THE JOURNAL OF FINANCE • VOL. XLII, NO. 3 • JULY 1987

Effects of Capital Gains Taxation on Life-Cycle Investment and Portfolio Management

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ABSTRACT

We examine the impact of capital income taxation, both accrual forms of taxation and taxation of realized capital gains, on total savings and the demand for corporate financial instruments. We find that investors may hold both debt and equity in the face of effective collection of capital gains taxation even in a flat tax system. We also find that the two taxes will have substantially different effects on saving and consumption behavior, making it unlikely that the tax structure can be summarized by any single equivalent accrual tax rate.

I. Introduction

THIS PAPER EXAMINES THE impact of income taxation on the demand for various corporate financial instruments, focusing on unique features due to taxation of realized capital gains. We take a life-cycle view of investment behavior and examine how accrual and realization taxation jointly affect an individual's allocation of intertemporal resources. We shall consider the impact on his demand for debt and equity, showing that taxation of realized capital gains generates an *intrapersonal* tax clientele effect: at any one point of time, an individual will strictly prefer either to buy debt or equity, but that choice may be different at different ages. Therefore, many individuals will simultaneously hold both debt and equity. We also find that any attempt to measure the "effective" capital gains tax rate will be difficult and that existing approaches appear flawed since they do not correspond to the pattern of distortions which we find in this explicit model of asset demand and saving.

This analysis deviates from earlier work on capital gains taxation in several substantial dimensions. Whereas much of the capital gains taxation literature focuses on how capital gains tax regulations generate arbitrage opportunities in perfect capital markets, the analysis used below essentially assumes effective enforcement of the intent of capital gains taxation. This is desirable since, contrary to the perfect capital market implication, capital gains tax revenues are nontrivial. Therefore, in examining the nature of asset demand and choice, it is reasonable to assume somewhat effective collection of the capital gains tax liabilities. Our analysis will assume completely effective enforcement to highlight the impact of capital gains taxation; it will be clear that several types of leakages will not substantially alter the qualitative conclusions.

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The crucial feature of our analysis of capital gains taxation which yields novel results is the *duration dependent* net return structure generated by capital gains taxation. By duration dependence we mean that the current instantaneous rate of return to holding an asset depends on the length of time the asset has been held. In the case of capital gains taxation by realization, this net return is increasing in the holding period. Since debt is not subject to such duration dependence, its after-tax rate of return to holding is constant if nominal tax rates and before-tax returns are constant. Hence, when we take the equilibrium approach described in Miller (1977), the possibility arises that debt dominates equity over short holding periods because of the deductibility of interest at the corporate level, but that equity dominates over longer holding periods because of the deferral advantage associated with capital gains. One of the costs of explicitly considering duration-dependence is that tractability forces us to assume a deterministic technology and thus a deterministic evolution of security prices. While ignoring risk abstracts from important aspects of reality, it will be clear that the considerations on which we focus below will continue to be as important in a risky world.

When assets have duration dependent returns, the theory of optimal portfolio management and consumption plans must be reconsidered. Balcer and Judd (1985) examined the optimal life-cycle investment and consumption plan of an individual investing in assets with duration dependent returns, and found that it obeyed a LIFO structure: assets purchased early in life should be consumed late in life, whereas the last assets purchased should be the first ones sold during any decumulation phase. In the presence of both debt and equity, this implies that a life-cycle saver will first purchase equity planning to sell it late in life, then possibly switch to buying bonds for the remainder of the saving period, planning to sell the bonds before any equity. In particular, we see that an individual will simultaneously hold both bonds and equity during parts of his life, even in the face of constant nominal tax rates. Our results show that the standard static analysis of capital structure and taxation generates an excessively sensitive view of capital structure. A static analysis of asset demand would imply that individuals specialize in one kind of asset; in particular, only one kind of security would survive in equilibrium under a flat income tax system. In our explicitly dynamic model we find both kinds of assets will survive even when nominal tax rates are constant across individuals and time. This study will examine the implications of capital gains taxation for capital structure at equilibrium.

In evaluating a tax structure, it is often desirable to construct an index which summarizes its economic effects. The considerations on which we focus also will show that it is very difficult to develop a measure of the "effective" capital gains tax rate, and that standard approximations will be seriously flawed. One purpose of such an index is to indicate the extent to which investment decisions are affected by the tax. We will explicitly compute the accrual tax system which would generate the same investment decisions. We find that the effective tax rate varies substantially over life, being zero in early and late life but being substantially greater in midlife, even reaching the full nominal level in some cases. This indicates that empirical analyses which need such an effective rate will also be much more problematic than studies involving only accrual tax rates.

We also provide several examples in which the economy's response to a change in capital gains taxation differs substantially from its response to changes in a pure accrual tax. In particular, we provide examples where saving is decreasing in accrual tax rates if there is no realization taxation, but is increasing in both accrual and realization tax rates when both taxes are present. These considerations lead us to conclude that attempting to find an accrual equivalent will likely be futile.

Section II reviews some of the literature related to the exercises which we conduct. Section III develops the basic model. Sections IV and V reviews a number of exercises which indicate the qualitative and quantitative dimensions of the effects we study. Section VI concludes.

II. Literature Review

Theoretical examinations of capital gains taxation have generally focused on its impact on an investor's optimal trading strategy. This literature has often argued that investors are able to use a combination of tax code provisions to eliminate capital gains tax liabilities and even eliminate other tax liabilities at the same time. Many of the arguments along those lines are summarized in Stiglitz (1983). These arguments can lead to unexpected results. For example, Constantinides and Scholes (1980) have pointed out in one context that, in the absence of transaction costs, an increase in capital gains tax rates is desired by investors because the value of various tax arbitrage strategies is thereby increased. However, Constantinides and Scholes showed that their particular strategy is easily swamped by transaction costs and point out possible legal problems.

This study differs from most earlier efforts by taking a dynamic life-cycle perspective of savings in the presence of effective capital gains taxation, that is, a collection of rules and regulations which do succeed in raising revenue. We assert that we are analyzing the tax that the government is trying to implement, and is doing so with some success. First of all, a casual examination of many of the provisions of the tax code (limit on interest deductions, limit on losses, no wash sales, deemed realization on future contracts) argues for that position. Second, large and increasing amounts of revenue are raised through taxation of realized gains, even though capital markets are becoming more perfect. Third, Poterba (1986) has shown that many investors do not play the sophisticated trading strategies which create the arbitrage opportunities, presumably because of transaction costs, both in resources and the fear of legal complications.

In many respects, our approach most resembles Constantinides (1983). He examined a model of asset trading where a Capital Asset Pricing Model (CAPM) of security evaluation holds when appropriate parameter adjustments are made to account for the taxes. He showed that investors would immediately realize losses and want to hold gainers indefinitely. However, in order to obtain a tractable problem, he assumed that consumption was financed by dividends, exogenous income, and random liquidation of assets independent of their basis value. Random liquidation ignores the vintage nature of portfolios, i.e., the fact that otherwise identical assets may have different basis values. He also assumed that the investor is assumed to have access to insurance which eliminates any risk premium arising from the random liquidation. Because of random portfolio liquidation and the related insurance, he was able to ignore the nonlinearities induced by vintage aspects of the portfolio. Constantinides' solution is elegant and imaginative, but we argue that the vintage element assumed away by this analysis is a crucial differentiating feature between capital income and capital gains taxation. In fact, all the interesting results below arise because we assume an optimal realization strategy for the investor which endogenously generates the crucial heterogeneity in personal tax rates.

However, Constantinides faced the basic arbitrage problem arising from the duration-dependence of returns: the presence of a realization tax implies two different prices for the same state-contingent security, leading investors to try to churn short positions in order to acquire cash at a low interest cost and invest the proceeds in assets held for a long period of time, yielding a high return due to the deferral. He assumed the presence of a brokerage fee for short transactions just large enough to eliminate such arbitrage opportunities for the investor. With uncertainty some short transactions will occur because of diversification objectives, but in our deterministic context assuming such a fee will be equivalent to ruling out short sales.

In summary, when we compare approaches to the study of capital gains taxation, one must compare objectives. If we are interested in security price formation, then it is valid to examine what prices must be in order to eliminate arbitrage profits for individuals with low transactions costs. However, they do not necessarily provide the majority of investment capital nor the bulk of tax revenues. Since our interest is the study of how capital gains taxation affects capital structure and effective tax rates, we must look at the total supply of capital from all investors and their intertemporal allocations given security prices. These issues depend on the marginal decisions of the average investor, not decisions of marginal investors. Therefore, our focus on enforceable capital gains taxation is the appropriate one.

Discussions of financial structure, as in Auerbach (1979), Miller (1977), and DeAngelo-Marsulis (1980), also assumed only accrual forms of taxation. These capital structure analyses were essentially static. Such models missed the dynamic features unique to capital gains taxation; bringing capital gains taxation explicitly into the analysis will generate a richer view of asset demand and capital structure.

III. Basic Model

In this section, we shall present the basic model of consumer optimization with realization taxation of capital gains and show how it relates to the standard model of consumer optimization with taxation of accrued gains.

We assume a single good for consumption and investment with a simple linear technology. In particular, we assume that a unit of the single good invested in one period yields $1 + r$ units in the next, which may be reinvested or used for consumption without any adjustment costs; r therefore represents net output per period per unit of investment. All production takes place in a corporation whose securities are held by our individual investors. The corporation may issue bonds and equity, which are traded among individuals and firms. The corporate income tax is assessed each period on corporate profits net of debt payments at the rate τ_a. The individual pays taxes on ordinary personal income at the rate τ_p, and pays taxes at the rate τ_d on the total realized net gain of any equity sale. (See Balcer and Judd (1986) for a treatment of taxation of nominal realized gains.) τ_a and τ_p are accrual taxes and τ_d is the realization tax.

Given this tax structure, a firm must be indifferent between issuing either asset since there is no difference between the two from the firm's point of view. We could generalize the analysis to include the effects of investment tax credits and accelerated depreciation, which were studied by DeAngelo and Masulis (1980). We choose not to do so here since the results would be qualitatively the same and we want to show that there will be a demand for both assets even without equity-specific tax deductions. Therefore, the net rate of return of an individual for holding a bond must be $r(1 - \tau_p)$. If that bond is held for s periods, the cumulative after-tax return is $RB(s) = [1 + r(1 - \tau_p)]^s$. Similarly, if a unit of equity is held by an individual for s periods, the after-tax total return is $RE(s) = [1 + r(1 - \tau_a)]^s(1 - \tau_d) + d$. We are assuming that the corporation pays no dividends. This is reasonable in this model since dividends are dominated by either debt or capital gains. This focus is more realistic than conventionally thought since in recent years the majority of all cash payments from corporations to stockholders were in the form of repurchases or sales to other corporations (see Bagwell and Shoven (1987) and Shoven (1987)).

Before continuing, we should note that this specification of technology has many important implications for our analysis. First, security prices are not affected by capital income or gains taxation since investment is perfectly fungible: arbitrage argues that one unit of the single good in its capital form must trade with one unit of the good in its consumption form. If the market value of a firm were less than its book value, another firm could use some of its capital to buy that firm, achieving an arbitrage profit. Similarly, if it were overvalued, the equityholders would sell the firm's operations and reinvest the proceeds in new physical capital. We are implicitly assuming that these options exist for firms; such an assumption is not inconsistent with reality nor with the other elements of our model.

Examining a model in which taxes and savings will have no effects on market value may initially seem odd. By contrast, equilibrium models of security prices, as in Lucas (1978), commonly assume that the amount of capital is fixed, implying infinite adjustment costs. For issues related to interactions among capital gains taxation, short-run stocks, and security prices, that approach would be appropriate. However, issues in capital structure are predominantly long-run in nature and the assumption of a flexible capital stock is most natural. Another desirable feature of our linear technology is that it is a general equilibrium analysis since it is essentially a Robinson Crusoe analysis. This eliminates any questions as to whether the analysis is consistent with all markets clearing.

Given the nature of capital gains taxation, some restrictions on capital market transactions must be made since unlimited shorting will lead to arbitrage strategies. Since any short sale in our model would be for tax avoidance purposes only, it is appropriate to rule them out. In practice, some shorting is allowed.

However, if we are to avoid the trivial, and empirically questionable, result that agents pay no capital gains taxes, then investors must face some binding limit on such shorting. Once such a limit is reached, the agent will be at a corner with regard to shorting possibilities and the prices which determine resource allocation will be those implicit in our no-shorting assumption. We will also assume that an individual cannot borrow against future wages. Such a restriction is realistic, but also inessential to our main points since it will not be binding in most of our examples.

One seemingly unreasonable aspect of our restrictions on borrowing is that prohibits an investor to use the step-up in basis at death to avoid capital gains taxation. In a deterministic model this seems to be a particular objectionable assumption since an individual could plausibly borrow to finance consumption in old age, and pay off that debt at death by realizing capital gains which then excape taxation. If the world were certain then we would agree. However, it is unclear just how valuable this scheme is in an uncertain world because of the mismatch between assets and liabilities. The debt would be riskless but the equity collateral would be risky, substantially increasing the riskiness of the net wealth position. While we are unaware of any explicit analysis of this problem, it is clear that the increased riskiness in net wealth would reduce the value of attempting to use the basis step-up at death to finance retirement consumption. One would avoid the matching problem if the debt could be made state-contingent, but such an arrangement would almost surely be deemed by the IRS to be a realization. Even though tractability considerations force us to examine a deterministic model, we would argue that the most reasonable modelling choice is to discount the basis step-up at death as an important element in any tax-minimizing strategy since it is of only limited value in an uncertain world.

Our model also represents cases in which capital gains taxation is not completely effective. Suppose that an individual felt that he could fail to report $5000 of capital gains and avoid taxation or, if caught, avoid serious penalties. (Poterba found substantial underreporting of capital gains.) Such an "opportunity" would be equivalent to an increase in his endowment. At the margin, an investor would still face effective capital gains taxation. Since marginal decisions determine resource allocation and effective tax rates, it is appropriate to assume that any arbitrage opportunities of bounded value are already included in the endowment income. Similarly, if he felt that he could "forget" about ten per cent of his gains, then the effective tax rate is reduced, but still affects marginal incentives in the same fashion.

We now move to the analysis of the consumer's problem. Since all positions are assumed to be long, his choice of investment vehicle during periods of saving will depend on how long he intends to hold the asset. The critical fact for our purposes is that the choice of asset may change as the saving horizon changes. If an asset is held for one period, then there effectively is no deferral advantage to equity and the effective tax rate on the investment, $1-(1-\tau_a)(1-\tau_d)$, includes both the full nominal corporate and personal tax rates, whereas a bond investment of one period faces a net tax rate of τ_p. On the other hand, if an investment is held for a long time, equity dominates whenever τ_a is less than τ_p. The presence of capital gains taxation on a realization basis creates a nontrivial asset choice problem if bonds tend to dominate when investments are of short duration and equity tends to dominate for long-term investment.

Note that the structure of returns displays duration dependence. In fact, the instantaneous return for continuing to hold an asset purchased s periods previously, $R'(s)/R(s)$, is increasing in holding period s whenever $R(s) = RE(s)$ and is constant otherwise. Balcer and Judd (1985) showed that in the presence of such a relation between holding period and return, one can not aggregate an individual's portfolio and focus on its market value in examining his investment problem; instead, we must regard the assets purchased in each period as distinct.

Given these considerations and assumptions, we can now state the investor's problem. Let x_{it} represent the amount saved in period i which is liquidated in period t for $i < t$. The liquidation can be for either consumption or reinvestment, although the latter will be suboptimal in our deterministic model. The consumer in our model therefore faces the following maximization problem:

$$(C) \quad \text{Max}_{x_{it}} \sum_{t=1}^{T} \beta^t U(c_t)$$

$$s.t.\ w_t - c_t = \sum_{j=t+1}^{T} x_{tj}$$

$$- \sum_{i=1}^{t-1} x_{it} R(t-i),\ t = 1, \ldots, T$$

$$x_{it} \geq 0, \quad i, t = 1, \ldots, T$$

where w_t is the endowment (his wage, say) in period t, c_t is consumption, x_{it} represents the savings generated in period i to be dissaved in period t. The nonnegativity constraint on the x's reflect the restriction that no short sales, including borrowing against future earnings, are allowed.

Under capital gains taxation, a quick look at the first-order conditions of the consumer's optimization problem will not reveal which x's are zero and which ones are positive. We can, however, determine some properties of the optimal investment strategy. Balcer and Judd (1985) demonstrated that a "HIFO" strategy—highest basis units are the first sold—is generally optimal. In a deterministic world with rising equity value this is equivalent to LIFO mangement. In particular, the last assets purchased in the saving phase of life should be the first sold when the consumer begins liquidating assets to finance consumption.

One desirable feature of our model the optimal strategy is a relatively simple LIFO one. Initially, the consumer's problem looks so complex that it may appear unreasonable to believe that investors actually do act in accordance with intertemporal optimization. The fact that the optimal policy reduces to an intuitive LIFO policy, together with the observation that people with capital gains tend to have enough wealth to make complex computations cost-effective, reduces the strength of this criticism.

Theorem 1 summarizes what we know about the solution to the consumer's problem in a deterministic world.

THEOREM 1: *There exists a unique solution to problem* (C). *Furthermore, optimal portfolio management of assets subject to taxation of realized capital gains obeys a LIFO rule. In particular, when faced with a choice between equity and debt, there is a duration* t' *such that savings invested for a period less than* t' *will* *be put into bonds and savings invested for a period exceeding t' will be put into equity.*

Proof: Existence and the LIFO characterization follow from Balcer and Judd (1985). The existence of t' follows from comparing $RB(s)$ and $RE(s)$. Q.E.D.

We can compare this pattern of asset holding to other predictions. For example, Merton (1971) showed that if an investor has increasing relative risk aversion, a popular assumption, then the desired ratio between safe and risky assets rises as he ages. Since debt is usually safer than equity we see that both tax considerations and increasing relative risk aversion argue for a rising allocation of wealth to debt in early life, but that they differ as to the appropriate decumulation strategy.

In summary, this model has a number of valuable features. Demand for assets is generated from utility-maximizing life-cycle savings-consumption behavior. We give the investor full use of the timing option implicit in capital gains taxation by realization, never forcing him to liquidate assets at inopportune times. Yet, we assume that the investor is not allowed to *abuse* the timing option to avoid taxation. The result is a model where an individual faces a capital gains tax law which is effective at the margin, but allows him to put off the realization of profits until such profits are desired to finance consumption. We now examine the qualitative impact of such a tax system on investment behavior and intertemporal allocation.

IV. Effective Tax Rates

In studying complex tax structures, it is often desirable to compute an effective equivalent accrual tax rate in order to summarize its net impact on resource allocation and incentives. Such effective tax rate calculations often play a role in evaluating proposed tax changes as, for example, by the Office of Tax Analysis (1985) in its analysis of the 1978 changes. They also would help in assessing the impact of capital gains taxation on the cost of capital. In general, the construction of such indices is only an approximation but deemed useful in summarizing a collection of taxes.

The complexity created by taxation of realized capital generates makes such an index valuable, but also difficult. This section discusses a sense in which this is possible in our model. We will argue, however, that in the case of capital gains taxation it is unlikely that there is any reliable accrual approximation because of the substantial differences between accrual and realization taxation.

There have been earlier attempts to compute an effective tax rate, but never based on a choice-theoretic structure. Protopapadakis (1983) computed an effective average tax rate on an equity asset held s periods to be equal to the accrual tax rate on bonds which would make the investor indifferent between the two assets held both for s periods. More formally, the Protopapadakis effective tax rate is

$$\tau_P^{\text{eff}} = 1 - (1/rs)\ln[e^{rs} - \tau_d(e^{rs} - 1)]$$

King and Fullerton (1984) chose an alternative way to compute the average effective tax rate. They assumed that an investor liquidated a constant fraction of all vintages of his asset in each period, an approach similar in spirit to Constantinides. If λ is the rate of realization of unrealized capital gains, the King–Fullerton effective tax rate is

$$\tau_{KF}^{\text{eff}} = \frac{\lambda}{\lambda + r(1 - \tau_a)}\,\tau_d$$

The advantage of a model in which consumption and investment decisions are analyzed completely and jointly is that we can compute the impact on intertemporal prices the investor effectively faces along his optimal path as expressed in his marginal rates of substitution, determining in turn the effective distortion generated by taxation. Before discussing the equivalent accrual tax, we should examine the structure of demand in our model. The LIFO rule tells us much about the marginal rate of substitution among various consumption periods. Let $MRS_{t,s} = u'(c(t))/u'(c(s))$. If m is the last period in which the consumer saves, m' is the first period in which the consumer dissaves, and $m < m'$, the intuitive case, then LIFO implies

i) for $t < m$, if there exists a $t' > m$ such that $x_{tt'}$, and $x_{t+1,t'}$ are positive, then $MRS_{t,t+1} = R(t' - t - 1)/R(t' - t)$.
ii) for $t > m'$, if there exists a $t' < m$ such that $x_{t',t}$ and $x_{t',t+1}$ are positive, then $MRS_{t,t+1} = R(t + 1 - t')/R(t - t')$.
iii) if $t = m$ and $x_{t,t+1}$ is positive, then $MRS_{t,t+1} = R(1)$.
iv) if $m < t < m'$, then $c(t) = w(t)$.
v) if $t < t'$, then $MRS_{t,t'} R(t' - t)$

(i) reflects the fact that if an agent saves in both periods t and $t + 1$ for consumption in period t', then he must be indifferent between marginal consumption decisions in periods t and $t + 1$ which leave the total ability to consume later unchanged. (ii) follows from a similar logic for consecutive periods of dissaving. We will assume below that the optimal consumption path is such that all dissavings follows all savings,[1] a focus which we shall maintain in this study.

Note that the collection of arbitrage conditions which hold with equality for our model is far smaller than if all taxation were accrual-based. For example, if no capital gains were realized then $R(s) = RB(s)$, we could allow arbitrary long and short transactions, and we would have $MRS_{t,t'} = R(t' - t)$ for all periods t and t'. Whenever equity is competitive, however, we have to generally settle for the obvious and less informative general condition (v), which just states that at the optimum one does not want to save more at t for consumption at t'. In general, equality in the arbitrage condition between t and t', $t < t'$, holds only if one saves at t for consumption at t'.

This description of marginal rates of substitution suggests that the appropriate way to define the effective tax rate at any time t is to find that accrual tax rate which would leave the marginal rate of substitution between times t and $t + 1$ unchanged. This is done by computing the gap between r and $MRS_{t,t+1}$ at any t. We first do this for the case of an investor liquidating some assets for consumption purposes. For purposes of exposition it is best to take the continuous-time

[1] This condition will be true if for instance w is constant during the working life, an assumption which we make in our examples; for a more detailed discussion, see Balcer and Judd (1985).

expression. Then the marginal decision to sell is governed by an effective tax rate

$$\tau_m^{\text{eff}} = (r - R'(s)/R(s))/r = \tau_d[e^{rs}(1 - \tau_d) - \tau_d]^{-1}$$

This measure of effective tax rate is substantially different from both Protopodakis' and King and Fullerton's. In general it will be less than both approaches since they are average return indices and this is a marginal return measure. The marginal tax on holding an asset will be less since the marginal return to holding an asset is increasing in holding time.

The choice of index to measure depends on the intended use. If one wants to examine the impact of capital gains taxation on the holding of existing assets, then the marginal tax rate on holding existing assets is the correct one since past returns are "sunk" and play no role on current decisions. This marginal effective tax rate on holding is sometimes the appropriate one. In Judd (1985), the marginal effective tax rate is shown to be the correct effective tax rate for an economy where repurchase is the dominant form of cash flow from the corporate sector to households, retained earnings are sufficient to finance capital needs, and the economy can be modelled as a representative agent.

On the other hand, if one is interested in the impact on prospective savings and investment in a life-cycle context, then the marginal liquidation measure will be inappropriate since in a life-cycle model there are always some households investing cash in the corporate sector. However, despite their intention to represent such investment disincentive aspects of capital gains taxation, it is not clear that any average measure along the lines of King and Fullerton or Protopapadakis will be correct. The exact nature of effective tax rates is endogenous, depending on the chosen pattern of investment, consumption, and holding times. The latter aspect will be particularly important since LIFO implies that there will be substantial heterogeneity in holding times, depending on when the asset is purchased, and both the average and marginal effective tax rates are highly nonlinear in holding time.

In order to get an idea of the equilibrium pattern of effective tax rates, we will examine a series of examples of optimal consumption patterns and effective tax rates for various parameterizations of tastes, technology, and tax rates. Balcer and Judd (1985) describe an algorithm which can be used to calculate optimal consumption patterns in the presence of duration-dependent returns. The basic model introduced in section three is evaluated under the assumptions that the investor "lives" for T years, earns a wage of unity until age RET, after which his wage is zero. We take $T - RET$ to be 15, and T to be 60 or 45. The point in time represented by $t = 21$ is meant to be neither actual birth nor entry into the labor market. It should be interpreted here as the time at which retirement savings become feasible and desirable since an individual first spends money to accumulate consumer durables. Instead of modeling those decisions explicitly, we fix the date of initial retirement saving. Since we are ignoring the early period in life when earnings are low, the assumption of constant wage income is an acceptable approximation.

To examine accrual equivalents to realization taxation, we examine consumption paths which are chosen when all savings are subject to capital gains taxation (implicitly assuming that tax rates are such that equity always dominates) and ask what kind of accrual tax system would generate the same pattern of savings and consumption. Figure 1 displays the pattern of *age-dependent* accrual personal tax rate structures (assuming that in this alternative world there is no corporate taxation and debt dominates) which will, possibly along with some lump-sum transfers, reproduce the same consumption pattern for a variety of capital gains tax rates. We assume logarithmic utility with a four per cent per annum discount rate, 45 years of working life and 15 years of retirement, and a six per cent before-tax return. Note that the equivalent accrual tax must vary with investor's age to reflect the LIFO nature of the investor's strategy under capital gains taxation. Assets purchased early in life are held for a long time, nearly eliminating the capital gains tax. However, assets purchased later are held for a far shorter period of time, reducing the effectiveness of deferral in reducing the capital gains tax liability. In some cases the last unit of equity purchased will be sold almost immediately, making the effective rate equal to the full nominal rate.

In Figure 1, this pattern is represented for the cases where the initial capital gains tax rate is .1, .2, .3, .4, and .5. In all cases the accrual equivalent varies substantially over time, starting and ending at essentially zero. In most cases it moves up to the full nominal rate. In the $\tau_d = .4$ case the top effective rate is .32 because between saving and dissaving there is a period where consumption equals the constant wage. During this period the investor is essentially at a corner, where new investment has a low return since it would be held for a short period, but old investments are not liquidated since the marginal return to continued holding is relatively high. In this case the implicit accrual tax rate is .32. In the $\tau_d = .5$ case there is a similar corner within the saving phase, again generating a period where the accrual equivalent is .32.

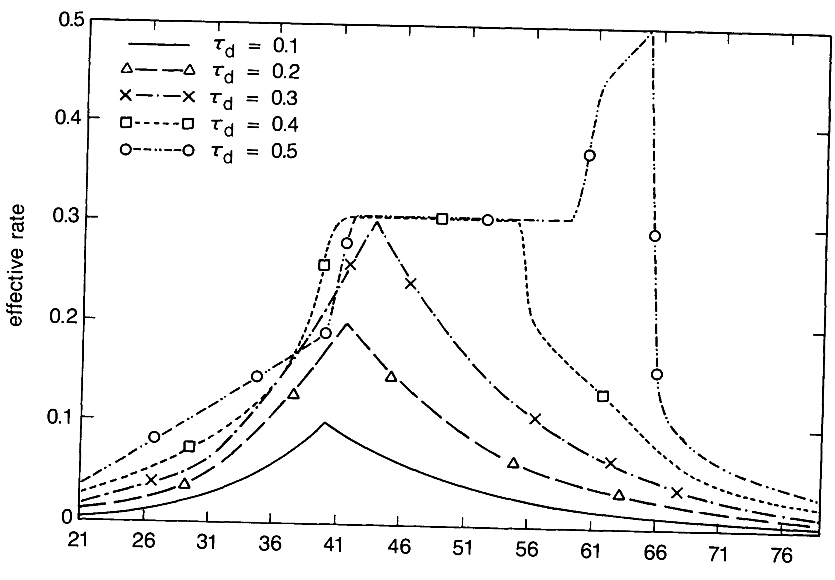

Figure 1. Effective tax rates, log utility, $r = 0.06$, $\beta = 0.96$.

These examples show the complexity in saving behavior generated by capital gains taxation by realization. In particular, since the accrual-equivalent depends so critically on the age of the investor, it clearly is very difficult to speak of any particular accrual equivalent for the economy as a whole. Some other indications of the futility of such an exercise arise when we examine the nature of asset demand volumes.

V. Steady-State Asset Demand

Next, we want to examine cases where the investor will hold both assets, and how the total demand for assets and the allocation across debt and equity are affected by changes in the underlying parameters. Suppose that a constant tax rate is applied to all individuals independent of age and income. Furthermore, assume that population is constant with births and deaths occurring at constant and identical rates. While any individual is either demanding only one asset at any moment, there may be a demand for both assets since different individuals may be at different points in their life cycle. Since r is constant, aggregate asset demand will converge to a steady state. We assume that the utility discount factor, β, is .96, representing utility discounting at an annual rate of four per cent. In addition, the utility of current consumption is $U(c) = c^{(\gamma+1)}/(\gamma + 1)$. The calculations will be performed for four before-tax interest rates, $r = .06$, .075, .09, and .12, and for five values for the inverse of the elasticity of intertemporal substitution in consumption, $\gamma = -.5, -1, -2, -5$, and -10.

We make no claim that these calculations are definitive estimates of the effects of capital gains and corporate income taxation in the U.S. economy. The calculations in this and the following section are offered to explore crucial questions in a completely specified model of intertemporal savings and investment behavior.

Table 1 examines the impact of the tax structure on the steady-state ratio between the demand for debt and equity. Each block corresponds to a choice of working and retirement periods and tax rates. Each column represents a choice of r and each row represents a choice of γ. The numbers in each block are the debt-equity ratios in the steady state.

We should make some obvious observations. Some comparisons need no table. First, if the personal tax rate is lower than the corporate rate, then debt always dominates. Second, if the joint corporate and capital gains tax rate is less than the personal tax rate on ordinary income then equity dominates. Table 1 makes some interesting points. For the choices of tax rates in Table 1, neither asset dominates at all points of an investor's life cycle, making steady-state comparisons of capital structure nontrivial. First, as the marginal product of capital increases there is greater reliance on equity. This is expected since a greater return will cause life-cycle consumption to have a greater rate of increase, implying more saving in early periods when savings go into equity. Second, as the length of the working period is reduced, there is greater reliance on debt. Again, this is intuitive since a shorter period of time to save for retirement will imply that debt will be the dominant investment for a greater proportion of that time. Third, the concavity of utility affects the steady-state debt-equity ratio substantially but the impact is not monotone.

Table 1

Debt-Equity Ratios for Steady-State Security Demand

$t_p = .5$, $t_d = .25$, $t_c = .4$:

	$T = 60$,	$RET = 45$				$T = 45$,	$RET = 30$		
γ	$r = .12$	.09	.075	.06	γ	$r = .12$	.09	.075	.06
−10.00	.0087	.0781	.1467	.3225	−10.00	.1099	.2477	.4290	.8473
−5.00	.0058	.0673	.1566	.3864	−5.00	.0883	.2412	.4396	.8414
−2.00	.0109	.0584	.1898	.5809	−2.00	.0740	.2230	.4743	1.0067
−1.00	.0121	.0252	.2578	1.5472	−1.00	.0392	.1970	.4795	1.5609
−.50	.0123	.0071	.3980	7.0391	−.50	.0300	.1555	.5540	7.0391

$t_p = .4$, $t_d = .2$, $t_c = .3$:

	$T = 60$,	$RET = 45$				$T = 45$,	$RET = 30$		
γ	$r = .12$	.09	.075	.06	γ	$r = .12$	.09	.075	.06
−10.00	.0017	.0224	.0565	.1262	−10.00	.0308	.0952	.1388	.2681
−5.00	.0031	.0058	.0535	.1380	−5.00	.0141	.0831	.1394	.2661
−2.00	.0050	.0043	.0353	.1425	−2.00	.0065	.0586	.1411	.2595
−1.00	.0047	.0090	.0115	.2350	−1.00	.0088	.0515	.1381	.3221
−.50	.0034	.0077	.0041	.4587	−.50	.0100	.0220	.1092	.4637

$t_p = .35$, $t_d = .175$, $t_c = .3$:

	$T = 60$,	$RET = 45$		
γ	$r = .12$	.09	.075	0.6
−10.00	.0053	.7243	1.8140	Debt Only
−5.00	.0694	.5682	1.6804	Debt Only
−2.00	.1221	.5364	1.3827	Debt Only
−1.00	.1462	.4929	1.2776	Debt Only
−.50	.1247	.4527	1.3597	Debt Only

These calculations show that capital income taxation has an impact on capital structure, but not as stark a one as typically hypothesized. Individual investors will demand both assets over the life cycle, choosing the one which is best for investment at the moment. In particular, it is clear that this diversity of assets will continue even in a flat tax system.

The coexistence of debt and equity in our model resembles in spirit earlier arguments by Miller (1977). In his analysis, each individual would specialize in one type of asset with progressive taxation leading different individuals to choose different assets. However, in our model, the crucial tax rate heterogeneity occurs even in a nominally flat tax system because the effective tax rates change endogenously across time for each individual.

The next set of calculations show how the interaction of accrual and deferral taxation can generate surprisingly perverse behavior. It is well known that the impact of interest taxation on saving behavior is generally ambiguous since the income and price effects operate in opposite directions. We will find that realization and accrual taxation apparently generate different patterns of income and substitution effects. In Table 2 we examine the change in lifetime savings as a result of an increase in the tax rate on realized capital gains and compare it to the effect of a marginal change in the corporate income tax rate. Cumulative savings is defined to be the average asset holdings of an individual over his lifetime and equals the total steady state wealth of society if there is no population growth. In columns labeled ϵ_s we indicate the percentage change in cumulative

Table 2

Savings Elasticities and Accrual Equivalents for Capital Gains Tax Changes

	$r = .12$		$r = .09$		$r = .0.75$		$r = .06$	
γ	ϵ_s	a_e	ϵ_s	a_e	ϵ_s	a_e	ϵ_s	a_e
$\tau_p = .5$,	$\tau_d = .25$,	$\tau_a = .4$:						
			$T = 60$,	$RET = 45$				
−10.00	0.06	−13.381	0.79	−.621	1.20	.151	1.96	.291
−5.00	0.31	−2.606	1.33	−.257	1.43	.254	2.35	.080
−2.00	1.75	−.219	2.19	−.135	2.60	.274	3.98	.274
−1.00	3.71	.063	3.92	.174	5.03	.205	4.74	.489
−.50	6.18	.079	6.42	.177	9.78	.273	1.76	1.000
			$T = 45$,	$RET = 30$				
−10.00	0.38	−.718	1.70	−.165	1.98	.284	3.83	.401
−5.00	1.00	−.188	1.91	−.083	2.05	.309	3.90	.404
−2.00	1.67	−.219	2.52	.077	2.27	.377	3.63	.325
−1.00	2.65	.151	3.46	.240	1.53	.562	4.08	.502
−.50	4.66	.190	5.18	.315	4.47	.370	7.61	1.000
$\tau_p = .5$,	$\tau_d = .25$,	$\tau_a = .35$:						
			$T = 60$,	$RET = 45$				
−10.00	−0.28	3.502	−0.12	6.412	−0.09	8.110	0.00	187.2
−5.00	0.33	−2.424	0.33	−2.150	0.28	−2.210	0.28	−1.680
−2.00	1.75	−.234	1.26	−.316	1.40	−.134	1.32	−.058
−1.00	3.45	.004	3.07	.087	3.33	.148	3.15	.261
−.50	5.75	.102	5.89	.210	6.92	.271	3.43	.354
			$T = 45$,	$RET = 30$				
−10.00	−0.18	4.494	−0.04	19.532	−0.02	25.479	0.04	−11.364
−5.00	0.18	−4.040	0.20	−2.887	0.17	−2.850	0.20	−1.929
−2.00	1.04	−.411	0.94	−.309	0.76	−.312	0.74	−.111
−1.00	2.31	.004	2.02	.079	1.79	.128	1.84	.287
−.50	4.17	.143	3.86	.240	3.90	.303	0.95	.157
$\tau_p = .4$,	$\tau_d = .20$,	$\tau_a = .3$:						
			$T = 60$,	$RET = 45$				
−10.00	−0.25	3.670	0.03	−25.700	0.28	−1.886	0.46	−.835
−5.00	0.33	−2.339	0.46	−1.345	0.59	−.849	0.83	−.283
−2.00	1.60	−.281	1.35	−.239	1.53	.041	1.99	.303
−1.00	3.40	−.014	2.89	.063	3.46	.180	4.31	.224
−.50	5.47	.085	5.11	.180	4.97	.234	6.73	.503
			$T = 45$,	$RET = 30$				
−10.00	0.08	−8.613	0.27	−1.387	0.78	.119	0.93	.395
−5.00	0.41	−1.476	0.63	−.457	0.98	.148	1.06	.476
−2.00	1.09	−.352	1.34	−.093	1.58	.192	1.64	.491
−1.00	2.34	.033	2.02	.089	2.41	.199	2.70	.339
−.50	3.97	.129	3.68	.225	4.05	.263	4.78	.555
$\tau_p = .35$,	$\tau_d = .175$,	$\tau_a = .3$:						
			$T = 60$,	$RET = 45$				
−10.00	3.27	−.038	5.04	−.088	4.16	.511	0.00	Undef.
−5.00	2.75	−.226	5.96	.081	4.41	.540	0.00	Undef.
−2.00	4.00	−.036	6.60	.055	5.08	.605	0.00	Undef.
−1.00	5.40	.002	7.47	.063	5.48	.636	0.00	Undef.
−.50	6.79	.035	8.70	.132	5.57	.641	0.00	Undef.

savings if τ_d is decreased by .01. In columns labeled a_e we indicate the accrual equivalent, i.e., the change in τ_a, multiplied by 100, which would generate the same change. The first values corresponding to $\gamma = -10$, $\tau_p = .5$, $\tau_d = .25$, $\tau_a = .4$, $T = 60$, $RET = 45$, and $r = .12$ are $\epsilon_s = .06$ and $a_e = -13.4$. Conventional reasoning would say that both numbers should be positive since a reduction in capital income taxation "should" increase saving and the capital stock. Already in the first entries we find perverse behavior with a .01 decrease in capital gains taxation increasing savings by .06 per cent but a .01 decrease in corporate taxation reducing savings, but an amount more than thirteen times less. We even find cases where ϵ_s is negative but a_e is positive, indicating that a decrease in either tax will reduce savings. These perversities generally occur when γ is relatively large in magnitude, that is, when the utility function is highly concave, and price effects tend to be small and overwhelmed by income effects. However, they do not occur only when $\gamma = -10.0$. It appears that in the context of a mixed capital structure, the income effects associated with changes in the taxation of capital income acquire a greater strength, making perverse responses more likely. Since cumulative savings may respond in different directions to changes in the different taxes and since Table 2 indicates that there are few reliable patterns obeyed by the accrual equivalent, it is doubtful that there is any useful accrual equivalent for capital gains taxation.

While the parameter values for these calculations were chosen to correspond to real world values, they should only be taken to be suggestive. This does not reduce their value, however, since they do show that we must be very careful in our analysis of capital gains taxation and the demand for assets differentiated by tax regulations.

VI. Conclusions

This study examined the impact of effective capital gains taxation on investment behavior and equilibrium capital structure. We first formulated a model in which investors use the timing option implicit in realization taxation, but do pay capital gains taxation because of effective restrictions on tax avoidance through manipulation of capital gains taxation. We found that in this case investors will demand only one type of asset at any moment, but that that choice will change over time. The examples in sections IV and V made two important points. First, the concept of an effective capital gains tax rate is extremely elusive, even in a simple model. Therefore, studies which depend crucially on such a parameter will face severe problems. Second, we saw that individuals will demand both equity and debt even in a nominally flat income tax system, indicating that equilibrium capital structures will be less responsive to tax changes than implied by static analyses. These effects arise in our model because of its ability to simultaneously represent both accrual and realization taxation, something which is impossible in a static model. Third, we saw that demand for assets may move in perverse directions in response to tax changes, apparently because income effects are stronger for changes in accrual taxation than for changes in realization taxation.

While many elements of reality were ignored, we would argue that the effects on which we focused will be robust to the addition of many such considerations. In general, our results on resource allocation in the presence of effective capital gains tax regulations indicates that it is important to explicitly model the set of feasible transactions if we are to engage in realistic examinations of capital gains taxes.

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DISCUSSION

CHESTER S. SPATT*: I liked this paper very much. It is a careful and high quality treatment of the effect of capital gains taxation in a long-run model of the life-cycle portfolio behavior of investors.

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