changes. When borrowing restrictions are allowed for, theoretical predictions about the efficiency effects of altering the relative reliance on various tax bases or the degree of progressivity of the income tax are no longer clear.

We consider fiscal reforms in both the income tax and the social security system. With respect to the former, we show that arguments that reductions in capital income taxation financed by increased labor income taxation are efficiency-improving depend on a substantial interest sensitivity of saving in the life cycle framework and on the ability of consumers with low current earnings to borrow. With borrowing restrictions, the gains from higher saving rates and output must be weighed against the efficiency losses from the reduced consumption of constrained individuals. For some plausible parameter values, elimination of capital income taxation compensated by higher labor income taxation can reduce the welfare of a representative individual.

In a similar vein, recent analyses of progressive taxation focus on the disincentive effects on work effort and saving of high marginal tax rates. A

move toward proportional taxation would indeed reduce these effects, but, in the presence of the borrowing constraints discussed here, would also reduce the consumption of constrained low-income individuals. We find that the use of an exemption and a higher marginal tax rate can in some cases improve efficiency relative to a proportional tax. That is, there may be significant efficiency gains from using exemptions.

Our framework for reconsidering the efficiency effects of tax reform suggests the potential importance of other preexisting distortions in influencing the outcome of changes in fiscal policy. In particular, many government programs, most notably social insurance programs, affect agents' lifetime budget constraints to the extent that private insurance against uncertainty over length of life, job loss, catastrophic illness, and so on, is incomplete. When social insurance is examined in the framework of precautionary saving, its provision will in general affect lifetime consumption and not just consumption during the periods in which payments are received.

In the context of lifetime uncertainty, we show within our life cycle model that the introduction of even an actuarially fair, fully funded social security system can be efficiency-improving. Of course, as in the case of tax reform, the extent to which consumers can spread the benefits from social security annuities over their lifetime depends on the degree to which capital markets permit consumption smoothing when current resources are insufficient. More important, the proportional payroll tax used to finance social security depresses consumption dollar for dollar when borrowing constraints are binding. With realistic limitations on borrowing, increasing payroll-tax-financed social security benefits may decrease utility to individuals from lifetime consumption, even though the system would be efficiency-improving under perfect capital markets. Indeed, the steady-state efficiency cost of social security under payroll tax finance can be substantial. Consistent with our analysis of progressive income taxation, we show that a progressive payroll tax could be efficiency-improving.

Finally, theoretical research should probe more closely the origins of borrowing constraints in consumer finance and their implications for intra-generational and intergenerational issues in optimal taxation.²⁴ Realistic consideration of imperfections in the markets in which taxpayers carry out their plans and the implications of these imperfections for "second-best" policy design is an important research strategy.

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24. See for example CALOMIRIS and HUBBARD [1987] and HUBBARD and JUDD [1987].

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Elastic Labor Supply and the Life-Cycle Model

For the case of variable labor supply, we rewrite the individual's lifetime utility function in equation (1) as being additively separable in consumption, c , and leisure, l :

$$(15) \quad \int_0^T [U(c) - V(1-l)] e^{-\rho t} dt,$$

where the labor endowment is normalized to unity (that is, labor supply, L —"hours"—is equal to $1-l$). Again assuming an isoelastic utility function, we let

$$(16) \quad U(c) = \frac{1}{1-\beta^{-1}} c^{1-\beta^{-1}},$$

and

$$(17) \quad V(1-l) = \frac{\gamma}{1+\eta} (1-l)^{1+\eta},$$

where γ measures the intensity of leisure (that is, regulates the marginal rate of substitution between consumption and leisure) and η is the compensated elasticity of labor supply.

Individual net earnings (E in the text) can now be decomposed as the product of the after-tax wage rate (per efficiency unit), individual productivity per hour, and hours worked. Denoting the gross wage, labor tax rate, and productivity by w , τ_L , and e , respectively, we have

$$(18) \quad E = (1 - \tau_L) w e (1-l) = \bar{w} L.$$

Arbitrage conditions yield

$$(19) \quad U'(C) = \lambda,$$

and

$$(20) \quad V'(L) = \bar{w} \lambda.$$

Hence labor supply is equal to

$$(21) \quad L = \min \left[1, \left(\frac{\bar{w} \lambda}{\gamma} \right)^{1/\eta} \right],$$

so that if $L < 1$,

$$(22) \quad \bar{w} U'(c) = V'(L),$$

or

$$(23) \quad L = \left(\frac{\bar{w}c^{-\beta-1}}{\gamma} \right)^{1/\eta}.$$

In our parameterizations, we vary the intertemporal elasticity of substitution and the labor supply elasticity across a range suggested by various empirical studies. In varying the parameters, we also vary γ so as to keep the consumption of leisure in the first period of life, if constrained, equal to 0.6. This is done to model the observation that 40-50% of available hours are spent on work. When we vary the interest rate we also adjust the wage profile to keep the wage-rental ratio constant. This adjustment ensures that we are examining the same production function as we vary our parameters.

Our specification of utility makes the special assumption of additive separability between consumption and leisure. This specification was assumed by MACURDY [1981]. Also, MACURDY [1983] estimates a more general utility function, but could not reject separability.

For the human capital vector, e , we took DAVIES's [1981] fourth-degree polynomial estimate of earnings.