Equilibrium Incentives in Oligopoly

By Chaim Fershtman and Kenneth L. Judd*

We examine the incentives that owners of competing firms give their managers. We show that, in equilibrium, each manager will be paid in excess of his decision's marginal profit in a Cournot-quantity game, but paid less than the marginal profit in a price game. In the Cournot case, deviations from profit maximization are reduced by ex ante cost uncertainty and increased by correlation in the firms' costs.

Orthodox economic theory treats firms as economic agents with the sole objective of profit maximization. Some have criticized this view of the firm as being simplistic, arguing that real firms may consistently strive toward a different goal. For example, William Baumol (1958) suggested sales maximization as a possible objective function of firms. Later, when economists more seriously considered the fact that the modern corporation is characterized by a separation of ownership and management, their attention focused on managerial objectives (see Herbert Simon, 1964; Oliver Williamson, 1964; Michael Jensen and William Meckling, 1976; and the principal-agent literature, such as Stephen Ross, 1973).

It is generally argued that a proper analysis of the firm's objective function should be based on the analysis of the owner-manager relationship. A manager's objective depends on the structure of the incentives that his owner designs to motivate him. Owners often index managerial compensation to profits, sales, output, quality, and many other variables. Even if we accept the traditional view that owners want to maximize profits, the incentive scheme they design may imply managerial incentives different from profit maximization. For a monopolistic firm, the owner-manager relationship can be described as a standard principal-agent problem. Such analyses have yielded rich insights into the structure of agents' incentives.[1] For example, Bengt Holmstrom (1977) showed that compensation in optimal contracts would likely use information other than final profits.

However, when we discuss oligopolistic markets, the individual owner-manager relationships must be examined within the context of *rivalrous* owner-manager pairings. More generally, whenever the profit accruing to a principal-agent pair depends on decisions that other rational agents make, the potential interactions must be considered. In this paper we examine the incentive contracts that principals (owners) will choose for their agents (managers) in an oligopolistic context, focusing on how competing owners may strategically manipulate these incentive contracts and the resulting impact on the oligopoly outcome. This analysis will yield different insights as to why managerial compensation contracts may not depend solely on realized profits, and also examine

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[1] The principal-agent approach (Ross, 1973; James Mirrlees, 1976; Bengt Holmstrom, 1977; Milton Harris and Artur Raviv, 1979; Roger Myerson, 1982; and many others) assumes that a principal chooses an incentive structure for agents which maximizes his welfare subject to information constraints and adequate compensation for the agents.

interactions between the structure of internal incentives within a firm and market structure elements external to the firm.

Even though it comes as no surprise that the strategic use of incentives can be important, little work has been done on the problem. Once one begins to think of incentives as strategic tools, it is clear that there may be value to the owner of distorting his manager's incentives away from maximizing the owner's welfare if the reaction of the owner's competitors is beneficial. In the case of a monopoly firm, the optimal incentive structure is obviously the regular principal-agent problem since there are no opponents and, in the absence of risk-sharing and asymmetric information considerations, such an owner will motivate his manager to maximize profits. In the case of an oligopolistic market, the optimal incentive structure is not so clear a priori. For example, Chaim Fershtman (1985) showed that nonprofit-maximizing firms may enjoy more profits than profit-maximizing firms in a duopoly. The strategic trade analysis of James Brander and Barbara Spencer (1983, 1985) is also an example of a "principal," in their case a government, distorting the incentives faced by an "agent," the local firms, in order to change the behavior of a competing "principal-agent" pair, a foreign government and its firms, in a fashion that advances the principal's objectives. Our analysis also expands on their insights on international trade policy by allowing uncertainty in critical parameters. More recent work by Brander and Tracy Lewis (1986) and Vojislav Maksimovic (1986) showed that a firm's owners may alter its capital structure in order to alter their incentives and the competitors' behavior.

In this paper we examine equilibrium incentive contracts in an oligopoly. We show that profit-maximizing owners will almost never tell their managers to maximize profits when each firm's managers are aware of other managers' incentives since each manager will react to the incentives given to competing managers. For example, if one firm's manager is told to maximize sales revenue instead of profit, he will become a very aggressive seller. Since his payoff is thereby affected, there will be a different equilibrium outcome in the competition among the managers. Also, the other managers' equilibrium behavior will be affected if they are aware of the firm's new incentive for sales maximization or learn of it through repeated play. This reaction in the competing firms' managers' behavior gives each owner an opportunity to be a Stackelberg leader vis-à-vis the other firm's managers when he determines his managers' incentives. We find that this interaction causes owners to twist their managers away from profit maximization even though the owners care only about profits.

We find, however, that the nature of the desired distortion critically depends on the nature of oligopolistic competition. In the case of Cournot-quantity competition, we prove that each owner wants to motivate his manager toward high production in order to get competing managers, who are aware of these incentives, to reduce their output. Therefore, in equilibrium owners will give a positive incentive for sales. On the other hand, if firms are selling differentiated products and compete in price, each owner will want his manager to set a high price, thereby encouraging competing managers to also raise their prices. Therefore, with price competition owners will pay managers to keep sales low.

This paper also determines the impact of uncertainty and heterogeneity on the oligopoly outcome. In the Cournot-quantity game, we find that the equilibrium outcome with incentive contracts is more efficient than the simple Cournot outcome not only because of the increase in output but also because the low-cost firm's share of output is greater. Furthermore, if the firms' costs are uncorrelated, *ex ante* uncertainty at the time the incentive contracts are written will reduce the deviation from profit maximization.

Even though our models will be specific, it will be clear that the idea of strategic manipulation of agents' incentives is of general interest. For example, we could similarly analyze a sales manager's decisions when he establishes incentives for his salespeople, and show that he overcompensates his salespeople at the margin if that will cause competing salespeople to work less.

Also, many of our results continue to hold when incomplete information and a moral hazard manager determine the information available for contracting purposes (see Fershtman and Kenneth Judd, 1987).

Section I describes the general framework we examine. Section II examines the results for a Cournot industry with random demand, whereas Section III examines the case of random costs. Section IV studies the case of n Cournot firms. Section V examines price competition in a differentiated product market. Section VI summarizes this study's results.

I. The Basic Model

Our model assumes two firms, each with an owner and a manager. When we say "owner," we mean a decision maker whose objective is to maximize the expected profits of the firm. This could be the actual owner, a board of directors, or a chief executive officer. "Manager" refers to an agent that the owner hires to observe demand and cost conditions and make the real-time decisions concerning output and/or price. While we will refer to the profit-maximizing agent as the owner, he in turn could be an employee who has been given incentives to maximize firm profits.

We examine a two-stage game. In the first stage, the owners of each firm simultaneously determine the incentive structure for its manager, knowing the true probability distributions governing demand and costs. Each owner must offer his manager a contract under which the manager expects to receive his opportunity cost of participation; at this stage the manager shares his employer's uncertainty about demand and costs and the belief about the incentives under which the opposing manager will work. In the second stage, the competing managers play an oligopoly game, with each firm's manager knowing his incentive contract and those of competing managers. In the second stage, the realized nature of demand and costs facing all firms will be perfectly known and common knowledge among the managers. After all sales have been made, each owner observes the costs and sales, and hence profits, of his firm.

Before continuing, we should note that our analysis is equivalent to another view of the market for managers.[2] Some will argue that instead of shareholders hiring managers, it is managers who propose incentive structures to the capital market, which then chooses among the competing managerial proposals. Even if one views the managers as making the first move, the resulting game is equivalent to our game as long as any contract which the managers can propose can also be proposed to the managers in our game, and vice versa. The order of who proposes the incentive contract, firm or potential managers, is not important. The crucial assumption is that the firm gets all the rent from the relationship, an outcome that will occur in either situation as long as there are a large number of potential managers per firm: if the firm proposes an incentive scheme in a take-it-or-leave-it fashion, it need offer a manager only his opportunity cost; whereas if there are many managers with similar opportunity costs making proposals to the shareholders, competition among them will leave the winner only with his opportunity cost, and in both cases an optimal scheme from each firm's point of view will be proposed and accepted. In some respects, this alternative formulation is attractive since a crucial assumption of our analysis is that the "owners" observe only profits and sales, and do not bother learning about the day-to-day details of the firm's operation, an assumption which is a plausible description of shareholders. In any case, we will stick with the more common theoretical structure of an owner proposing a contract and the manager responding.

The assumption that each firm's manager in stage two knows the other firm's manager's incentive contract and costs is a natural one in this context. We view the manager's contracts as being infrequently altered and in force for a substantial amount of time. Repeated play would presumably cause

[2]We thank an anonymous referee for pointing out this alternative view of the interaction between the capital and managerial markets.

managers eventually to learn one another's true incentives even if they were not initially common knowledge. However, despite this appeal to repeated play, we are assuming a single-shot game with common knowledge in stage two among managers about their incentives. A true repeated play specification of the managers' game would clearly generate many interesting new possibilities, but because of the intractable inference problems and the multiple-equilibria problems that arise in repeated games, it is beyond the scope of this paper to move beyond our two-period specification. Moreover, it will be clear in this two-stage game that each owner will want its manager's incentives to be common knowledge. For these reasons, we regard this critical information specification as appropriate and the two-period specification a reasonable one in which to study the issues on which we want to focus.

We assume that the incentive structure takes a particular form: risk-neutral managers are paid at the margin in proportion to a linear combination of profits and sales. More formally, firm i's managers will be given incentive to maximize

$$O_i = \alpha_i \pi_i + (1 - \alpha_i) S_i,$$

where π_i and S_i are firm i's profits and sales.[3] This formulation is moderately general in that it is equivalent to maximizing linear combinations of profits and costs or sales and costs. We make no restrictions on α_i, allowing even negative values. We are assuming that, after the managers have acted and sales and production have been realized, the firms' owners can (or choose to) observe only profits and sales figures, not realizations of demand parameters or number of units sold. We allow managers to do whatever is in their best interest given their options and incentives, making the owner-manager relationship a delegation relationship, not a team relationship. The linearity restriction is not descriptively unreasonable. Furthermore, tractability demands that some restriction be put on the space of contracts since in similar generalized principal-agent problems it is known that equilibrium may not exist in unrestricted contracts (see Roger Myerson, 1982). While this is an unfortunate limitation of our analysis, it will be clear that it is not the reason for the qualitative nature of our results.[4]

Another crucial element of our model will be the assumption that there is uncertainty about crucial market parameters describing demand and costs at the time the incentives are determined. Such uncertainty from the owners' perspective is natural and also gives the managers a role as observers of these random variables. Uncertainty is also crucial to our focus on equilibria in which incentives are distorted away from profit maximization. We will argue that if we had no uncertainty about the *ex post* state of the market, then our analysis would be unconvincing since there would be no justification for ignoring quantity- or price-indexed contracts that would force the usual Cournot and Bertrand outcomes. However, simple deterministic forcing contracts will not be desired by owners when they face nontrivial uncertainty since each owner will want his manager to react to the eventual environment. Therefore, uncertainty is necessary to make the use of linear contracts in profits and sales reasonable and superior to contracts which yield the usual oligopoly outcomes.

The implicit restriction that i's manager's compensation depends on only firm i's sales and profits, not its competitor's, is motivated by a couple of realistic considerations. First,

[3] O_i will not be a manager's reward in general. Since his reward is linear in profits and sales, he is paid $A_i + B_i O_i$ for some constants A_i, B_i, with $B_i > 0$. Since he is risk-neutral, he acts to maximize O_i and the values of A_i and B_i are irrelevant.

[4] Recent work has indicated that the restriction to linear contracts is reasonable and does not mislead us. Bengt Holmstrom and Paul Milgrom (1987) show that linear contracts are optimal in some realistic continuous-time principal-agent problems. Fershtman and Judd (1987) showed that the basic insights of this paper continue to hold when moral hazard considerations also enter into the contracting problems of a duopolist. The focus on linear contracts here allows us to address questions that are intractable when we combine shirking by agents with a more complex dynamic structure.

a firm has much better information about its profits and sales than about its competitor's. Second, giving one's manager any incentive to increase a competitor's profits could possibly be illegal because of its clear role as a device to facilitate collusion. Third, even if we did allow cross effects in compensation, it will be clear that our main result of incentive manipulation would continue to hold true since each firm wants the other manager to operate in a cooperative fashion, but not its own manager.[5]

We examine the subgame-perfect Nash equilibrium of our two-stage game. In the second stage, we compute the Nash equilibrium that results when the firms' managers make simultaneous choices of their strategic variables, knowing one another's incentive contract and the realized nature of demand and costs. Below, we will examine cases in which the strategic variable is either price or quantity and make various assumptions concerning the information each firm has in the contract-writing stage about the eventual costs and demand. In the first stage, each owner simultaneously chooses its α_i, the relative weight it forces the manager to give to profits, with Nash equilibrium describing the outcome. In this game among the owners, each knows the payoff structure of each possible second-stage game as a function of the α's. We will refer to the stage-one equilibrium choice of the α_i and the resulting probability distribution of output and prices as the *incentive equilibrium*. We now move to the determination of incentive equilibria in several contexts.

II. Incentive Equilibrium with Cournot Competition and Random Demand

We first examine the issue of oligopolistic incentive structures for managers in a model of duopoly Cournot competition in a homogeneous good market. For reasons of tractability, demand is assumed to be linear:

$$(1) \qquad p = a - bQ, \qquad a, b > 0,$$

where p is market price and Q is total output. q_i denotes the output of firm i, $i = 1,2$. Firm i will have constant unit cost $c_i \geqq 0$, $i = 1,2$. Both a or b are possibly unknown to all in stage one, but revealed to the managers at the beginning of stage two. We will make no special assumptions about the distribution of a and b other than assumptions on the support of their distributions necessary to assure that each firm's output will be positive in equilibrium. We see no reason to burden the reader with the extra algebra that would be needed when zero output is a possible equilibrium outcome, particularly since our interest is in the study of active oligopolies. The exact nature of such assumptions will be made explicit in the statements of the theorems below. In this section, c_1 and c_2 are known perfectly by all in both stages.

We solve for the incentive equilibrium in the standard backward fashion. In stage two, the manager of each firm observes a, b, c_1, c_2, α_1, and α_2, and chooses q_i to maximize O_i. In this case, O_i becomes

$$(2) \qquad O_i = \alpha_i(a - bQ - c_i)q_i + (1 - \alpha_1)(a - bQ)q_i.$$

Given α_1 and α_2, the Cournot reaction functions in quantity are

$$(3) \qquad q_1 = \frac{a - bq_2}{2b} - \frac{\alpha_1 c_1}{2b},$$

and symmetrically for firm two. Note that α_1 just affects the manager's perspective on costs. If $\alpha_1 < 1$, that is, firm one's manager moves away from strict profit maximization toward including consideration of sales, then firm one's reaction function moves out in a parallel fashion since the managers view $\alpha_1 c_1$ as the marginal cost of production. Therefore, as the owner of firm one changes α_1, he essentially changes his manager's reaction function. Symmetric results hold for firm two. These facts play the crucial role in the results below.

For values of α_i, $i = 1,2$, inherited from the outcome of stage one, stage-two equilibrium in terms of demand, cost, and

[5] Fershtman and Judd (1987) demonstrate this in a simple model.

incentive parameters is

$$p = (a + \alpha_1 c_1 + \alpha_2 c_2)/3, \tag{4a}$$

$$q_1 = (a - 2\alpha_1 c_1 + \alpha_2 c_2)/3b, \tag{4b}$$

$$\pi_1 = (a + \alpha_1 c_1 + \alpha_2 c_2 - 3c_1) \times (a - 2\alpha_1 c_1 + \alpha_2 c_2)/9b, \tag{4c}$$

and similarly for q_2. Note that as α_1 and α_2 are smaller, p is smaller, reflecting the fact that providing incentives for sales results in output beyond the profit-maximizing level.

Given the outcomes in stage two, firm one's owner chooses α_1 in stage one so as to maximize his expected profits net of his manager's opportunity costs. Since the cost of hiring a manager is fixed and unaffected by risk, this is equivalent to maximizing expected profits.[6] We first address the case in which only b is unknown *ex ante*. In this case, the reaction function for firm one's owner's choice of α_1 as a function of α_2 is

$$\alpha_1 = \frac{3}{2} - \frac{a}{4c_1} - \frac{\alpha_2}{4}\frac{c_2}{c_1}, \tag{5}$$

and similarly for firm two's owner's choice of α_2.

The case of uncertain b is particularly easy to examine since b is simply a parameter for the scale of the market and, as seen in (5), does not enter into the owners' choice of α_1 and α_2. Note that if firm two's manager maximizes profits, that is, $\alpha_2 = 1$, and costs are equal, then firm one's owner will choose $\alpha_1 < 1$. Therefore, profit-maximizing contracts generally do not arise in equilibrium. In fact, the final outcome is

$$\alpha_1 = 1 - \frac{a + 2c_2 - 3c_1}{5c_1}, \tag{6a}$$

$$q_1 = (2a - 6c_1 + 4c_2)/5b, \tag{6b}$$

and similarly for firm two. Since $\alpha_1 < 1$, if and only if $q_1 > 0$, we find that if both firms produce output, both will twist their manager's incentives away from strict profit maximization toward sales incentives as well.

The intuitive explanation for these results is given in Figure 1. For a particular b, R_i is firm i's reaction function, yielding q_i as a function of q_{3-i}. First, take the point of the owner of firm one. His choosing $\alpha_1 < 1$ pushes R_1 out and pushes the Nash equilibrium down firm two's manager's reaction function from A to B. The fact that α_1 is communicated to the manager of firm two means that firm one's owner acts as a Stackelberg leader with respect to firm two's manager. Here, however, each owner is a leader vis-à-vis his opponent's management. This dual leadership causes both owners to make their managers more aggressive sellers, leading both owners to choose an α_i less than unity, pushing both reaction curves out, and finally causing the stage-two Cournot equilibrium to shift from A to C. Theorem 1 summarizes.

THEOREM 1: *In a Cournot market, if a, c_1, and c_2 are known in stage one and both firms always produce positive quantities in equilibrium for any value in the support of b, then, for $i, j = 1,2$,*

$$\alpha_i = 1 - \frac{a + 2c_j - 3c_i,}{5c_i}, \qquad i \neq j \tag{7a}$$

$$q_i = (2a - 6c_i + 4c_j)/5b, \qquad i \neq j \tag{7b}$$

$$p = (a + 2(c_1 + c_2))/5, \tag{7c}$$

implying that owners always give incentives for sales and may even penalize for profits if costs are sufficiently low.

There are several interesting comparisons between our incentive equilibrium outcome and the Cournot outcome. Total output in the incentive equilibrium always exceeds Cournot output, and profits and prices are lower. For example, if $c_1 = c_2 = c$, then Cournot price is $(a + 2c)/3$, whereas the

[6] Recall, from fn. 3, that i's manager is paid $A_i + B_i O_i$. Since managers are risk neutral, the owners can first set the optimal α's, yielding the O_i, and then promise A_i's, which set the means of $A_i + B_i O_i$ equal to the manager's opportunity cost. Hence, owners' profits equal expected profits minus managerial opportunity costs.

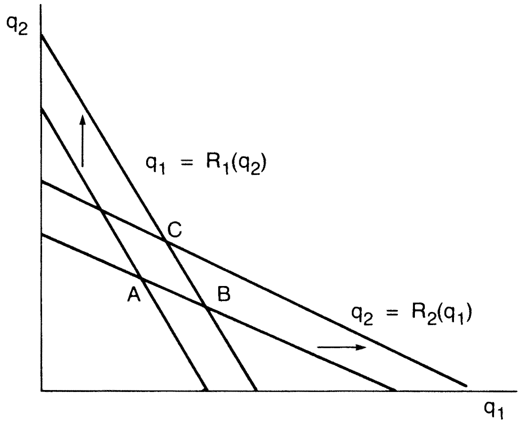

FIGURE 1

incentive equilibrium price is lower and equal to $(a+4c)/5$. Similarly, Cournot profits equal $(a-c)^2/9b$, whereas in our incentive equilibrium they are $2(a-c)^2/25b$, a lesser amount.

The incentive equilibrium outcome also has strong performance implications relative to the usual Cournot-quantity analysis. Since output is increased and oligopoly rents are lower, efficiency is improved. However, the incentive equilibria are more efficient not only because price is closer to marginal cost but also because production rises relatively more at the low-cost firm. If firm one is the low-cost firm, straightforward calculations show that its market share is $1+(c_2-c_1)/(a-2c_1+c_2)$ times greater in the incentive equilibrium than in the Cournot equilibrium. Corollary 1 summarizes.

COROLLARY 1: *Under the assumptions of Theorem* 1, *incentive equilibria in the quantity game generates greater output, lower rents, lower prices, and a more efficient allocation of production than the usual Cournot equilibria.*

The case of uncertain a is similarly examined. Let $\bar{a}$ denote the mean of a. Proceeding as above, we find Theorem 2.

THEOREM 2: *If b, c_1, and c_2 are known in stage one and if the minimum value of a with nonzero probability exceeds $2c_2-c$ and $2c_1-c_2$, then in the incentive equilibrium*

$$\alpha_1 = 1-(\bar{a}+2c_2-3c)/5c_1, \tag{8a}$$

$$q_1 = (2a-6c_1+4c_2)/5b, \tag{8b}$$

and similarly for firm two.

Before continuing, we should discuss one alternative formulation and its relationship to our game, particularly since it will give an argument as to why the addition of uncertainty was important to our analysis. Suppose that owners could write only contracts that force managers to produce a certain level of output and that there were no uncertainty in the level of demand. Such a world is equivalent to the usual Cournot game. Now suppose that the firms could write both these forcing contracts and the linear contracts we studied above. If firm one wrote a contract forcing its Cournot level of output, the best firm two could do would also be to write a contract that specifies its Cournot output since it could not manipulate the performance of firm one's manager. In Figure 1, such a forcing contract would cause R_1 to be vertical, a graphical representation of its nonmanipulability. Of course, the same argument applies to firm one. Therefore, the usual Cournot outcome is also an equilibrium if we assume no uncertainty and allow quantity-forcing contracts.

This observation does not, however, immediately eliminate the incentive equilibrium we computed above since it also remains an equilibrium in this extended game. The crucial fact is that, without uncertainty, a firm can choose any point along its opposing manager's reaction curve by choosing a quantity-forcing contract or a linear contract. If firm one believed that firm two was going to write the incentive equilibrium contract with its manager, then firm one is indifferent between writing a contract that forces its manager to produce the best point along firm two's manager's reaction curve and giving its manager the incentive equilibrium contract that will also produce that outcome. Similarly, if firm two's owner believed that one's manager was going to write the incentive equilibrium contract, it could do no better than to write its incentive equilibrium contract. Therefore, multiple equilibria often result if both forcing and linear incentive contracts were possible.

In many cases of multiple equilibria, there is no reason to choose one over the other. However, the incentive equilibria would not be the natural one to focus on here in the absence of uncertainty. To argue this, we appeal to focal point considerations. Since our incentive equilibrium often results in less profit for both firms (and surely will if costs are identical), the incentive equilibrium would often be strictly Pareto inferior. In such cases, focal point considerations argue that the owners would realize that it is in their mutual interest to act according to the simple Cournot allocation implemented by forcing contracts. Therefore, the incentive equilibria lose much of their appeal in deterministic versions of our model.

However, if there are nontrivial levels of uncertainty, then such *noncontingent* quantity-forcing contracts would not be desirable since the owner would want the manager to be able to respond to contingencies that the owner does not observe, but which do affect his profits. Such flexibility could be partially attained in this context by a profit-maximizing contract. If both firms chose profit-maximizing contracts, then the state-contingent Cournot outcomes would result. However, once firms chose such contracts for their managers, each manager will react to deviations in the other's incentive contract. Therefore, by assuming uncertainty, we have both given a function to the manager and also increased the plausibility of our incentive equilibrium relative to one important perturbation of our game.

These comments also apply to the trade policy analyses in the papers by Brander and Spencer (1983, 1985). Their models will also have additional equilibria in similarly extended strategy spaces, with the extra equilibria sometimes being mutually preferable to both nations; however, uncertainty about the underlying profit opportunities will again make the linear contract equilibria the more plausible ones. The nature of our results also generalizes, implying that the strategic trade interventions will tend to be less valuable in the presence of uncertainty.

Section II has demonstrated the basic insight in our analysis: profit-maximizing owners may not want to give profit-maximizing incentives to their managers because an owner can influence the outcome of the competition between the managers in his favor by distorting his manager's incentives. This result does not rely on asymmetric information considerations as in Holmstrom, since a firm in this model will choose profit-maximizing contracts if it faces no competition. This result shows that internal relationships and incentives can be distorted and manipulated for interfirm strategic reasons, giving a new and fundamentally different role for internal contracts. In the following sections we elaborate on this theme for the cases of random costs, multiple-firm oligopoly, and price competition in differentiated markets.

III. Incentive Equilibrium with Cournot Competition and Random Costs

The case of random costs is substantially different. We examine it because new results concerning the impact of inter-firm heterogeneity are obtained.

Suppose that c_1 and c_2 are identically distributed with mean μ, variance σ^2, and correlation coefficient r. Let $v = \sigma/\mu$ be the coefficient of variation. Again, we will assume that the cost randomness is contained so that output for each firm is positive in each state of the world. We assume that each manager knows the other's costs in stage two. In this section, we assume that a and b, the demand parameters, are known perfectly in both stages. Therefore, the stage-two reaction functions are given by (3). In stage one, owner i chooses α_i to maximize expected profits given his expectation of α_{3-i}. Expected profits are given by *ex ante* expectation of (4c). Stage-one reaction functions are

$$(9)\quad \alpha_i = \frac{3}{2} - \frac{a}{4\mu}\frac{1}{v^2+1} - \frac{\alpha_{3-i}}{4}\frac{1+rv^2}{1+v^2}, \qquad i=1,2.$$

Understanding the dependence of this reaction function on r, v, and μ is crucial to understanding the equilibrium results. If there were no reaction by firm two's manager to firm one's incentive structure, there would be no gain to the owner from distorting his manager's incentives. The marginal gain to firm one's owner of increasing α_1 by $d\alpha_1$, assuming firm two's manager does *not* react to this change in his opponent's incentives, is

$$2(1-\alpha_1)(3b)^{-1}E\{c\}^2 d\alpha_1,$$

which is zero at $\alpha_1 = 1$, the profit-maximizing contract. However, since the manager of firm two will react in the stage-two equilibrium by increasing output as α_1 is increased, the marginal loss of increasing α_1 arising from this reaction is

$$\left(a\mu + \alpha_2 r\sigma^2 - 2\alpha_1(\mu^2+\sigma^2)\right)(3b)^{-1} d\alpha_1,$$

which is positive if q_1 is positive for all c_1, c_2 realizations. The reaction function chooses α_1, which equates the marginal gain and loss of an increase in α_1. As the variance, σ^2, increases, marginal losses due to deviations from profit-maximizing incentives increase, pushing the optimal α_1 toward 1. Also, if α_1 is near its optimal value given α_2, the gains from such deviations fall as σ^2 rises. Hence, we see that as σ^2 rises, firms move toward a profit maximization. Similarly, as costs are more correlated, the benefits of deviations from profit-maximizing incentives rise, implying that the optimal α_1 falls. Also, as a/μ rises, that is, the choke price rises relative to mean cost, the profit margin is greater and firms move away from profit-maximization incentives, as was the case for deterministic c.

Theorem 3 follows directly from an examination of the reaction functions.

THEOREM 3: *With ex ante uncertain and identically distributed costs, if q_1 and q_2 are nonnegative in equilibrium for all realizations of (c_1, c_2), then in equilibrium,*

$$(10) \quad \alpha_1 = \alpha_2 = \alpha = \frac{6(v^2+1) - a/\mu}{(4+r)v^2+5}.$$

Therefore, (i) α rises as a/μ falls and as v and r rise, and (ii) $\alpha < 1$ for r sufficiently close to 1 and v sufficiently close to zero.

The case of random costs is somewhat richer but more difficult to analyze completely. If the equilibrium α is less than unity, then an increase in the uncertainty of costs and their correlation will cause firms to move closer to profit maximization because it is more difficult to choose the right α conditional on the realized costs. We are not able to prove that α is always less than 1, but we know of no case in which it is not. The formula for α would seem to indicate that α could exceed one, but only if the variance of costs is large and costs are not perfectly correlated. This situation could possibly lead to negative output according to (6b) and violating the nonnegativity condition on output. To determine whether this occurs, one would have to impose specific distributions on the random variables. We want to confine the analysis in this study to cases in which examination of the random variables' first and second moments and weak conditions on their support is sufficient. Since the nonnegativity constraints on output are satisfied for r close to one or when the support for costs is small, yielding a v nearly zero, the formula for α in Theorem 3 is valid in these cases and (ii) of Theorem 3 holds, showing that Theorem 3 applies for a nontrivial set of cases.

This section shows how cost shocks affect the equilibrium nature of incentives. If cost shocks are commonly experienced, as in the case of an uncertain price for a common input, then the owners decide to distort incentives. However, if shocks are not commonly experienced, then deviations from profit maximization are reduced. Similarly, if there is too much variance in costs, then owners are not as willing to distort incentives away from profit maximization.

IV. Equilibrium with Many Firms

We saw above in a duopoly that owners may distort their managers' incentives if each firm's manager reacts to distortions in the competing managers' incentives. It is natural to ask next if these distortions of owners' incentives disappear as the industry is less concentrated. We establish this formally in the case of perfectly correlated uniform costs. The same results for the cases of uncertain a and b are easily proven.

THEOREM 4: *As n approaches ∞, the firms' managers become profit maximizers, that is, $\alpha_i \to 1$, if a is uncertain, b is uncertain, or costs are equal but uncertain in stage one.*

PROOF:

Firm i's objective function is

$$(11) \quad \alpha_i(a-bQ-c)q_i + (1-\alpha_i)q_i(a-bQ).$$

The first-order condition for choosing q_i is

$$(12) \quad (a-2bq_i-b\overline{Q}_i)-\alpha_i c=0,$$

where $\overline{Q}_i = Q - q_i$. The second-order conditions are clearly satisfied. Thus, the ith firm's reaction function is

$$(13) \quad q_i = \frac{a-b\overline{Q}_i-\alpha_i c}{2b}, \quad Si=1,\dots,n.$$

Summing (13) yields

$$\sum_{j=1}^{n} q_j = \frac{1}{2b}\left(na-b(n-1)Q-\sum_{j=1}^{n}\alpha_j c\right).$$

Since $\Sigma q_i = Q$, then $b(n+1)Q = na - \Sigma\alpha_j c$. Therefore,

$$(14) \quad Q = \frac{1}{b(n+1)}\left(na - c\sum\alpha_j\right).$$

Substituting (14) into (13) $(\overline{Q}_i = Q - q_i)$ yields

$$2bq_i = a-\alpha_i c - \frac{n}{n+1}a + \frac{c\Sigma\alpha_j}{n+1} + bq_i,$$

$$q_i = \frac{1}{b(n+1)}\left(a + c\sum_{j\neq 1}\alpha_j - nc\alpha_i\right).$$

From (14) we can calculate the price

$$p = a - \frac{1}{n+1}\left(na - c\sum_{j=1}^{n}\alpha_j\right) = \frac{1}{n+1}\left(a + c\sum_{j=1}^{n}\alpha_j\right).$$

The ith firm's expected profit when c is unknown in stage one is

$$(15) \quad \pi_i = \frac{1}{b(n+1)^2} \times E\left\{\left(a - c\sum_{j\neq i}\alpha_j - nc\alpha_i\right) \times \left(a + c\sum_{j=1}^{n}\alpha_j - (n+1)c\right)\right\}.$$

Maximizing the above profit functions for each i yields

$$(16) \quad E\left\{c\left(a + c\sum_{j\neq i}\alpha_j - nc\alpha_i\right) - nc\left(a + c\sum_{j=1}^{n}\alpha_j - (n+1)c\right)\right\} = 0.$$

Since at the symmetric equilibrium $\alpha_i = \alpha$ for all i,

$$(17) \quad \alpha = 1 - \frac{n-1}{n^2+1}\left(\frac{a\mu - \sigma^2 - \mu^2}{1+\sigma^2+\mu^2}\right),$$

where $\mu = E\{c\}$ and σ^2 is the variance of c. Thus $\lim_{n\to\infty}\alpha = 1$. This proves Theorem 4.

This result is intuitively appealing because it coincides with our understanding of the perfectly competitive market. In the traditional theory of perfect competition with free entry, firms cannot afford to do anything other than be profit maximizers. If all firms have the same technology, the long-run equilibrium price is identical to minimum average cost. If one firm deviates from its profit-maximizing output, its average cost is going to increase above the market price, implying that the firm loses money.

Since monopolists want their managers to maximize profits, we find that managers in both monopolized and competitive sectors will be told to maximize profits. Nonprofit-maximizing incentives will be given only in oligopolistic industries, showing that the relationship between market structure and managerial incentives will likely not be monotonic.

V. Price Competition and Incentive Equilibrium in a Differentiated Product Duopoly

The analysis of incentive equilibrium in a differentiated market is similar to the analysis in Section IV with one exception—now we assume price competition between firms selling differentiated products instead of Cournot-quantity competition. We assume that the demand is given by

$$q_i = A\tilde{\varepsilon} - bp_i + ap_{3-i}, \qquad i=1,2, \tag{18}$$

where $\tilde{\varepsilon}$ is a common shock to demand. We assume $\bar{\varepsilon}=1$. Also $b>a$, implying that the effect of a firm's own price on sales is greater than the effect of its rival's price. This is equivalent to concavity in the implicit linear-quadratic consumer utility function.

Owners know that the strategic variable in the competition between managers in the second stage is price. Thus, given an incentive structure which is a linear combination of profits and sales, firm i's manager will act so as to maximize

$$O_i = \alpha_i(p_i - c)(A\tilde{\varepsilon} - bp_i + ap_j) + (1-\alpha_i)p_i(A\tilde{\varepsilon} - bp_i + ap_j). \tag{19}$$

THEOREM 5: *When price is the strategic variable in the second stage of the competition among differentiated producers facing linear demand, $\alpha_i > 1$, that is, the incentive equilibrium is such that managers are overcompensated at the margin for profits.*

PROOF:

The reaction function of firm i's managers is given by

$$p_i = \frac{A + ap_j + \alpha_i cb}{2b}, \qquad i \neq j,\ i, j=1,2, \tag{20}$$

and the stage-two equilibrium prices, as a function of incentive and demand parameters, are

$$P_i(\alpha_i, \alpha_j, \tilde{\varepsilon}) = \frac{2bA\tilde{\varepsilon} + aA\tilde{\varepsilon} + a\alpha_j cb}{4b^2 - a^2} + \frac{2b^2\alpha_i c}{4b^2 - a^2}, \qquad j \neq i, \quad i, j=1,2. \tag{21}$$

Given the equilibrium in the second stage, the owners can compute their expected profits, π_i, $i=1,2$, as a function of the incentivestructures in their own firm as well as in the rival's firm:

$$\pi_i = E\left\{ \frac{2bA\tilde{\varepsilon} + aA\tilde{\varepsilon} + a\alpha_j cb - 4b^2c + a^2c + 2b^2\alpha_i c}{4b^2 - a^2} \times \left[A\tilde{\varepsilon} - bP_i(\alpha_i, \alpha_j, \tilde{\varepsilon}) + aP_j(\alpha_i, \alpha_j, \tilde{\varepsilon}) \right] \right\}. \tag{22}$$

By differentiating (22) with respect to α_i and equating it to zero, we find the reaction function of firm i's owner to firm $j=3-i$ to be

$$\alpha_i = m + \beta\alpha_j, \tag{23}$$

where

$$m = \frac{2ba^2A + a^3A - 6a^2b^2c + a^4c + 8b^4c}{4b^2(2b^2 - a^2)c},$$

$$\beta = \frac{a^3}{4b(2b^2 - a^2)}.$$

The equilibrium in the first stage of the game is a pair (α_1^*, α_2^*) such that (23) is satisfied for $i=1,2$. Substituting (22) into

(23) and solving for α_i yields

$$(24)\quad \alpha_i^* = 1 + \frac{(A-(a-b)c)(a^3+2a^2b)}{bc(8b^3-4a^2b-a^3)} > 1, \quad i=1,2$$

since $a<b$. The overcompensation for profits can also be interpreted as an owner's tax on the manager for his input expenditures. This tax disciplines the manager and prevents him from being too aggressive in his pricing strategy.

An immediate corollary of Theorem 5 is that the price in the incentive equilibrium is above the equilibrium price in an industry in which managers maximize profit, the usual specification of behavior in a differentiated market. This can be illustrated by a reaction function analysis. The crucial difference between this case and the Cournot-quantity case is that here the reaction curves in prices slope upward, that is, the greater a firm's expectation about its opponent's price, the greater will be the price he chooses. Under the profit-maximization hypothesis the equilibrium prices are (p_1^*, p_2^*) in Figure 2. By penalizing managers on sales at the margin, as occurs here since the equilibrium of the owner's game implies that $\alpha>1$ for both firms, managers will price less aggressively than under the regular profit-maximization hypothesis. This pushes their reaction functions upward, a shift evident from (20) which describes i's reaction function. This mutual restraint results in equilibrium moving outward, a direction favorable to both, and leading to prices of $(\hat{p}_1, \hat{p}_2)$, which are higher than the equilibrium prices under the profit-maximization hypothesis.

Comparing the above result with the equilibrium in the quantity competition case[7]

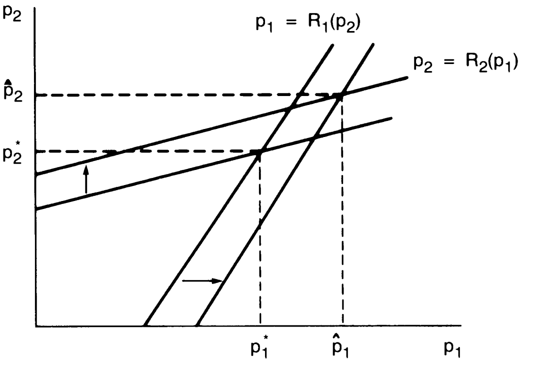

FIGURE 2

demonstrates that the equilibrium incentive structure depends on the way firms compete in the market. In the quantity competition case, $\alpha<1$ and owners motivate managers to behave aggressively and to produce beyond the profit-maximization level, whereas in the price competition case $\alpha>1$ and managers behave nonaggressively. In the quantity competition case each owner, acting as a Stackelberg leader with respect to the opposing manager, recognizes the negative slope of its rival manager's reaction function and therefore wants his manager to expand output. In the price competition case, each owner knows that any credible increase in its own price will be followed by an increase in its rival's price, therefore motivating its manager to be less aggressive and charge a price above the profit-maximizing price.

Moreover, the performance implications of incentive equilibrium differ in the two cases. In the quantity competition case, the incentive equilibrium increases efficiency and reduces oligopoly rents since the outcome is closer to perfect competition than the outcome in the regular Cournot competition. However, in the price competition case, incentive equilibrium essentially pushes the price toward the monopolistic price. The structure of our incentive contract game would therefore be one of mutual advantage to the firms and hence one toward which they would like to move instead of the decision-making structure implicit in the usual Bertrand analysis.

[7]Had we assumed our differentiated producers competed in quantities, then the results would have resembled those of the nondifferentiated Cournot analysis. Therefore the comparisons we make here result from the mode of competition, not from product differentiation. The product differentiation feature was added to the analysis of Section IV in order to keep price competition from always resulting in marginal cost pricing.

Finally, the argument that incentive equilibria in the deterministic quantity competition case with no uncertainty were not plausible for focal point reasons when we expand the space of possible contracts does *not* apply here. Since firm profits are increased by incentive contracts, such considerations argue in favor of the incentive contracts over forcing contracts (which here would be interpreted to force the manager to choose a certain price) even when both were equilibria in the expanded contract space. These arguments indicate that our theory of incentive equilibria may be more relevant for the case of differentiated markets.

VI. Concluding Comments

This paper has examined the interactions of internal contracting and external strategic considerations. We found a principal (firm owner) will want to distort the incentives of his agents (firm managers) in order to affect the outcome of the competition between his agent and competing agents. The general implications of our analysis are clear. In general, the owner of a firm will alter his managers' incentives in that direction which will cause opposing agents to change their behavior in beneficial directions. For example, if advertising will cause opposing firms to reduce their advertising, then a firm's owner will give his managers extra incentive to advertise. This can be implemented by explicit incentives or by hiring agents who are known to be inclined to aggressively advertise. In some cases, various asymmetries may cause the owners to distort their managers' behavior in opposing directions. For example, if in the differentiated products case firm one is a price setter, but firm two, for some technical reason, fixes his quantity, then firm one's manager will be paid to overproduce in order to get firm two's manager to reduce his output, but firm two's manager may be paid to keep his price high and output low in order to encourage firm one's manager to allow the market prices to be high. The variety of problems that can be analyzed by focusing on this joint determination of internal incentives and external environment is obvious.

There are a variety of directions which further research should pursue. The major weakness of the analysis above is the assumption of linear contracts and the absence of a detailed asymmetric information structure which motivates the existence of contracts in the first place. This study is offered as an imperfect but intuitive and suggestive analysis of the possibilities that arise when we jointly examine managerial incentives and market structure. A more recent paper by the authors (Fershtman and Judd, 1987) examines a model with a more standard incomplete information and moral hazard structure, which demonstrates that the intuitive results derived above continue to hold within a more standard principal-agent structure.

We also assumed that the managers play a simple Nash noncooperative equilibrium when they compete. An alternative theory of their behavior would be to have them bargain toward some outcome that is cooperative from their point of view. While this may substantially affect the outcome of the managers' game for any given set of managerial incentives, the owners would still take into account the impact their decisions have on the outcome of the managers' decision-making process. For example, if the managers were known by the owners to bargain in accordance with a "split the gains from trade" rule, then each owner will strive to increase his profits by demanding a large bond from the manager and then contract to give a large portion of it back whether bargaining succeeds. This will raise the manager's threat point, making the agreement point more favorable to his firm, and increase the owner's profits. In any case, strategic manipulation of managerial incentives will be valuable to owners as long as the manager's incentives affect the joint allocation of profits in the manager's game, a feature which appears in most cooperative modes of interaction as well as noncooperative.

This paper has demonstrated that competing firms' owners will often distort their managers' objectives away from strict profit maximization for *strategic* reasons. This initial analysis made several simplifying assumptions including linear payoffs to managers, absence of the usual moral hazard problems, and linear demand. Further research should generalize our analysis. However, it is clear from the basic intuition that distortions of managers' incentives are potentially important strategic instruments for owners of competing firms and point to the importance of external competitive conditions for the determination of internal relationships.

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