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DEBT AND DISTORTIONARY TAXATION IN A SIMPLE PERFECT FORESIGHT MODEL

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We examine the real effects of a temporary substitution of debt for distortionary income taxation. We find that such a policy may reduce consumption, increase investment, and reduce interest rates. These results arise even though we examine the same model used by Barro, where temporary substitution of debt for lump-sum taxes was shown to have no real effects. We show that these anti-Keynesian effects on consumption are as significant as various pro-Keynesian effects, such as finite lives. Hence, the impact of debt is an empirical issue and Ricardian neutrality appears to be a reasonable benchmark.

1. Introduction

The impact of debt financing on current consumption and savings has long been an important issue in macroeconomic analysis. Some have argued that replacing current taxes with debt causes consumers to feel wealthier because of their increased bond holdings and increase their consumption. This positive wealth effect of debt on consumption plays a major role in the Keynesian analysis of expansionary fiscal policy, as in Blinder and Solow (1973).

On the other hand, Barro (1974) has shown that if debt replaced current lump-sum taxation and if preferences were intergenerationally altruistic, there would be no effect on any current or future real variables. Several deviations from Ricardian equivalence case have been examined. Barro (1974) noted the importance of the particular bequest motive he used, argued that finite lives will induce intergenerational income effects inducing Keynesian-like effects, and also discussed the sensitivity of Ricardian neutrality to imperfections in the capital market. However, he argued that there were many other real-world phenomenon which would counter the Keynesian biases, that the neutrality of debt was an empirical matter, and the neutral case on which he focussed was a reasonable benchmark.

The debate has centered on whether the neutral case was a natural benchmark. Tobin (1980) examines a long list of real-world elements, arguing that

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Keynesian biases were overwhelmingly present. Recent work by Chan (1983) and Barsky, Mankiw and Zeldes (1986) has elaborated on the sensitivity to the perfect capital markets assumption, generally arguing that there was a substantial Keynesian bias when one makes reasonable assumptions concerning restrictions on market transactions and tastes. Blanchard (1985) elaborated on the finite-life arguments for Keynesian non-neutrality.[1]

However, all this work assumes taxes which are essentially lump-sum in nature, or would be if capital markets were perfect. This assumption of distortion-free taxation is unrealistic, especially at the margin [see, for example, the discussion of average marginal tax rates in Barro and Sahuskal (1983)]. The case of distortionary taxation was not explicitly discussed in Barro (1974). Tobin (1980) discusses it, asserting:

> The bias [of non-neutralities] is invariably in one direction: compared to current taxation, debt finance of government expenditure increases current consumption, reduces the saving available to purchase assets other than government securities. These conclusions are reinforced if real-world taxes are considered in place of lump-sum taxes.

Tobin and Buiter (1982) also argued that 'the nature of real-world tax systems create a presumption that debt finance of government spending increases current consumption'. This study takes up this issue in a dynamic general equilibrium model, wherein these assertions are shown to be false.

In this paper, we examine the short-run impacts on consumption, investment, output, and interest rates when a debt issue partially and temporarily replaces income taxation which must be increased later to keep the government within its dynamic budget constraint. We demonstrate that the distortionary character of income taxation cannot be ignored in discussing this issue. Most striking is that, outside of perverse cases, a temporary balanced-budget shift to debt followed by a tax increase would *depress* consumption in our model, contrary to the standard Keynesian arguments. In our model, such a twist in government finances also stimulates capital formation and output in the short run. Furthermore, we find that the non-equivalence of debt and

[1]Barsky et al., in particular, focus on a two-period model of economics where an individual cannot insure against uncertain second-period wage income. They show a reduction in first-period taxes financed by a second-period tax proportional to the realized wage. While they claim that their results concern non-lump-sum taxes, their taxes have no allocative effect since labor supply is inelastic and capital income is exempt from their taxes. Their taxes are best described as being state-contingent lump-sum taxes since they do not affect the marginal rate at which agents can trade state-contingent goods. As Varian (1980) noted, this kind of tax structure amounts to a public provision of the missing insurance market, resulting in a positive lifetime welfare effect which increases consumption in the first period if first period consumption is normal. Their results concerning the impact of debt is surely exaggerated since they block the private provision of a service, assume that the government has no problem providing the same service, and end up comparing allocations under an incomplete market structure with allocations arising from a partial completion of that market.

income taxes is quantitatively non-trivial. For example, the magnitude of the anti-Keynesian effect is greater than the finite-life effect modelled by Blanchard (1985) when the latter is realistically parameterized. All of this holds even though the preference structure for the infinitely-lived representative agent is that assumed by Barro.

Intuitively, this is due to price and income effects acting in the same direction. The price effect arises since a reduction in the taxation of interest income reduces the price of future consumption, stimulating saving and capital formation. The income effect arises since a shift in taxation into the future increases the excess burden of taxation, reducing lifetime utility and initial consumption demand. Also, these results do not depend on (empirically untested) third-order properties of utility, a feature of the Chan and Barsky et al. analyses, but are determined solely by elasticities of substitution in consumption and production.

We also examine the relationship between deficits and interest rates. When we add adjustment costs to our analysis, we find that a temporary tax cut will induce investment but immediately *reduce* interest rates, despite the extra demand for savings, if it is expected that future tax increases will be imposed to balance the budget. However, Judd (1985b) showed that capital decumulation will likely arise in this model if future cuts in government consumption balance the budget. In that case, we find that the deficit would be accompanied by a rise in interest rates. Therefore, the impact of deficits on interest rates depends on how the deficit is expected to be financed.

While this analysis is very limited, it does demonstrate that the Keynesian presumption, forcefully argued by much recent work, is substantially blunted when the analysis includes the distortionary character of real world taxation. This returns the debate to the initial assertion that the impact of debt on current consumption is an empirical question, depending on the relative strengths of several factors.

The paper is organized as follows. Section 2 contains a description of the basic model. Section 3 discusses a graphical analysis of a current tax reduction financed by a future tax increase. In section 4, the basic short-run quantitative analysis of perfect foresight models is developed. Section 5 applies these results to the debt versus tax issue. Section 6 discusses the extension of the model to adjustment costs and the implications for interest rates. Section 7 summarizes the paper's main points.

2. The model

Since the model we use is a special case of Brock and Turnovsky (1981), we will only describe the model and intuitively derive the equilibrium condition, referring the reader to Brock and Turnovsky for the formal analysis. Assume an economy composed of a large fixed number of identical, infinitely-lived individuals. The common utility functional is assumed to be additively separable in time with a constant pure rate of time preference, ρ:

$$U = \int_0^{\infty} e^{-\rho t} u(C(t))\,dt,$$

where $C(t)$ is consumption of the single good at time t. One unit of labor is supplied inelastically at all times t by each person, for which he receives a wage of $w(t)$.

There are two assets in this economy, government bonds and capital stock, each with the same net rate of return since they will be perfect substitutes. Let $F(k)$ be a standard neoclassical CRTS production function giving output per capita in terms of the capital–labor ratio, k. At $t=0$, k_0 is the endowment of capital for each person. Capital depreciates at a constant rate of $\delta > 0$. $f(k)$ will denote the net national product, that is, gross output minus depreciation. σ will denote the elasticity of substitution between capital and labor in the net production function.

We will keep the institutional structure simple. Think of each agent owning his own firm, hiring labor and paying himself a rental of $r_E(t)$ per unit of capital at t, gross of taxes, credits, and depreciation. It is straightforward that the alternative assumption of value-maximizing firms would be equivalent [see Brock and Turnovsky (1981)]. The gross return on bonds at t is denoted $r_B(t)$.

It will be convenient to use consumption defined as a function of p, the marginal utility of consumption, so define $c(p)$:

$$u'(c(p)) \equiv p. \tag{1}$$

Also, let $\beta \equiv u'(C)/Cu''(C) = c'(p)p/c(p)$ denote the intertemporal elasticity of consumption demand.

At time t, the government taxes capital income net of depreciation at a proportional rate $\tau_K(t)$, taxes labor income at a proportional rate of $\tau_L(t)$, assesses a lump-sum tax of $l(t)$ per capita, consumes $g(t)$ units of output, pays interest on outstanding debt, and floats $\dot{b}(t)$ new bonds. Any revenues not needed to pay off debt of finance government consumption are returned to agents via a negative lump-sum tax. The bonds are assumed to be continuously rolled over, allowing us to ignore effects due to the term structure of debt. Since the U.S. debt is largely short-term, this is a reasonable simplification.

The representative agent will choose his consumption path, $C(t)$, capital accumulation, $\dot{k}(t)$, and bond accumulation, $\dot{b}(t)$, subject to his budget constraint, taking the wage, rental, and tax rates as given:

$$\max_{C(t),\, b(t),\, k(t)} \int_0^{\infty} e^{-\rho t} u(C(t))\,dt, \tag{2}$$

subject to

$$C + \dot{k} + \dot{b} = w(1 - \tau_{\mathrm{L}}) + ((r_{\mathrm{E}} - \delta)k + r_{\mathrm{B}}b)(1 - \tau_{\mathrm{K}}) - l,$$

$$k(0) = k_0, \qquad \lim_{t\to\infty} k + b \geq 0.$$

(Time arguments are suppressed when no ambiguity results.) The last inequality prohibits the individual from running up an unbounded indebtedness in the long run. It is convenient to define

$$q(t) \equiv \int_t^{\infty} \mathrm{e}^{\rho(t-s)}((r_{\mathrm{E}} - \delta)(1 - \tau_{\mathrm{K}}) + \delta\theta)u'(c)\,\mathrm{d}s, \tag{3}$$

where $q(t)$ is the current marginal utility value of an extra unit of capital at time t. The basic arbitrage relation between consumption and q is $u'(C(t)) = q(t)$. That is, each individual is indifferent between an extra unit of consumption and the extra future consumption that would result from an extra unit of investment. This can also be expressed as

$$C = c(q). \tag{4}$$

The arbitrage condition for investment in bonds is similar:

$$u'(C(t)) = \int_t^{\infty} \mathrm{e}^{\rho(t-s)}u'(C(s))r_{\mathrm{B}}(s)(1 - \tau_{\mathrm{K}}(s))\,\mathrm{d}s. \tag{5}$$

Since these equalities hold at all times t, differentiation implies that

$$\rho - \dot{q}/q = r_{\mathrm{B}}(1 - \tau_{\mathrm{K}}) = (r_{\mathrm{E}} - \delta)(1 - \tau_{\mathrm{K}}). \tag{6}$$

Eq. (6) tells us what r_{B} must be in terms of r_{E} and the tax parameters. In what follows, r_{B} will therefore be regarded as the function of r_{E} and τ_{K} implied by (6). We will assume that the transversality conditions at infinity hold for both assets:

(TVC_{∞})

$$\lim_{t\to\infty} q(t)k(t)\mathrm{e}^{-\rho t} = 0, \qquad \lim_{t\to\infty} p(t)b(t)\mathrm{e}^{-\rho t} = 0.$$

This condition is needed to insure that p, q, and k remain bounded as $t \to \infty$ and is a necessary condition for the agent's problem if $u(\cdot)$ is bounded, which can be assumed without loss of generality here since the net production function is bounded [see Benveniste and Scheinkman (1982)]. In the case of bonds, the content of these conditions is most clear: the government is not allowed to play a Ponzi game with consumers, i.e., it cannot succeed forever in paying off interest on old bonds by floating new bonds.

To describe equilibrium, impose the equilibrium conditions

$$r_{\mathrm{E}} = F'(k), \tag{7a}$$

$$w = f(k) - kf'(k), \tag{7b}$$

$$\dot{b} = g - \tau_{\mathrm{K}} kf'(k) + br_{\mathrm{B}}(1 - \tau_{\mathrm{K}}) - \tau_{\mathrm{L}}(f(k) - kf'(k)) - l(t), \tag{7c}$$

on (2) and the budget constraint, yielding the equilibrium equations

$$\dot{q} = q(\rho - (1 - \tau_{\mathrm{K}})f'(k)), \tag{8a}$$

$$\dot{k} = f(k) - c(q) - g, \tag{8b}$$

where $f(k) = F(k) - \delta k$, the net product. Note that these equations describe only the real activity of the economy, the path of bond holdings being determined as a residual obeying eq. (7c). The transversality condition insures that in equilibrium

$$0 < \lim_{t \to \infty} q(t), \qquad \lim_{t \to \infty} k(t) < \infty. \tag{9}$$

[See Judd (1985) for a proof of this.] Eqs. (8a) and (8b) describe the equilibrium of our economy at any t such that q and k are differentiable. To determine the system's behavior at points where q may not be differentiable, we impose the equilibrium conditions on (2), yielding

$$q(t)\mathrm{e}^{-\rho t} = \int_t^{\infty} \mathrm{e}^{-\rho s} q(s) f'(k(s))(1 - \tau_{\mathrm{K}}(s))\,\mathrm{d}s, \tag{10}$$

which shows that $q(t)$ is a continuous function of time. The system of relations given by eqs. (8) and (10) and the inequality (9) will describe the general equilibrium of our economy.

3. Graphical analysis

One can partially analyze the impact of a temporary substitution of debt for taxes on the equilibrium in a graphical fashion using a phase diagram. For the purpose of this example we will assume that both capital and labor income is taxed at the rate τ; this makes the graphical analysis more transparent and later we will see that the implications are not substantially altered.

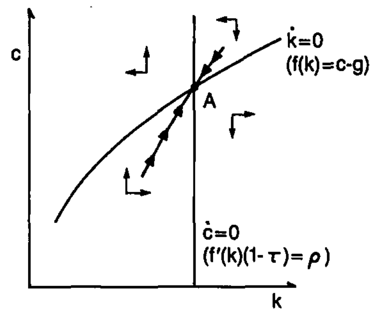

Fig. 1. k is the capital stock and c is consumption. The $\dot{k}=0$ locus is the collection of (c, k) points where net investment is zero, and the $\dot{c}=0$ locus is the combination of points where agents desire constant consumption.

Eqs. (8) can be represented qualitatively by a phase diagram as in fig. 1. Note that this phase diagram is in the $c-k$ space instead of the $q-k$ space. This representation is equally simple, more intuitive, and is derived from eqs. (8) by using the equality $C=c(q)$. The $\dot{c}=0$ curve is the locus in the $c-k$ space where consumption is constant and is derived from (8a); the $\dot{k}=0$ line represents the locus where there is no investment, being derived from (8b).

Within each of the four regions defined by these curves, the arrows indicate the general movement of the system described by eqs. (8). This differential equation system displays a saddle point structure, having a stable and an unstable manifold, the former being the set of points such that if the system starts there it converges to the steady state, point A. Note that the $\dot{k}=0$ locus depends only on g and that the $\dot{c}=0$ locus depends only on τ. Therefore, changes in τ affect only the $\dot{c}=0$ locus.

With these tools, fig. 2 analyzes the effects of a tax cut followed with a lag by a tax increase sufficient to balance the dynamic budget of the government. In a moderate τ regime the phase diagram is determined by the $\dot{k}=0$ and $\dot{c}=0$ loci intersecting at A, the corresponding steady state. If τ were reduced, then the new $\dot{c}=0$ locus is to the right and the new steady state would be a point like C with a stable manifold BC. If τ were increased, the $\dot{c}=0$ locus moves left and the new system would have a steady state with less capital, such as E with a stable manifold DE.

Now suppose that the economy is taxed moderately and is at its steady state A when, at $t=0$, there is an unanticipated decrease in τ. Furthermore, it is known that there will be a lag of T periods between the immediate cut in τ and the future increase in τ sufficient to balance the intertemporal budget. Furthermore assume that the initial reduction moves the $\dot{c}=0$ locus to C and that the eventual increase moves the $\dot{c}=0$ locus to E and the long-run steady state to D. In the time before the increase of τ, the economy is governed by

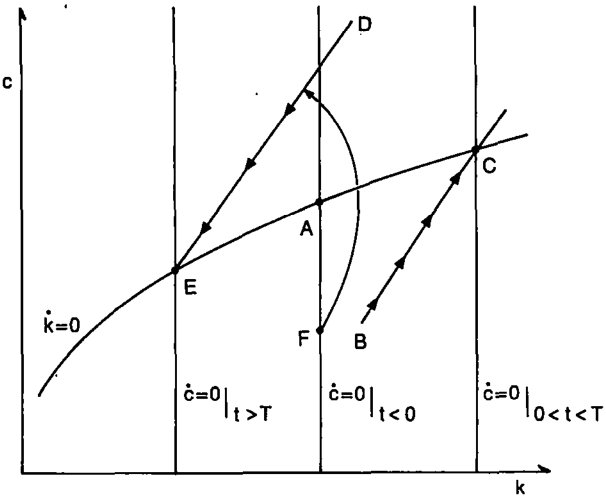

Fig. 2. k is capital and c is consumption. The three $\dot{c}=0$ loci correspond to the three tax policies: The initial policy ($t<0$), while the tax cut is in effect ($0<t<T$), and after the final budget-balancing tax increase ($t>T$).

the system with steady state at C, but eq. (10) implies that in equilibrium there are no jumps in c at $t=T$. Also, after T, the system is autonomous and stability implies that it be on DE converging to E. Therefore, the system between $t=0$ and $t=T$ must move, obeying the system with steady state C, to a point on DE. At $t=0$, the capital stock is given by the initial capital stock, but consumption must jump so as to move the economy to a point, F, such that the flow determined by the system with steady state at C moves the point F to some point on DE at $t=T$.

This example illustrates the basic principles of the model in a transparent graphical fashion, but also shows that such graphical analysis is inconclusive for this problem. For example, we drew our phase diagram so that consumption dropped at the time of the tax decrease. This was not dictated by the indicated flows and their changes. This shows that while qualitative phase diagrams can be illuminating, they often cannot settle an issue. We will return to this problem after developing the necessary analytical tools.

4. Local quantitative analysis

Since the graphical analysis cannot determine the impact of debt on the economy's evolution, we quantitatively determine these effects. We will use a locally valid linear approximation of our non-linear system to determine the response of the economy to a 'small' substitution of debt for taxation.

More precisely, we suppose that the tax rates are τ_K and τ_L, government consumption has been $\bar{g}$, and that the economy has reached the corresponding steady state with bonds at the level consistent with budget balance. Next suppose that, at $t=0$, the government announces (or does something which causes agents to expect) a future tax policy which possibly differs from the preceding constant policy. The government could choose among many combinations which are consistent with budget balance. We would like to compute the economy's response to any possible policy. For example, suppose that the government announces that it will immediately reduce the capital income tax rate from 0.5 to 0.3, but raise it in four years to balance the budget. To compute this policy's effects we would have to solve the non-linear equilibrium equations with $\tau_K = 0.3$ for the initial four years and $\tau_K = 0.5$ thereafter, with the economy starting with the steady-state capital stock associated with $\tau_K = 0.5$. We cannot do this analytically. However, a plausible indicator of what those effects would be is the response of the economy to an 'infinitesimal' cut in τ_K from an initial level of 0.5 followed by a comparably small budget-balancing increase later. To perform this marginal calculation, we consider a one-dimensional family of policies. In particular, for each ε consider the announcement that τ_K at $t>0$ will be $\varepsilon h_K(t)$ greater and that τ_L will be $\varepsilon h_L(t)$ greater. This representation of the policy choices separates the magnitude of a policy from its intertemporal complexity, with ε giving the magnitude of a policy change and the h_K and h_L functions representing the intertemporal structure of a policy, i.e., the timing and the relative sizes of the various changes, etc. For example, if h_K is 1 for $t \in [4,6]$, then we are examining a policy which will raise τ_K for two periods beginning four periods in the future. With this representation, any announcement is just an announcement of the value of ε given knowledge of h_K and h_L.

For any fixed ε, the equilibrium of our model is therefore given by the solution to the differential equations

$$\dot{q} = q(\rho - (1 - \tau_K - \varepsilon h_K) f'(k)), \tag{11a}$$

$$\dot{k} = f(k) - c(q) - \bar{g}, \tag{11b}$$

$$0 < \left| \lim_{t \to \infty} k(t) \right| < \infty, \qquad k(0) = k_0. \tag{11c}$$

For any particular ε, we will denote the solutions to (11a) and (11b) as $k(t, \varepsilon)$ and $q(t, \varepsilon)$, making explicit the dependence on ε.

With this decomposition of a policy between its magnitude and intertemporal structure and the resulting family of solutions indexed by the scalar parameter ε, we can begin our formal analysis. Since we assume that the capital stock is the steady-state stock for the previous constant tax policy, an announcement of $\varepsilon = 0$ will mean that there will be no change in tax policy and that the economy will stay at the original steady state. Hence, for $\varepsilon = 0$ we can solve the equilibrium response. We are ideally interested in the equilibrium response for an arbitrary ε; that, however, would be equivalent to solving the non-linear system (11), which we noted was not feasible. It is, however, feasible to compute the impact of changing ε away from 0 through linearization of (11) around $\varepsilon = 0$. Such an approach will allow us to determine the character of solutions for announcements of small ε.

Since the system (11) is smooth in ε, the solution of (11) for any fixed values for the initial values of k and λ will also depend differentiably on ε. [See Coddington and Levinson (1955).] We can therefore express the response of k and investment at any $t > 0$ for small ε by their derivatives:

$$\frac{\partial k}{\partial \varepsilon}(t,0) \equiv k_\varepsilon(t), \qquad \frac{\partial}{\partial \varepsilon}\left(\frac{\partial k}{\partial t}(t,0)\right) \equiv \dot{k}_\varepsilon(t).$$

Also of interest will be the similarly defined impacts, $q_\varepsilon(t)$, $\dot{q}_\varepsilon(t)$, on q and its time rate of change, respectively.

Differentiation of the equilibrium system (11) with respect to ε at $\varepsilon = 0$ yields a linear differential equation in the variables k_ε, q_ε:

$$\begin{pmatrix} \dot{q}_\varepsilon \\ \dot{k}_\varepsilon \end{pmatrix} = \begin{pmatrix} 0 & -q(1-\tau_K)f'' \\ -\beta c q^{-1} & f' \end{pmatrix} \begin{pmatrix} q_\varepsilon \\ k_\varepsilon \end{pmatrix} + \begin{pmatrix} q h_K(t) f' \\ 0 \end{pmatrix}, \tag{12}$$

where the steady state values of q, c, and k are used in (12). Let J denote the matrix in (12), a linear system with constant coefficients. The only initial condition which we have is the shock to the initial capital stock, which we assume to be zero. In intertemporal optimizing models such as this, the initial value of q, the shadow price of capital, is endogenous. If $\varepsilon = 0$, then we know that q will remain at its steady-state value. To determine how the equilibrium responds to other small values of ε, we need to find the value of $q_\varepsilon(0)$, the rate of change in the initial shadow value of capital as ε increases. This is determined by the fact that the response of k to a change in ε, $k_\varepsilon(t)$, must be bounded. This is intuitive since (11c), the equilibrium condition implied by the individual's transversality condition, says that the capital stock must be bounded for any particular choice of ε. See the appendix in Judd (1985) for a proof of the boundedness of k_ε.

We will solve for the unique bounded solutions of (12) by Laplace transform techniques. The Laplace transform of a function $m(t)$ defined for positive t is another function $M(s)$ defined for sufficiently large positive s, where

$$M(s) = \int_0^\infty e^{-st} m(t)\,dt.$$

Let $Q_\varepsilon(s)$, $K_\varepsilon(s)$ be the Laplace transforms of $q_\varepsilon(t)$, $k_\varepsilon(t)$, respectively. Similarly, let $H_K(s)$ and $H_L(s)$ be the Laplace transforms of h_K and h_L, respectively. A Laplace transform of a function is quite natural in this context since it is just the possible present values of that function, and many indices of economic interest, such as the change in utility or revenues, are expressed in discounted value terms.

The eigenvalues of J are of critical importance in our linearization. Let $\mu > 0 > \lambda$ be the eigenvalues of J. They are given by

$$\mu, \lambda = \frac{f'}{2}\left(1 \pm \sqrt{1 - \frac{4(1-\tau_K)\theta_L\theta_c\beta}{\theta_K\sigma}}\right), \tag{13}$$

where θ_c is the steady-state share of net output going to private consumption and f' is the steady-state marginal product of capital. Since $\mu + \lambda = f' = \rho/(1-\tau_K)$, if the effective tax rate is positive, i.e., if $f' > \rho$, then $\mu > \rho$ since $\lambda < 0$. This implies that (12) is a saddlepoint stable system.

The boundedness of k_ε together with saddlepoint stability of J implies that the solutions to $K_\varepsilon(s)$, $Q_\varepsilon(s)$, and $q_\varepsilon(s)$ are given by

$$\begin{pmatrix} Q_\varepsilon(s) \\ K_\varepsilon(s) \end{pmatrix} = (sI - J)^{-1}\begin{pmatrix} qH_K(s)f' + q_\varepsilon(0) \\ 0 \end{pmatrix}, \tag{14a}$$

$$q_\varepsilon(0) = -qH_K(\mu)f'. \tag{14b}$$

The details are provided in the appendix.

This solution leads to several important formulas. First, the initial impact on investment is given by evaluating (12) at $t = 0$, yielding

$$\dot{k}_\varepsilon(0) = cf'H_K(\mu)\beta. \tag{15}$$

Next, we need to know the relationship between the initial decrease in taxation and the later increase which is imposed by the government's budget constraint. Brock and Turnovsky (1981) showed that intertemporal budget balance required that

$$b_0 = \int_0^\infty (\tau_K kf' + \tau_L(f - kf') + l)\mathrm{e}^{-\int_0^t \psi(s)\,\mathrm{d}s}\,\mathrm{d}t,$$

where

$$\psi(t) = \rho - \beta^{-1}\dot{c}/c.$$

Judd (1985b) showed that the family of policies indexed by ε comprises a balanced family of policies only if

$$0=\left\{\frac{Q_\varepsilon(\rho)}{q}\rho-\frac{q_\varepsilon(0)}{q}\right\}b_0+L(\rho)+\tau_K K_\varepsilon(\rho)(f'+kf'')$$

$$+kf'H_K(\rho)-\tau_L kf''K_\varepsilon(\rho)+H_L(\rho)(f-kf'), \tag{16}$$

where $L(\rho)$ is the present value of $l(t)$ discounted at ρ. If b_0, the initial stock of bonds, were zero this expression states that extra revenue equals extra spending discounted at the rate ρ, the steady state real net return. If $b_0>0$, the real rate of interest which must be paid on bonds when they are rolled over changes and the net discounted value of the altered interest bill per unit of existing debt is $(q_\varepsilon(0)-\rho Q_\varepsilon(\rho))/q$. This term would be different if the term structure were non-trivial. It would disappear if bonds were consols, whereupon bearers may experience a capital gain or loss at $t=0$.

With these formulas for budget balance and the initial investment impact investment, we can now move to the analysis of the temporary substitution of debt for distortionary taxation.

6. The non-equivalence of debt and taxes

In this section, we analyze balanced budget changes in the tax instruments τ_L and τ_K. In particular, we assume that τ_L and/or τ_K are reduced at $t=0$, being raised later to balance the budget. We determine the immediate impact on consumption and investment and the short-run impact on output.

First suppose that initially $\tau_K=\tau_L=\tau$ and that τ at t is changed by $\varepsilon h(t)$. We will assume $b_0=0$ and $\theta_c=0$ for this section to focus on the direct effects of the taxation. The balanced budget condition becomes

$$\tau f'(k)K_\varepsilon(\rho)+H(\rho)f(k)=0, \tag{17}$$

where $h=h_K=h_L$ and H is the Laplace transform of h. The case where taxes are cut today and raised to the higher permanent level at $t=T$ can be represented by $h(t)=-1+\zeta H_T(t)$ where the tax increase ζ is unknown initially, being determined by the dynamic budget constraint, and where H_T is a Heaviside function, which is 1 for $t>T$ and zero otherwise. If there are initially neither taxes nor bonds, the dynamic budget constraint (17) reduces to $H(\rho)=0$ which implies that $\zeta=e^{\rho T}$. This is intuitively clear since the debt will grow at the rate ρ and, since taxes are lump-sum in nature for small tax rates, the eventual tax rate need only be equal to the ratio of the capitalized value of the debt to the capitalized value of net national product. Then the impact on investment is found to be

$$\dot{k}_{\varepsilon}(0) = -\beta\rho c\left(\frac{1-\mathrm{e}^{(\rho-\mu)T}}{\mu}\right) > 0. \tag{18}$$

Since $\mu > \rho$, we have proved Theorem 1.

Theorem 1. If $\tau_{\mathrm{K}} = \tau_{\mathrm{L}} = \tau = 0$ *initially, then a temporary income tax cut financed by future taxes will stimulate investment. Furthermore, that stimulus is greater as* T*, the lag in the tax increase, is greater, and as* β*,* θ_{K}*, and* σ *are smaller.*

Proof. Eq. (18) shows $\dot{k}_{\varepsilon}(0) > 0$. The dependence on β, θ_{K}, σ, and T follow from inspection of (18) and the eigenvalue formula (13).

Theorem 1 shows that the price effects above induce a determinate shock to investment in this model, in sharp contrast to the zero effect caused by a shift in lump-sum taxes. Also, the shock is greater as the wait for the tax increase is greater, as utility is less concave, and as factors are less substitutable. This dependence on taste and technology is intuitive since both make the economy more willing to adjust rapidly to changes.

If the economy is initially taxed, solving for ζ yields

$$\zeta = \mathrm{e}^{\rho T}\left(\frac{1+\dfrac{\tau}{1-\tau}\dfrac{\rho}{\rho-\lambda}\dfrac{f'\beta}{\mu}}{1-\dfrac{\tau}{1-\tau}\dfrac{\rho}{\rho-\lambda}\dfrac{\beta f'}{\mu}\dfrac{\rho}{\mu-\rho}\left(\mathrm{e}^{(\rho-\mu)T}-\dfrac{\mu}{\rho}\right)}\right). \tag{19}$$

The impact on capital accumulation is therefore equal to

$$\dot{k}_{\varepsilon}(0) = \frac{\rho}{(1-\tau)}\frac{c|\beta|}{\mu}\left(1-\zeta\,\mathrm{e}^{-\mu T}\right). \tag{20}$$

From this we immediately see that the temporary tax cut is more stimulative for large T if ζ is positive, the non-perverse case, since $\zeta\,\mathrm{e}^{-\mu T}$ is positive but decreasing in T.

The focus of the original debate has been the impact of debt on aggregate consumption. We next address that issue. Let *MPS* be the general equilibrium impact marginal propensity to save out of extra disposable income at $t=0$ for this policy. More specifically, *MPS* is the ratio of the change in asset demand at $t=0$ to the extra disposable income which both result from the debt–tax substitution. For example, if *MPS* equals 0.25, a tax cut which initially reduces tax revenues by one dollar and increases bond sales by one dollar, causes private savings, including both the purchase of newly issued government debt as well as capital formation, to rise by 0.25. If *MPS* is less than unity, then capital decumulation occurs at $t=0$ since the extra private savings are not sufficient to absorb extra government dissaving. *MPS* is

$$MPS = -\frac{\rho\beta}{\mu}(1-\zeta\,\mathrm{e}^{-\mu T})+1. \tag{21}$$

Theorem 2. If $b_0=0$, $\theta_c=0$ and a permanent cut in τ_K at $t=T$ would require a positive lump-sum tax for budget balance, then $MPS>1$, and capital formation at $t=0$ is stimulated.

Proof. Straightforward calculation shows that the value of extra government revenue from a permanent τ_K increase at $t=T$ is proportional to the denominator of ζ. Since $\rho<\mu$, if the denominator of ζ in (19) is positive, so is its numerator which is also less than $\mathrm{e}^{(\mu-\rho)t}$ times the numerator. Hence, $\zeta\,\mathrm{e}^{-\mu t}$ is less than unity proving that *MPS* exceeds unity.

We next show that the effect of Theorem 2 is substantial. Table 1 lists values of *MPS* over a wide range of values for β, σ, T, and τ. ρ is set at 0.01, making one period of time that duration over which utility is discounted by 1%. If the annual rate of discount is 4%, T equals the number of quarters between the tax cut and the budget-balancing increase. Many attempts have been made to estimate the intertemporal rate of substitution between goods and the elasticity of substitution between capital and labor. The estimates turn out to vary substantially across studies [see Weber (1970) and (1975), Ghez and Becker (1975), Hansen and Singleton (1982), Berndt and Christensen (1973), Nerlove (1967), and Lucas (1969)]. The values for β and σ used in table 1 represent these estimates. Note that σ here is the elasticity of substitution in the *net* production function, which is less than that of the gross production function. Since estimates for these parameters vary substantially, the results are reported for a large range of possible values for σ and β. Casual examination of national income accounts suggest that we take capital share to be 0.25 and government consumption to be 0.2 of net production. These values are acceptable for our purposes since *MPS* is insensitive to reasonable changes in these parameters compared to its sensitivity to σ and β.

Examination of table 1 indicates how *MPS* is affected by parameter changes. *MPS* increases as the utility function is more linear and as the lag between tax cuts and tax increases grows, just as Theorem 1 showed when $\tau=0$. The dependence on σ, however, is ambiguous in table 1. Also, *MPS* exceeds 1 at all entries in table 1 because the hypothesis of Theorem 2 is satisfied by all of the tabulated parameter values.

Table 1[a]

σ	0.3		0.5		1.0	
τ	0.2	0.4	0.2	0.4	0.2	0.4
$\beta = -2.00$	1.03	1.03	1.03	1.03	1.03	1.03
	1.13	1.13	1.13	1.13	1.13	1.13
	1.21	1.19	1.22	1.21	1.22	1.22
	1.31	1.27	1.39	1.33	1.52	1.44
$\beta = -1.00$	1.02	1.02	1.01	1.02	1.01	1.01
	1.07	1.07	1.06	1.07	1.06	1.06
	1.11	1.11	1.11	1.11	1.11	1.11
	1.21	1.18	1.26	1.22	1.34	1.28
$\beta = -0.33$	1.00	1.00	1.00	1.00	1.00	1.00
	1.02	1.02	1.02	1.02	1.02	1.02
	1.04	1.04	1.04	1.04	1.03	1.03
	1.11	1.10	1.13	1.11	1.16	1.13
$\beta = -0.10$	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.01	1.00	1.00
	1.01	1.01	1.01	1.01	1.01	1.01
	1.05	1.04	1.06	1.04	1.06	1.05

[a]Each column of five numbers lists *MPS* for T equalling 2, 10, 20, and 400, respectively. All entries of 1.00 actually exceed 1.00 but by less than 0.005. σ is the elasticity of substitution between capital and labor in the aggregate production function. β is the intertemporal elasticity of substitution in consumption. τ is the tax rate on all income.

Another interesting fact to note is the size of *MPS*. Suppose that bonds were net wealth. In an intertemporal optimizing model such as this with inelastic labor supply, if an individual receives $1 more in wealth, he will save all of it, consuming the net interest income. For our model, that would have consumption per period increase by $0.01 since $\rho = 0.01$ is the steady-state net return on capital. We find that consumption actually *drops*, and usually the drop exceeds 0.01. In fact, for lags in excess of 10 periods the drop is substantially larger than 0.01 when β is less than -0.3. After allowing for multiplier effects, the magnitude of the effect of bond financing in this model is close to that in the typical IS-LM model while the sign is different. Therefore, under distortionary taxation, temporary deficit financing *depresses* consumption by a non-trivial amount.

One of the more often-discussed non-neutralities is the birth of new individuals who can be compelled to pay the future taxes. Blanchard's (1985) study of this non-neutrality never gave examples of the magnitude of the non-neutralities implied by his model, but Hubbard and Judd (1986) computed some cases. The 'finite-life' non-neutralities they calculated were less (and of opposite sign) than the distortionary tax non-neutralities discussed above. Again, the anti-Keynesian non-neutrality of interest taxation appears to be an important effect among non-neutralities.

Why is there this stimulus to capital formation and drop in consumption? It is not only due to an income or wealth effect but there is a substantial price effect since the temporary tax cut reduces the real price of goods tomorrow, inducing substitution away from consumption today. Generally, the impact on welfare is proportional to $H(\rho)f(k)$. If there is no tax initially, then the budget balance condition reduces to $H(\rho)=0$, implying that there is no income effect, intuitively because in an initially undistorted equilibrium there is no deadweight loss from a small tax. Hence the entire shock to investment in Theorem 1 is due to the cumulative price effect. At first this doesn't appear to be quite correct since goods in the distant future may be more expensive due to the later tax hike. We can compute the marginal price and income effects of our policy in the same fashion that we computed the general equilibrium impacts. Since the procedure is the same, we will just outline the steps here. If we take the investor's maximization problem, (2), in which the after-tax return is exogenous, we can linearize around the initial steady state to compute the individual's response to the induced change in the after-tax return. Such a calculation will yield the change in utility, expressed in terms of the initial shock to personal assets which would generate the same change in lifetime utility. If we then compute the impact on consumption at time zero of a compensating change in initial assets[2] and add that to the uncompensated impact, we would have the cumulative price effect of the policy on consumption demand at $t=0$. That cumulative compensated price effect is given by

$$\beta c\left(\frac{\rho}{1-\tau}\right)^2 \frac{H(\rho)-H(\mu)}{(\rho-\lambda)(\rho-\mu)}\left(1-\frac{\tau\theta_{\mathrm{K}}\sigma}{\theta_{\mathrm{L}}}\right)\frac{\theta_{\mathrm{L}}}{\sigma},$$

which is negative for small τ since $\lambda<\rho<\mu$ and $H(\rho)<H(\mu)$. Note that this price is larger in magnitude as β and τ are greater in magnitude. The negative income effect will also be greater as τ increases. Hence, as the tax system becomes more distortionary, the anti-Keynesian effects increase.

Standard public finance considerations help us see why investment is stimulated. If there is initially some positive tax, the income effect of a temporary tax cut followed by a tax increase sufficient to balance the budget is negative (assuming that we are not in a perverse region where tax cuts raise revenue). This is because an immediate and temporary tax *increase* is partially a lump-sum tax on capital in place and any revenues thereby raised in the short run would allow us to reduce future distortionary taxation.[3] Hence, a temporary tax cut would reduce total utility, and the wealth effect initially reduces consumption and induces extra investment. Also, it is clear from the

[2] Since the changes are marginal, compensating and equivalent variations are the same.

[3] For an extended analysis of the excess burden of unanticipated and anticipated, permanent and temporary tax changes in this model, see Judd (1981, 1984, 1985a, 1987).

continuity of the cumulative price effect that it, too, would be substantial for small τ.

7. Adjustment costs, interest rates, and deficits

Next we add adjustment costs to our analysis in order to examine the implications of debt for interest rates. To model adjustment costs, we adopt the specification

$$\dot{k} = k\phi(I/k), \tag{22}$$

where I is gross investment expenditures and ϕ is a concave function of $I/k \equiv x$. Therefore, if an individual spends I units of input on investment, then his capital stock, k, is augmented at the rate $k\phi(k)$. Furthermore, assume

$$\phi(\delta) = 0, \qquad \phi'(\delta) = 1, \tag{23}$$

implying that, if gross investment equals depreciation, then net capital formation is zero and there are no adjustment costs at the margin.

In order to analyze equilibrium with adjustment costs we need to be more detailed about the firm's problem. In the following we shall extend and adapt Abel and Blanchard's (1983) treatment of adjustment costs for the case of capital income taxation. We assume that firms maximize their value, taking their tax rates (we make the clarifying and inessential assumption that capital income is taxed at the corporate level) and the path of interest rates as given. This problem is

$$\max_x \int_0^\infty \mathrm{e}^{-\int_0^t r(z)\,\mathrm{d}z}\left(F(k) - w - xk - \tau_K(F(k) - w - \delta k)\right)\mathrm{d}t,$$

$$\dot{k} = \phi(x)k.$$

In this problem we are assuming that the firm is allowed depreciation at the economically true rate, but is allowed to neither expense nor depreciate adjustment costs. This assumption will not affect any important result, only keep our formulas simpler. In particular, the steady state will be unaffected because there are no adjustment costs at the margin in the steady state.

If q is the current shadow price of capital to the firm, the conditions for the dynamic optimum are

$$\dot{q} = rq - F'(k)(1 - \tau_K) - \delta\tau_K - q(\phi(x) - x\phi'(x)), \tag{24}$$

$$0 = -1 + q\phi'(x). \tag{25}$$

Differentiating (25) with respect to time and using (25) shows that (24) is equivalent to

$$\varepsilon_\phi \frac{\dot{x}}{x} = F'(1-\tau_K)\phi' + \delta\tau\phi' + \phi - x\phi' - r, \tag{26}$$

where $\varepsilon_\phi = x\phi''/\phi' < 0$ is the elasticity of adjustment costs.

From the consumer's point of view nothing has changed. He still faces the interest rate pattern $r(t)$, implying that consumption and interest rates are related by

$$r = \rho - \frac{\dot{p}}{p} = \rho - \frac{\dot{c}}{c}\beta(c). \tag{27}$$

We need to tie these two basic equations – one for the supply price of k, (23), and one for demand behavior, (26) – together to get an expression for equilibrium. This is done by noting that

$$F(k) = c + xk. \tag{28}$$

Differentiation of (28) shows that

$$\frac{\dot{x}}{x} = -\frac{F - xk}{xk}\frac{\dot{c}}{c} + \frac{kF' - xk}{xk}\frac{\dot{k}}{k}. \tag{29}$$

Substituting (29) and (27) into (26) yields the following equation for the consumption path:

$$\frac{\dot{c}}{c} = \left(\beta^{-1} + \varepsilon_\phi \frac{c}{I}\right)^{-1}\left(\rho - F'(1-\tau_K) - \delta\tau_K\phi' - (\phi - x\phi') + \varepsilon_\phi \frac{\theta_K F - I}{I}\phi\right). \tag{30}$$

Eqs. (30) [in which we implicitly make the substitution $I = F(k) - c$] and (22) together comprise the differential equation system in the variables c and k which describes general equilibrium. Note that if ε_ϕ were zero then the general equilibrium system reduces to our earlier one where we assumed no adjustment costs. Hence, this system generalizes of our earlier analysis.

We could linearize as we did above. Since the steady state of (22) and (30) is independent of ε_ϕ and its Jacobian is clearly continuous in ε_ρ, then our formula for $k_\varepsilon(0)$, for example, would be continuous in ε_ϕ. It turns out that adding reasonable parameters for adjustment costs dampens the impact just as the decrease in intertemporal substitutability in consumption would. Hence, it is unnecessary to repeat those calculations if we just want to examine the initial impact of debt on capital formation. What we can do here that was not possible above is examine the initial impact on interest rates and the market value of firms. To do this we first obtain a more intuitive representation of the 'costate' equation, (30). Note that, if we combine (26) and (27) and rearrange terms, we obtain

$$\frac{e^{-\rho t}}{\phi'}\left(\varepsilon_{\phi}\frac{\dot{x}}{x}+\rho-\frac{\dot{p}}{p}\right)=p\,e^{-\rho t}\left(F'(1-\tau_{K})+\delta\tau_{K}+\frac{\phi-x\phi'}{\phi'}\right). \tag{31}$$

If we let $Z=p\,e^{-\rho t}/\phi'$, then the LHS of (31) is $-\dot{Z}$. Regarding the RHS of (31) as a function of time, we can integrate (31) and obtain

$$u'(c)e^{-\rho t}=\phi'\int_{t}^{\infty}e^{-\rho s}u'(c)\left\{F'(1-\tau_{K})+\frac{\phi-i\phi'}{\phi'}+\delta\tau_{K}\right\}ds. \tag{32}$$

Eq. (32) is quite intuitive. It states that the marginal utility of consumption at time t must equal the marginal addition to the capital stock which it could generate if saved instead of consumed multipled by the marginal utility of the after-tax income flow which would be generated by an additional unit of capital. That after-tax income flow not only includes the gross income after taxes, $F'(1-\tau_{K})$, plus the credit for allowed depreciation, $\delta\tau_{K}$, but it also must be altered by the future change in adjustment costs, $(\phi-x\phi')/\phi'$, which arises due to the greater capital stock. This formula is just a marginal cost equals marginal benefit equation for the investor.

We next find a formula relating the initial impact on interest rates and the impact on capital formation and the tax rate. Noting that (27) divided by $1-\tau_{K}$ expresses the before-tax interest rate on bonds, we can combine that with (30), differentiate with respect to ε, and evaluate the result at the steady state to find the initial impact on before-tax interest rates. That procedure yields

$$r_{B\varepsilon}(0)=\frac{1}{1-\tau_{K}}\frac{\varepsilon_{\phi}}{1+\beta\varepsilon_{\phi}c/I}\left((F'-\delta\tau_{K})\frac{\dot{k}_{\varepsilon}(0)}{I}+\frac{c}{I}\beta f'h_{K}(0)\right). \tag{33}$$

This formula decomposes the impact on interest rates today into several parts. First, capital decumulation is associated with higher interest rates. Capital accumulation is stimulated, but only temporarily, by the tax cut in the case of no adjustment costs. With adjustment costs, this process of accumulation followed by decumulation is wasteful. A lower interest rate will encourage less of the deficit to be financed by foregone consumption and reduce this temporary stimulus.

Second, the larger the tax cut today is, the lower the interest rate is. This is also clear since this instant's tax cut has no wealth effect and would therefore be invested completely unless the price of today's goods are made cheaper, that is, the interest rate declines. This cheapening occurs because the adjustment costs discourage such short-run movements in investment, causing interest rates to decline to stem these investment flows.

Now that we have a model where interest rates are not tied to the capital stock, we can examine the impact of deficits on interest rates. We find in this model that we cannot draw any definite conclusions, because the interest rates and the deficits may move independently. If the deficits are due to a temporary substitution of debt for distortionary taxation, then we saw that capital accumulation would result and that interest rates would be reduced. However, if the deficits were due to a tax cut today to be followed by future reductions in government consumption, then there often will initially be capital decumulation [see Judd (1985b)] and a rise in interest rates. Since the relation between interest rates and deficits is so sensitive to the manner in which the government's budget will eventually be balanced or expected to be balanced, any empirical relation between deficits and interest rates would be due to an historical accident that one type of policy or expectation of policy occurred more frequently than the other.

8. Conclusions

The primary accomplishment of this paper was the analysis of the debt-versus-tax issue in a model with distortionary taxes. A temporary substitution of debt for taxes on income generally stimulates capital formation and depresses consumption initially. Such a policy change generates a negative impact on the consumers' lifetime utility, causing a decrease in current consumption, with this decrease being accentuated by a price effect encouraging the consumption of future goods.

We also found that there was no relationship between current deficits and investment and interest rates. If future taxes are used to finance a deficit due to a current tax cut, interest rates rise, whereas if future spending cuts are used to balance the budget, interest rates will rise. This shows strikingly how important anticipations are in determining the impact of current policy.

In general, this analysis points to the importance of considering actual non-lump-sum taxation when discussing fiscal policy issues. We also find such analysis to be tractable.

Appendix

In this appendix we review the basic mathematics of Laplace transform calculations. Suppose that we have the linear differential equation

$$\dot{z} = Jz + h, \quad z_2(0) = z_{20}, \quad z(t) \text{ bounded}, \tag{A.1}$$

where $z(t)$ is in R^2, J is a 2×2 matrix, and h is a known two-dimensional function of time. The Laplace transform of z is Z, a function defined by

$$Z(s) = \mathscr{L}[z](s) \equiv \int_0^\infty e^{-st} z(t)\, dt. \tag{A.2}$$

Notice that Z is defined only for those s for which the integral exists. If a function is known to be bounded, then Z must be defined for any positive s. Also note that Z is a linear operator on measurable functions.

The critical fact for our purposes is that

$$\mathscr{L}[z](s) = s\mathscr{L}[z](s) - z(0). \tag{A.3}$$

Applying the Laplace transform to (A.1) yields the following equation for Z in terms of H, the Laplace transform of h, and the initial values of z:

$$sZ(s) - z(0) = JZ(s) + H(s). \tag{A.4}$$

Note that, for any particular value of s, this is just a linear equation defining $Z(s)$. Combining terms containing $Z(s)$ we find that

$$Z(s) = (sI - J)^{-1}(H(s) + z(0)). \tag{A.5}$$

When the system is saddlepoint stable, J will have one positive, μ, and one non-positive, λ, eigenvalue. Since we have only one initial condition, $z_1(0) = z_{10}$, we are looking for a value for $z_1(0)$ which will yield a bounded solution for z in (A.1). A bounded solution for z must have a Laplace transform, Z, defined for all positive s; in particular, $Z(\mu)$ must be finite. This fact must be reconciled with the observation that, for $s = \mu$, $sI - J$ is singular, making the denominator implicit in (A.5), $(s - \lambda)(s - \mu)$, equal to zero. The only way for $Z(\mu)$ to be consistent with (A.5) is for the numerator in (A.5) to also be zero. That linear condition involves elements of J, $H(\mu)$, and $z(0)$, and defines a value for $z_1(0)$. It does so uniquely since $\mu I - J$ is singular. In particular,

$$\begin{aligned} z_1(0) &= -J_{12}(H_2(\mu) + z_2(0))/(\mu - J_{22}) - H_1(\mu) \\ &= (J_{11} - \mu)(H_2(\mu) + z_2(0))/J_{21} - H_1(\mu). \end{aligned} \tag{A.6}$$

We have therefore completely solved for the unique bounded solution to (A.1).

References

Abel, Andrew. B., 1984, The aggregate effects of a change in a lump-sum tax: A violation of Ricardian equivalence, Mimeo.

Abel, Andrew B. and Oliver J. Blanchard, 1983, An intertemporal model of saving and investment, Econometrica.

Barsky, Robert B., N. Gregory Mankiw and Stephen P. Zeldes, 1986, Ricardian consumers with Keynesian properties, American Economic Review.

Barro, Robert J., 1974, Are government bonds net wealth?, Journal of Political Economy.

Barro, Robert J. and Chaipat Sahuskal, 1983, Measuring the average marginal tax rate from the individual income tax, Journal of Business.

Benveniste, Lawrence and Jose Scheinkman, 1982, Duality theory for dynamic optimization models: The continuous time case, Journal of Economic Theory.

Berndt, Edwin R. and Laurits R. Christensen, 1973, The translog function and the substitution of equipment, structures and labor in U.S. manufacturing, 1929–1968, Journal of Econometrics.

Blanchard, Olivier J., 1985, Debt, deficits, and finite horizons, Journal of Political Economy.

Blinder, Alan S. and Robert M. Solow, 1973, Does fiscal policy matter?, Journal of Public Economics.

Brock, William A. and Stephen Turnovsky, 1981, The analysis of macroeconomic policies in perfect foresight equilibrium, International Economic Review.

Chan, Louis K.C., 1983, Uncertainty and the neutrality of government financing policy, Journal of Monetary Economics.

Coddington, E.A. and N. Levinson, 1955, Theory of ordinary differential equations (McGraw-Hill, New York).

Hall, Robert E., 1971, The dynamic effects of fiscal policy in an economy with foresight, Review of Economic Studies.

Hansen, Lars and Kenneth Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, Econometrica.

Hubbard, R. Glenn and Kenneth L. Judd, 1986, Liquidity constraints, fiscal policy, and consumption, Brookings Papers on Economic Activity.

Judd, Kenneth L., 1981, Dynamic tax theory: Exercises in voodoo economics, Mimeo.

Judd, Kenneth L., 1985a, Marginal excess burden in a dynamic economy, Economics Letters.

Judd, Kenneth L., 1985b, Short-run analysis of fiscal policy in a simple foresight model, Journal of Political Economy.

Judd, Kenneth L., 1987, Welfare cost of factor taxation in a perfect foresight model, Journal of Political Economy.

Lucas, Robert E., 1969, Labor–capital substitution in U.S. manufacturing, in: A.C. Harberger and M.J. Bailey, eds., The taxation of income from capital (Brookings Institution, Washington, DC).

Nerlove, Marc, 1967, Recent studies of the CES and related production functions, in: M. Brown, ed., The theory and empirical analysis of production (Columbia University Press for NBER, New York).

Tobin, James, 1980, Asset accumulation and economic activity (University of Chicago, Chicago, IL).

Tobin, James and Willem Buiter, 1976, Long-run effects of fiscal and monetary policy on aggregate demand, in: J.L. Stein, ed., Monetarism (North-Holland, Amsterdam).

Tobin, James and Willem Buiter, 1982, Fiscal and monetary policies, capital formation and economic activity, in: James Tobin, ed., Essays in economics: Theory and policy (MIT Press, Cambridge, MA).

Varian, Hal, 1980, Redistributive-taxation as social insurance, Journal of Public Economics.

Weber, Warren E., 1970, The effect of interest rates on aggregate consumption, American Economic Review.

Weber, Warren E., Interest rates, inflation and consumer expenditures, American Economic Review.