Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax

By R. Glenn Hubbard and Kenneth L. Judd*

This paper examines the impact of Social Security on national saving and individual welfare in the presence of realistic capital market imperfections — market failure in the private provision of annuities and restrictions on borrowing against anticipated future wages. The introduction of Social Security increases lifetime welfare and reduces national saving if borrowing restrictions are absent. However, the increase in individual welfare is reduced, and in some cases eliminated, when borrowing constraints are taken into consideration. The substantial difference suggests the importance of reexamining the proportional payroll tax finance of Social Security.

It has been recognized for some time in applied public economics that discussion of the impact of taxation and public programs on individual welfare (as well as on such aggregate measures of interest as the saving rate or the capital stock) requires an explicit analysis of agents' intertemporal budget constraints. Since the pioneering paper by Martin Feldstein (1974), studies of the effects of Social Security on saving have considered behavior over the whole life cycle and not just in old age. Empirical work has tested the impact on consumption of the individual wealth transfers accompanying the introduction of a pay-as-you-go Social Security system. Our concern here is with the impact of Social Security retirement annuities on national saving and individual welfare in the presence of realistic capital-market imperfections. In particular, we consider the relationship between two: (i) market failure in the private provision of annuities; and (ii) restrictions on borrowing against anticipated future wages.

The first has been examined in isolation. Andrew Abel (1985) and Hubbard (1987) have shown in the context of lifetime uncertainty that even an actuarially fair, fully funded Social Security system can generate increases in individual lifetime welfare. Hence, previous estimates of the impact of Social Security on consumption drawn solely from the consideration of the intergenerational wealth transfers at the introduction of the system may even be too small.

However, the specification of a lifetime budget constraint may be too narrow a de-
scription of restrictions on individuals' optimizing behavior in the presence of capital-market imperfections. Actual limitations on borrowing appear in upward-sloping interest rate schedules, collateral requirements, and quantity restrictions. It is clear on a qualitative level that the presence of liquidity constraints can reduce substantially the welfare gains from introducing Social Security annuities under payroll tax finance.

While the qualitative findings above are intuitive, the potentially important interaction of the capital-market imperfections requires a model with substantial intertemporal disaggregation. Our approach differs from Abel in that we employ life cycle simulation models to analyze capital-market imperfections. We do so for two reasons. First, given our interest in alternative public policies, it is important to obtain realistic quantitative estimates of the effects of Social Security. Second, knowledge of the length of periods during which borrowing restrictions bind is important; even a three-period analytical model would be inadequate for our purposes.

Recent advances in the examination of efficiency gains from dynamic tax reforms have used life cycle simulation models to isolate intragenerational and/or intergenerational effects. The capital-market imperfections discussed above are typically missing in these models. First, most of these exercises have ignored insurance features of fiscal policies; social insurance programs affect agents' lifetime budget constraints to the extent that the private insurance markets (against uncertainty over length of life, job loss, catastrophic illness, etc.) are incomplete. We focus on Social Security retirement annuities both because of their importance in the existing literature on tests of the life cycle model and because of the way in which they are financed.

Second, liquidity constraints are likely to be important. The extent to which agents can spread the benefits from participation in Social Security annuities over their lifetimes depends on the degree to which capital markets permit consumption smoothing when current resources are insufficient. More important, since Social Security is financed through a proportional payroll tax on current earnings, payroll taxes depress consumption dollar for dollar when liquidity constraints are binding. Including realistic limitations on borrowing introduces the possibility that increasing the provision of Social Security coverage (financed by the payroll tax) may leave individuals worse off in terms of utility gained from lifetime consumption, while at the same time increasing potential lifetime resources. In general, one expects that an optimal tax structure should reallocate this burden over an individual's lifetime.

We organize our analysis of the importance of capital-market imperfections (market failure in the private provision of annuities and borrowing restrictions) in describing the impact of Social Security on individual welfare and the capital stock as follows. In Section I, we investigate the relevance of borrowing restrictions and uncertainty over longevity for the size of the capital stock. In particular, we find that the stock of "precautionary saving" is quantitatively important, lending further support to the claim that the perfect-certainty version of the life cycle model cannot explain observed saving behavior.

In Section II, we take up the partial-equilibrium effects on individual consumption.

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3 The focus of such analyses is generally on switching tax regimes, say from a proportional general income tax to a proportional consumption or wage tax. See for example Lawrence Summers (1981), Alan Auerbach and Kotlikoff (1983), Owen Evans (1983), Laurence Seidman (1984), and our paper (1986). Auerbach et al. (1983) and our paper (1986) have considered progressive taxation as well.

4 It is possible to think of the "event-conditioned" transfer programs that comprise social insurance as relaxing constraints on individual consumption. For example, one of the primary goals of the Social Security retirement program is the maintenance of consumption in old age. When social insurance is viewed in the framework of precautionary saving (see also Daniel Hamermesh, 1982), its provision will in general affect lifetime consumption, and not just consumption during the periods in which payments are received.
As noted previously by Abel and Hubbard (1987), for any plausible set of assumptions about underlying parameters, social security generates a significant increase in lifetime consumption and welfare accompanied by a reduction in the capital stock if borrowing restrictions are absent. However, the increase in individual welfare is reduced, and in some cases eliminated, when borrowing restrictions are also taken into consideration. The substantial difference suggests the importance of reexamining the proportional payroll tax finance of Social Security.

In Section III, we extend the model to general equilibrium, with endogenous factor prices. Partial-equilibrium gains in lifetime welfare from participation in Social Security are offset in the long run by the interaction of higher steady-state interest rates and binding liquidity constraints. In contrast to the large welfare gains found in previous studies, we find that the steady-state welfare cost of Social Security under proportional payroll tax finance is in general substantial. Section IV illustrates the ability of alternative proposals for financing Social Security to alleviate the problem created by the interaction of borrowing constraints and the proportional payroll tax. Age-specific tax schemes can restore much of the potential gain from participating in Social Security annuities. Conclusions and directions for future research are discussed in Section V.

1. Lifetime Uncertainty, Borrowing Restrictions, and Individual Saving Behavior

Our emphasis in this paper is on the impact of Social Security annuities on national saving and individual welfare when there are imperfections in capital markets. We first consider the role of precautionary saving against lifetime uncertainty in the absence of an annuity market. Menahem Yaari's (1965) seminal paper showed that with an uncertain lifetime, intertemporal utility maximization can dictate saving against the possibility of living longer than expected. More recent applications to public pension schemes have appeared in James Davies (1981), Eytan Sheshinski and Yoram Weiss (1981), Abel, and Hubbard (1987). To the extent that precautionary saving is significant, modifying the basic life cycle model to include uncertainty may account for much of the failure of the model to explain wealth-age profiles (particularly among the elderly).

Such precautionary saving is necessary because of market failure in the private provision of old-age annuities. This market failure is likely because of asymmetries of information between individuals and insurers, the classic adverse selection problem discussed by Michael Rothschild and Joseph Stiglitz (1976) and elaborated in the context of Social Security by Zvi Eckstein et al. (1985). Benjamin Friedman and Mark Warshawsky (1985) show that under plausible assumptions about risk aversion, annuity contracts actually offered in the market would not be purchased by optimizing individuals.

Following Abel and Hubbard (1987), we do not explicitly model the reason for the absence of annuity markets. We assume that private annuities are not available. This is an appropriate assumption given our focus on the importance of the interaction of borrowing constraints and the Social Security system. By ruling out all annuities, we make the impact unrealistically large and bias the results in favor of Social Security. If we were to add reasonable features such as overpriced annuities, manipulative bequest motives (B. Douglas Bernheim et al., 1985), or altruistic bequest motives, the value of Social Security annuities would decline. The importance of liquidity constraints would not be affected, however, implying that our negative results would be more likely to hold.

We begin with the following life cycle model. Agents are assumed to be selfish, in that no bequests are desired. Individuals live for a maximum of \( T \) years, working only for...
the first $R$ years; the retirement age of $R$ is taken as exogenous, and labor is supplied inelastically. The probability of surviving through period $t$ is $p(t)$ for each $t$. Our simulations begin at the beginning of individuals' working lives (assumed to be age 20). Retirement occurs at model age 45. The maximum model age to which one can survive is 90.

Following Yaari and Robert Barro and James Friedman (1977), we let utility be additively separable across periods, and let utility from consumption $U(c(t))$ be evaluated contingent on being alive at time $t$. That is, with no restrictions on borrowing, individuals choose $c$ so as to maximize

$$
\max_c \int_0^T p(t) U(c) e^{-\rho t} dt,
$$

subject to

$$\dot{A} = w + rA + B - c,$$

$$A(0) = 0, \quad A(T) \geq 0,$$

where $c$, $\rho$, and $r$ represent consumption and the subjective discount rate and interest rate, respectively. The variable $A$ represents the stock of accumulated assets. A dot over a variable denotes a time rate of change. The income stream $w$ represents labor earnings; $B$ includes resources from unplanned bequests from the previous generation.7

Assuming that the utility function is of the isoelastic form, we can rewrite (1) as

$$
\max_c \int_0^T \frac{1}{1-\beta^{-1}} pc^{1-\beta^{-1}} e^{-\rho t} dt,
$$

where $\beta$ is the intertemporal elasticity of substitution in consumption. Note that if $h(t) dt$ is the probability of death during $(t, t + dt)$ conditional on being alive at $t$, the hazard rate, the problem becomes

$$
\max_c \frac{1}{1-\beta^{-1}} \int_0^T e^{-\int_0^t (p + h(s)) ds} c^{1-\beta^{-1}} dt,
$$

subject to (2). Denoting the marginal utility of consumption conditional on being alive at $t$ by $\lambda(t)$ and the sum of the rate of time preference and the hazard rate reflecting lifetime uncertainty at $t$ by $\bar{\rho}(t)$, the differential equations describing the time paths of consumption and asset accumulation (in the absence of borrowing restrictions) are given by

$$
\dot{\lambda} = (\bar{\rho} - r) \lambda,
$$

$$\dot{A} = rA + w + B - C(\lambda),
$$

where $C(\lambda)$ is defined by $U'(C(\lambda)) = \lambda$, expressing consumption as a function of the current marginal value of assets. Furthermore, we impose the boundary conditions $A(0) = A(T) = 0$.

Within the framework of the model described above, we can simulate the effect of lifetime uncertainty on the size of the capital stock. The total capital stock is aggregated up from age-specific individual asset stocks assuming a population growth rate of 1 percent per annum. Individuals in the certain-lifetime case are assumed to die at the average age of death in the population. Data on average survival probabilities are taken from J. F. Faber (1982). The individual age-earnings profile is taken from Davies.8 The rate

7The corresponding problem for the certain-lifetime case would be to maximize

$$
\int_0^D U(c) e^{-\rho t} dt, \quad \text{subject to} \quad \dot{A} = w + rA - c,
$$

where $D$ is the expected date of death in $(0, T)$ in the uncertain-lifetime case.

8As in Davies (p. 572), the lifetime path of mean noninvestment income $E$ is approximated from ages 20 to 65 by a fourth-order polynomial:

$$
E(t) = 36,999.4 + 3520.22t - 101.878t^2 + 1.34816t^3 - 0.00706233t^4.
$$

An alternative earnings function would be the estimate by Finis Welch (1979) for full-time workers with a high-school education. We used that earnings profile as well; the simulation results differed little, and we do not report them here.
of time preference $\rho$ is assumed to equal 0.015 per annum.

There is some evidence on the value of $\beta$ in the literature. Summers (1982) reports estimates of the intertemporal elasticities of about unity, using postwar data. Robert Hall (1985) concludes from a set of results that $\beta$ is roughly zero. The results of the study by Irwin Friend and Marshall Blume (1975) of household portfolio allocation imply a $\beta$ of no more than 0.5. Henry Farber's (1978) estimation of preferences of workers from collective bargaining agreements yielded results consistent with a $\beta$ of about 0.3. Lars Peter Hansen and Kenneth Singleton (1983) found implied estimates of $\beta$ of at least 0.5. In our simulation exercises, we use four values of $\beta$ (1.10, 0.50, 0.25, and 0.20) along with three alternative values of $r$ (0.02, 0.04, and 0.06).

Bequests are incorporated in the simulations as follows. Using information on $p(t)$ over the life cycle, a distribution of initial bequests can be generated. We consider intergenerational transfers from a generation of single “parents” to the next generation. Iteration proceeds for a given set of parameter values until an individual would transmit (in expected present value) the same bequest he or she receives. As our principal concern is with the first-order effects of lifetime uncertainty (and later Social Security) on the aggregate capital stock (and output and consumption), we do not discuss the impact of lifetime uncertainty or Social Security on the steady-state distribution of bequests (see, for example, Abel). Given our emphasis on life cycle patterns of consumption and savings, it is important that we do not aggregate intertemporally. We must allow the individual to live for several periods if we are to get a quantitative idea of the importance of liquidity constraints. We use a year as our unit of time; we shall see that a year is large enough that any substantially greater unit would involve too much aggregation. However, given this fineness in our intertemporal consumption patterns, it would be numerically intractable to calculate a steady-state rational expectations distribution of bequests. Therefore, all individuals are assumed to receive the weighted-average bequest regardless of their particular family mortality history. The implicit assumption is that individual bequests are taxed away by the government and redistributed lump sum to individuals. To model the observation that these bequests are most likely to occur when the recipient is in early middle age, such receipts are assumed to be obtained after twenty periods.\footnote{We bias the results against the possibility of significant effects of liquidity constraints by allowing bequests to be received as early as the twentieth period of life. The results reported are not very sensitive to changes in the timing of the receipt of the bequest after twenty periods.}

An additional critique of standard life cycle models is that borrowing restrictions inhibit the ability of individuals to carry out their optimal age-consumption profile. One such limitation—and the one which we employ here—is a collateral restriction, so that net worth must be nonnegative at all times.\footnote{The source of borrowing restrictions can be important. Fumio Hayashi (1985b) and Toshiki Yotsuzuka (1986) have argued within the credit rationing model of Dwight Jaffee and Thomas Russell (1976) that neutrality prevails in the presence of liquidity constraints; that is, liquidity constraints adjust to changes in fiscal policy, restoring Ricardian equivalence. Bernheim (1987) shows that the Hayashi-Yotsuzuka results are sensitive to assumptions about the distribution of future tax liabilities. Hayashi and Yotsuzuka assume that taxes are fixed and lump sum. Bernheim considers the case in which taxes depend positively on earnings, and shows that Ricardian equivalence is in general violated. In our paper (1987), we emphasize the importance of capital-market imperfections such as those discussed here for analyses of short-run fiscal policy.}

We stress the form of borrowing restrictions we exploit by public policy. In addition, the sort of collateral restrictions on loans to borrowers under asymmetric information discussed in Charles Calomiris and Hubbard (1986) would be consistent with this.
That this restriction is most likely to be binding for the young implies that consumption will be shifted to later years in life for the representative individual, and the aggregate capital stock will be larger than it would have been if capital markets were not subject to this restriction.12

The imposition of a borrowing restriction of this form requires motivation. Here we rely on observed collateral requirements for borrowing in U.S. capital markets, that is, the restricted access to "consumption loans." Institutional motivations for the constraint include legal restrictions prohibiting the inclusion of human capital as an asset in bankruptcy proceedings or, also for our purposes, the assignment of future Social Security benefits. Allowing individuals to borrow some fixed "small" amount would increase significantly the numerical complexity of the problem, without qualitatively altering the results. Taken together, our assumptions about market failure in the private provision of annuities and borrowing restrictions should introduce no bias a priori, since the former magnifies the potential impact of Social Security on lifetime welfare, and the latter reduces it.

There is extensive empirical evidence on the importance of liquidity constraints (Fumio Hayashi, 1982, 1985a; Marjorie Flavin, 1984; Stephen Zeldes, 1985).13 Using data from the Federal Reserve Board’s 1983 “Survey of Consumer Finances,” in our paper (1986), we show that a nontrivial fraction of U.S. households have holdings of financial assets that would be too small to insulate consumption from even moderate decreases in current earnings in the presence of restrictions on borrowing against future earnings. It is, of course, possible that liquidity-constrained consumers could borrow against net nonfinancial assets such as equity in homes. However, the tapping at any significant level of housing equity to finance consumption is a recent response to capital gains on housing. Moreover, information from the 1983 “Survey of Consumer Finances” shows that homeownership rates and home equity are low for the young (liquidity-constrained consumers in the model described below).14

When we impose the constraint that net worth must be nonnegative at all times, we substantially change the nature of the consumer’s optimization problem. The budget constraint in (2) becomes

\[ \dot{A} = w + rA + B - c; A(t) \geq 0, \]

for all t.

The first-order conditions must be altered to take into account this state constraint (see Morton Kamien and Nancy Schwartz, 1982, for a discussion of such problems). The new analysis. Stephen Zeldes (1986) considers the case in which individuals are permitted to borrow against the “certain component” of future resources.

12The existence of borrowing restrictions in the form of collateral requirements does not, of course, necessarily imply that they will be binding. In our 1987 paper we discuss a case in which individuals begin their working lives with a common wage and experience a stochastic transition to a higher wage. Consumption is driven to its upper bound of current earnings when the prospective wage increase and its likelihood are high.

13Hayashi (1982) found that approximately 20 percent of all consumption in the United States could be accounted for by liquidity-constrained consumers. Additional evidence in support of liquidity constraints is provided by Hayashi’s (1985) analysis of household data from the 1963 Survey of Consumer Finances, in which the relationship between consumption and income movements differs significantly between “high saving” and “low saving” families. Zeldes (1985) also reports that the sensitivity of consumption to changes in income depends on the level of household wealth. Ben Bernanke (1984) found no evidence against the permanent income hypothesis in his examination of individual expenditures on automobiles. Such loans, however, are self-collateralized. Flavin finds that the estimated marginal propensity to consume out of transitory income is explained almost entirely by proxies for liquidity constraints.

14We would suggest that our characterization of borrowing restrictions actually understates the importance of liquidity constraints in the real world. We only disallow net consumption loans. Real-world restrictions in addition to collateral (for example, minimum-income requirements for debt service) would strengthen our conclusions.
arbitrage equations become

\[ (5a') \quad \dot{A} = (\hat{\rho} - r)A, \]

if \( A > 0 \) or \( \lambda > U'(w) \),

\[ \lambda = U'(w), \]

if \( A = 0 \) and \( \lambda \leq U'(w) \).

\[ (5b') \quad \dot{A} = 0, \text{ if } A = 0, \quad \lambda \leq U'(w), \quad \text{and} \quad \int_0^t B(s) \, ds \text{ is continuous at } t, \]

\[ \dot{A} = w + rA + B - C(\lambda), \quad \text{otherwise.} \]

If assets are positive or if wages exceed consumption, then equation (5a) still holds. Otherwise, consumption is limited to current earnings. This divides the consumer's problem into constrained and unconstrained periods of time. Equation (5b') governs how these intervals meet. At the moment when the constraint that \( A \geq 0 \) becomes binding, it imposes the tangency relation between \( A \) and the constraint.\(^5\) Intuitively, it just states that consumption must be continuous at the moment \( A \) becomes zero; concavity of \( U(c) \) implies that one does not suddenly alter consumption when assets become zero.

The importance of capital-market imperfections for the capital stock is illustrated clearly in Table 1, which reports values for the aggregate savings ratio \((K/Y)\) in the certain-lifetime and uncertain-lifetime cases, the value of average unplanned bequests as a fraction of lifetime earnings, and information on the incremental effects of liquidity constraints. As in Hubbard (1987), in the absence of borrowing restrictions, for all assumptions about the interest rate or the intertemporal elasticity of substitution in consumption, unplanned bequests accompanying lifetime uncertainty are significant relative to lifetime earnings, and capital-income ratios are substantially higher in the uncertain-lifetime case. Aggregate capital stocks implied by the life cycle model when lifetime is certain are small relative to those implied by the uncertain-lifetime case. The unrealistically low \(K/Y\) ratios in the certain-lifetime case lend further support to the finding in Kotlikoff and Summers (1981) that the basic life cycle model can explain only a small portion of the aggregate capital stock. The importance of precautionary saving could help to explain this discrepancy without relying on intentional bequest motives.

The second part of Table 1 points up the additional relevance of liquidity constraints for the size of the capital stock in the uncertain-lifetime case. In most cases, the constraint is binding for a significant interval;\(^6\) the aggregate capital-income ratio is higher in the liquidity-constrained regime.

The potential welfare effects of liquidity constraints in this framework stem from the excess of desired consumption over earnings when young. These effects could be mitigated, of course, to the extent that individuals have a "humped" pattern of desired consumption as well—at least, to finance the expenses of rearing and educating children in middle age (see, for example, Gilbert Ghez and Gary Becker, 1975). In our paper (1986), we show that the general results of the impact of liquidity constraints on saving and welfare are robust to the consideration of

\(^5\) We are, of course, considering liquidity constraints that occur in youth. We are not addressing random earnings per se (see for example our 1987 paper). Given our emphasis on the problem faced by individuals reacting to an upward-sloping lifetime earnings profile; the restriction is probably not serious. Since the marginal loss due to a tighter liquidity constraint is negligible if the constraint is light and greater when the constraint is tight, losses due to the liquidity constraint are convex in the tightness of the constraint. This indicates that the losses that arise are underestimated by examining an average earnings pattern, since the distribution of earnings patterns would include some with much tighter constraints as well as some with looser constraints.

\(^6\) Walter Dolde (1978) documents the important influence of liquidity constraints on age-specific marginal propensities to consume. With no capital-market imperfections, the life cycle model predicts that marginal propensities to consume would increase monotonically with age. He shows that borrowing restrictions are significant in determining effective planning horizons for young (constrained) individuals.
TABLE 1—CAPITAL-MARKET IMPERFECTIONS AND SAVING BEHAVIOR

<table>
<thead>
<tr>
<th>Role of Lifetime Uncertainty (No Borrowing Constraints)</th>
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<tbody>
<tr>
<td>( (K/Y)_{CL} )</td>
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<tr>
<td>( \beta )</td>
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<tr>
<td>1.10</td>
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<td>( r = 0.04 )</td>
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<tr>
<th>Liquidity Constraints and Lifetime Uncertainty</th>
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<tr>
<td>Periods Constrained</td>
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<tr>
<td>( (K/Y)_{UL} )</td>
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<tr>
<td>( \beta )</td>
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<td>( r = 0.02 )</td>
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<td>( r = 0.06 )</td>
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Note: "CL" and "UL" refer to the certain-lifetime and uncertain-lifetime cases, respectively. \( (K/Y)_{UL} \) refers to the case where borrowing restrictions are imposed.

changes in family size over the life cycle (using the family weights presented in Walter Dolde). We modified the utility function in (1) to \( \int_0^T n(t) pU(c)e^{-pt} dt \), where \( c \) now represents consumption per capita in the household and \( n \) represents the number of equivalent adults at time \( t \). In fact, the liquidity constraints were shown to be more binding and last longer. For reasons of economy, we limit our presentation below to the nonfamily case.

II. The Impact of Social Security on Individual Welfare and the Capital Stock

In previous papers, Abel and Hubbard (1987) have shown that public provision of annuities through compulsory public pensions (Social Security) leads to partial equilibrium increases in individual welfare and decreases in national saving. In the context of our multiperiod life cycle model, a brief outline of such a Social Security system follows. Individuals are compelled to pay a payroll tax \( t_s \) on gross wages, from which the Social Security system is funded. During retirement they receive an annuity benefit \( S \) in each period \( t \) until death. The asset accumulation constraint (in the absence of borrowing restrictions) becomes

\[
\dot{A} = rA + (1 - t_s)w + B + S - c;
\]

\( A(T) = 0 \).

If benefits are set according to a replacement rate of the terminal wage, then the economywide actuarially fair benefit \( S \) satisfies the condition that

\[
S \int_0^T pe^{-rt} dt = t_s \int_0^R pwe^{-rt} dt.
\]

In the absence of borrowing restrictions, the system generates an increase in the propensity to consume out of lifetime resources. This increase in lifetime consumption occurs even in a system which is actuarially fair and fully funded.

With respect to our focus on liquidity constraints, a potential welfare gain from the introduction of Social Security comes about because of increases in preretirement consumption made possible by the annuity provisions. With a nonnegativity constraint on net worth, however, the Social Security payroll tax depresses preretirement consumption as long as the constraint binds,
### Table 2—Social Security, the Capital Stock, and Individual Welfare

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<tr>
<th></th>
<th>$\beta$</th>
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<th>$r = 0.04$</th>
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<td>8.1</td>
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<td></td>
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<tr>
<td></td>
<td>(-4.3)</td>
<td>(-5.2)</td>
<td>(-5.6)</td>
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<td><strong>Number of Periods</strong></td>
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<tr>
<td>0.20</td>
<td>10</td>
<td>13</td>
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</tbody>
</table>

**Notes:** Numbers in parentheses represent the alternative result in the presence of liquidity constraints. $\Delta$ is expressed as a percentage of lifetime earnings. $\Delta$ and the percentage change in $K/Y$ are measured with respect to the corresponding no-Social Security cases under lifetime uncertainty. In all cases, $t_s = 0.06$.

and increases consumption after the constraint ceases to bind. Hence the effect of an actuarially fair Social Security system is to increase desired consumption of the young, while decreasing actual consumption due to the interaction of the payroll tax and restrictions on borrowing.\(^{17}\)

In Table 2, we simulate the partial-equilibrium impact on initial participants of the imposition of an actuarially fair Social Security system financed by a proportional payroll tax of 6 percent.\(^{18}\) Percentage changes in the capital-output ratio and the average consumption path is financed by his earnings; bequests received and given are not important for consumption decisions.

\(^{17}\)The extent to which the provision of retirement annuities depresses preretirement saving depends in part on bequest motives. It is not obvious that planned bequests weaken our analysis. If an individual plans to leave a bequest to his (her) child, his desired lifetime consumption is reduced. However, if he herself receives a bequest from a parent in some middle or late period, that bequest could be used to finance a bequest to his child. As long as the bequest he receives from his parent is obtained after his constrained periods and he plans on leaving a bequest of comparable size to his child, his

\(^{18}\)We chose a moderate payroll tax rate between the rate assessed at the beginning of the system and the much higher rate in place now. The results are not qualitatively sensitive to the choice of the payroll tax rate. When the wealth nonnegativity constraint is introduced in the next section, welfare losses from the interaction of the borrowing constraint and the payroll tax increase with the tax rate in a convex fashion.
bequests as a fraction of lifetime earnings are reported, as is the change in lifetime welfare, \( \Delta \), expressed in terms of a percentage change in lifetime earnings. As before, simulation results are conducted over four values of \( \beta \) and three values of \( r \). Results both in the absence of and in the presence of liquidity constraints; numbers referring to the constrained case are in parentheses. For the constrained case, the number of periods for which the constraint is binding is also reported.

In the absence of borrowing restrictions, the system generates very large initial declines in the capital stock. Given the dramatic reduction in the size of accidental bequests shown in the table, this is not surprising.\(^9\) Potential welfare gains to initial participants from introducing an actuarially fair Social Security system are significant for plausible parameter values.

Realistic borrowing restrictions mitigate these large welfare gains. Because the system is financed by a proportional payroll tax on earnings, forced saving occurs in youth. The last set of rows in Table 2 illustrates the importance of the restriction on borrowing against future net earnings even under a moderate Social Security tax, with consumption constrained to be no more than current resources for at least ten periods at an interest rate of 0.02. For moderate measures of the intertemporal elasticity of substitution in consumption (\( \beta = 0.50 \)), the gains in lifetime welfare reported are substantially smaller than the potential gains in the absence of borrowing restrictions. When \( \beta = 0.25 \), the gains are trivial when the interest rate is 0.02, and at interest rates of 0.04 or 0.06, the operation of the Social Security system actually leads to a loss in lifetime welfare, as it does for all cases when \( \beta = 0.20 \). These results suggest the importance of both uncertainty and the method used to finance the system in evaluating the impact of Social Security on individual saving behavior. Before returning to the issue of financing the system in Section IV, we take up in Section III the problem of considering these effects when factor prices are endogenous.


To examine seriously the welfare effects of Social Security under different assumptions about capital market imperfections, we must analyze the new steady state after the system is introduced.\(^20\) Changes in the steady-state capital stock will affect the level of output and consumption per head, and hence the lifetime utility of a representative agent. Members of the first generation in the system benefit both from the accidental bequest from the uninsured previous generation and from the gains from participating in Social Security annuities.\(^21\) The reduced value of accidental bequests permits smaller gains in consumption for succeeding generations. Hence, to consider the potential welfare gains from compulsory pensions, the tradeoff between the benefits to early participants and the costs of a lower capital stock to subsequent generations must be examined.

The partial-equilibrium effects of Social Security on individual saving will be dampened in a general equilibrium analysis of the impact on aggregate capital formation, once factor-price changes are taken into account. Such considerations have been examined in certain-lifetime models. For example, Kotlikoff (1979a) used a life cycle model.
with certain longevity and a Cobb-Douglas production technology to consider the impact of a pay-as-you-go Social Security system on the capital stock. For plausible parameter values, he found that the positive lifetime wealth increment traceable to Social Security caused a 20 percent decrease in the steady-state capital stock. While this effect is certainly substantial, it is roughly half of his calculated partial-equilibrium effect which is directly related to the excess of the present value of benefits over the present value of contribution.

To examine the impact of savings against lifetime uncertainty on aggregate saving, we assume that output is produced according to a Cobb-Douglas production function in capital and labor, with a capital share equal to $\alpha$. Factor markets are assumed to be competitive, so that capital and labor are paid their marginal products. That is, the interest rate ($r$) and wage rate ($w$) are such that

\begin{align}
    r &= \alpha k^{\alpha-1}, \\
    w &= (1-\alpha)k^\alpha,
\end{align}

where $k$ represents the capital-labor ratio.

Within this framework, the steady state can be solved for as follows. A guess is made for $k$. Solutions for $r$ and $w$ are then generated from the marginal productivity conditions to produce individual consumption and wealth profiles. The resulting aggregate consumption and capital stock per capita are compared with the initial guess, and iteration proceeds until convergence is reached.

As in the partial-equilibrium case, a second calculation of the value of the steady-state bequest must also be made. Within the routine described above, each parameterization of $r$ and $w$ generates a different expected bequest, which is then transferred to the child.

Our analysis of the general equilibrium impact of Social Security on the capital stock and individual welfare proceeds in four exercises, the results of which are reported in Table 3. In all cases, $\alpha$, capital's share, is assumed to equal 0.30, and $\rho = 0.015$. First, we compute the initial steady-state values of the interest rate, capital-output ratio, average bequest relative to lifetime earnings, and lifetime welfare in the absence of Social Security for each of the four values of $\beta$. Those results are reported in panel A of Table 3. As expected, lower levels of the intertemporal elasticity of substitution in consumption are associated with higher average bequests and capital stocks, and hence lower steady-state interest rates.

To obtain the results in panel B of Table 3, we introduce actuarially fair Social Security annuities financed by a proportional payroll tax of 6 percent, holding the interest rate and initial bequest constant at the levels from the original no-Social Security steady state. As the third column shows, average bequest relative to lifetime earnings, and lifetime welfare in the absence of Social Security for each of the four values of $\beta$. Those results are reported in panel A of Table 3. As expected, lower levels of the intertemporal elasticity of substitution in consumption are associated with higher average bequests and capital stocks, and hence lower steady-state interest rates.

To obtain the results in panel B of Table 3, we introduce actuarially fair Social Security annuities financed by a proportional payroll tax of 6 percent, holding the interest rate and initial bequest constant at the levels from the original no-Social Security steady state. These results represent impacts on the initial generation to participate in Social Security. As the third column shows, average

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$r$</th>
<th>$K/Y$</th>
<th>$b$</th>
<th>$\Delta$</th>
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<table>
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<tr>
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<th>$r$</th>
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<tbody>
<tr>
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<td>+3.8</td>
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</tr>
<tr>
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<td>0.020</td>
<td>12.23</td>
<td>24.3</td>
<td>-4.9</td>
</tr>
</tbody>
</table>

Note: In all cases, $\rho = 0.015$, and $\tau = 0.06$. $\Delta$ is measured with respect to changes from the initial no-Social Security steady state. $b$ represents the average bequest as a percentage of lifetime earnings.
bequests are reduced substantially, as the initial generation to participate in Social Security obtains the dual benefits of a high initial bequest and access to Social Security annuities. Partial-equilibrium welfare gains are recorded for the cases where $\beta = 0.50$ or $\beta = 1.10$. The added burden of payroll contributions to Social Security during youth causes welfare losses for the two lower values of $\beta$ ($\beta = 0.25$, $\beta = 0.20$). Moving to panel C, the interest rate is still fixed at its initial steady-state level, but the steady-state bequest available in future generations declines to its new steady-state level. As a result, capital supply and lifetime welfare continue to decline relative to their counterparts in the no-Social Security steady state.

Finally, in panel D of Table 3, the new steady state in the presence of Social Security is computed. As expected from the substantial reduction in bequests and the capital stock, steady-state interest rates increase considerably. These results are important for our analysis of the effects of Social Security annuities in the presence of capital-market imperfections. The interaction of the higher interest rates and lower earnings with the contribution of payroll taxes to binding liquidity constraints leads to significant reductions in lifetime welfare. For example, in the $\beta = 1.10$ case, a 3.8 percent partial-equilibrium increase in lifetime welfare becomes a 14.7 percent reduction in the new steady state.

We do not mean to imply that these calculations describe the historical impact of Social Security on the capital stock in the United States. Participants in Social Security have, for example, obtained returns much greater than the actuarially fair return. The assumption of complete market failure in the private provision of annuities leads to a large effect on desired lifetime consumption from the introduction of social security annuities. While the annuity market in the United States is very imperfect, it is not nonexistent; Kotlikoff et al. (1987) point to family risk-sharing arrangements, and Hubbard (1987) identifies the importance of private pensions as annuity substitutes. However, these qualifications only amplify our points about the limited welfare gains from introducing Social Security given payroll tax finance and liquidity constraints. In the next section, we illustrate the importance of liquidity constraints for these impacts by examining an alternative system of tax finance.

IV. "Progressive Taxation" and the Welfare Gains from Social Security

To examine the influence of the method of financing Social Security on its impact on the capital stock and individual welfare, we now remove the assumption of proportional payroll tax finance. In its place we institute a progressive Social Security tax in which the first fifteen working periods are exempt from payroll taxation. To preserve comparability with our previous results, the retirement benefit is kept the same as under the proportional tax case where $t_s = 0.06$. A new, higher flat tax rate is instituted in the sixteenth period to maintain the average actuarial fairness of the system. The use of an exemption (effectively, an "earned income credit") alleviates the added contribution to the Social Security payroll tax to liquidity constraints on consumption. By shifting the burden of the tax intertemporally through higher taxes later in life, the same present value of contribution can be collected with an increase in individual lifetime utility from consumption. Cross sectionally, the use of the exemption corresponds to progressive taxation.

In general, the shape of the optimal lifetime payroll tax schedule should reflect a tradeoff between the distortions in labor supply with distortions in individual consumption. Of course, the model presented here assumes that labor is supplied inelastically in all periods. A negative labor supply response to the higher payroll tax would necessitate still higher taxes later in life.22 We considered the relative desirability of proportional and progressive general income taxation with liquidity constraints and elastic labor supply in our 1986 paper. Even with substantial labor supply elasticity, the

22 For a discussion of the impact of Social Security on preretirement labor supply, see Richard Burkhauser and John Turner (1978).
TABLE 4—"PROGRESSIVE" TAXATION AND SOCIAL SECURITY: REASSESSING THE IMPACT ON INDIVIDUAL WELFARE

\[
\begin{array}{cccc}
\beta = 1.10 & \beta = 0.50 & \beta = 0.25 & \beta = 0.20 \\
\hline
r = 0.02 & \Delta & \Delta & \Delta & \Delta \\
4.2 & 6.3 & 4.3 & 1.4 \\
(3.5) & (4.5) & (0.1) & (-4.3) \\
r = 0.04 & \Delta & \Delta & \Delta & \Delta \\
5.1 & 5.3 & 2.7 & 0.6 \\
(5.0) & (3.9) & (-1.8) & (-5.2) \\
r = 0.06 & \Delta & \Delta & \Delta & \Delta \\
6.0 & 4.0 & 0.8 & 0.2 \\
(5.9) & (2.4) & (-3.5) & (-5.6) \\
\end{array}
\]

Notes: \( \Delta \) is measured with respect to the corresponding no-Social Security cases under lifetime uncertainty, with borrowing restrictions imposed. Individuals are exempt from Social Security taxation for the first fifteen working periods. Numbers in parentheses are the welfare gains from imposing a social security system of identical size financed by a proportional payroll tax.

effects of liquidity constraints were strong enough that intertemporal shifts in the burden of taxation over the lifetime were welfare improving.

Partial-equilibrium results for the impact of Social Security on lifetime welfare (comparable to Table 3) are presented in Table 4. Simulations are run over four values of \( \beta \) and three values of \( r \), as before. The top entry in each cell represents the gain in lifetime welfare (expressed as an equivalent percentage of lifetime earnings) from participating in the "progressive-tax-financed" Social Security system. The numbers in parentheses below are the corresponding gains from the "proportional-tax-financed" system in Table 4. This change in the structure of payroll tax finance has important implications for the magnitude of the welfare gains made possible by Social Security. Gains are positive in all cases in Table 3, and for many parameterizations are substantially higher than in the proportional-tax-financed case. That delaying the timing of the tax collections can restore much of the potential gain from participating in Social Security puts in a new light claims based on "perfect-markets" models that large welfare costs necessarily accompany progressive taxation.

We can use this framework to consider the relative burden of the payroll tax on different groups. Suppose for example that individuals differ in their holdings of initial assets. \( \ldots \)

V. Conclusions

One of the original goals of the Social Security old-age benefit program was the maintenance of consumption in retirement. Over the past decade, however, many theoretical and empirical studies have focused on the impact on preretirement consumption of the provision of Social Security annuities. The consideration of realistic capital-market imperfections is important here. For example, with uncertainty over longevity and imperfections in private annuity markets, the introduction of even an actuarially fair So-

\(23\) In our 1987 paper, we derive expressions for the marginal propensity to consume out of assets in a finite-horizon model with uncertain lifetime and earnings under various assumptions about initial assets.
Social Security system can generate a substantial increase in lifetime consumption and welfare. When borrowing against future resources is limited, however, the use of proportional payroll tax finance for Social Security increases the incidence of liquidity constraints on the consumption of individuals whose current resources are low relative to their future resources.

Using simulation models under various assumptions about individual preferences and technology, we analyze the impact of precautionary saving against lifetime uncertainty and borrowing restrictions on individual welfare and the capital stock in the presence and absence of Social Security annuities. Our principal conclusions are two. First, while the introduction of an actuarially fair Social Security system leads to a significant increase in lifetime consumption and welfare, accompanied by a reduction in the capital stock, the gain is reduced—and in some cases eliminated—in the presence of realistic restrictions on borrowing. Extending the model to general equilibrium, we find that the partial-equilibrium gains in lifetime welfare from participation in Social Security are offset by the interaction of higher steady-state interest rates and binding liquidity constraints. Indeed, the steady-state welfare cost of Social Security under proportional payroll tax finance can be substantial.

Second, replacing the proportional payroll tax with a progressive tax (essentially a linear tax with an exemption), we show that age-specific tax schemes can restore much of the potential gain from participating in Social Security. By modeling labor-supply responses to tax-induced changes in the net wage, more formal methods of optimal taxation can be applied (see, for example, our 1986 paper).

While we focus on the Social Security system, our approach can be extended to examinations of other fiscal policies within life cycle models. Social Security provides an appropriate starting point for analysis, since realizing the large potential welfare gains from the insurance features of the system depends importantly on agents’ ability to smooth consumption over the life cycle. An obvious application of the emphasis on precautionary saving is to types of uncertainty other than that over longevity, and appropriate social insurance programs.

More generally, a broader class of issues arises with respect to analyzing fiscal policies in imperfect capital markets. To the extent that borrowing constraints play an important role, conclusions about the welfare effects of such policy reforms as altering the progressivity of the income tax, changing the tax base from income to consumption, or lowering taxes on capital income while raising labor-income taxes will have to be reexamined (as in our 1986 paper). Second, the importance of debates over “finite horizons” for analyses of fiscal policy may have been overstated. In our 1987 paper, we argued that assumptions about capital-market imperfections were quantitatively more important than distinctions between finite and infinite horizons in assessing the neutrality of short-run fiscal policy changes. Here, focusing on long-run intergenerational fiscal policy, the same is again true. Previous research emphasizing finite horizons relied on the actuarial unfairness of the Social Security system. In the absence of borrowing restrictions, such effects are, however, likely to be small relative to effects traceable to imperfect private insurance markets. With borrowing restrictions, those results are weakened substantially.

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