**DYNAMIC MODELS OF TAXATION**

A Dynamic Theory of Factor Taxation

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Many important questions in macroeconomics concern the impact of taxation and spending policies on private resource allocation. One approach, typified by Robert Hall (1971) and William Brock and Stephen Turnovsky (1981), is to use dynamic general equilibrium models to address basic issues in fiscal and tax policy. The objective is to explicitly examine macroeconomic issues without embracing the market imperfections (fixed prices, missing markets, money illusion, etc.) found in conventional macroeconomic analysis.

There is currently great interest in dynamic fiscal policy problems. This decade has already seen two major adjustments in tax policy, and many attempts to substantially alter spending patterns. The result of this tumult has been unprecedented peacetime deficits and much uncertainty about what measures will ultimately be used to bring the budget into balance.

In this paper, I discuss the impact of alternative fiscal policies in a dynamic general equilibrium model. I examine the short-run effects of fiscal policy changes, the efficiency cost of alternative dynamic tax policies, the effects of uncertain policy formation, and the redistributive effects of factor taxation.

I. Model

I briefly sketch the details of the basic model, referring the reader to my paper (1987) for details. I assume an economy with a larger number of identical, infinitely lived individuals with a dynamic utility function

\[ U = \int_0^\infty e^{-\rho t} u(c, l) \, dt + \int_0^\infty e^{-\rho t} v(g) \, dt, \]

where \( \rho \) is the pure rate of time preference, \( c(t) \) is the rate of consumption of private goods, \( g(t) \) is public consumption, and \( l(t) \) is labor supply at \( t \), and \( u(c, l) \) and \( v(g) \) are felicity flows from private and public consumption, respectively. I assume one capital stock, which depreciates at a constant rate of \( \delta \). The \( K \) units of capital together with labor supply \( l \), both per capita, produces gross output at the rate of \( F(K, l) \) per capita.

I assume a simple tax structure where the marginal tax rate is \( \tau_K \) for capital income net of depreciation and \( \tau_L \) for labor income, and where firms receive an investment tax credit (ITC) for gross investment. Individuals may receive a lump sum subsidy, representing both nonproportionality in the income tax and public provision of goods which are perfect substitutes for private goods. Government also issues bonds, allowing taxes at one time to finance government consumption at another. Such bonds are perfect substitutes for private capital.

First, I must describe individual choices in such a dynamic world. They face a before-tax, but net of depreciation, return on physical assets of \( r(t) \) and a wage of \( w(t) \). If \( \lambda \) represents the private marginal value of capital, optimality implies that \( \lambda \) obeys the Euler equation,

\[ \dot{\lambda} = \lambda \left( p - (r(1 - \tau_K) + \delta \theta) / (1 - \theta) \right), \]

and that consumption demand and labor supply satisfy

\[ u_c(c, l) = \lambda = u_l(c, l) / (w(1 - \tau_L)). \]

In equilibrium, \( r = F_K \) and \( w = F_L \). Combining these conditions with the Euler equation,
consumption demand, labor supply equations, and the identity \[ K = F(K, l) - \delta K - c - g \] results in a pair of differential equations for \( K \) and \( l \) which describe equilibrium. Individual optimality also requires that neither consumption nor capital ever vanish. These conditions determine the economy's dynamic equilibrium course for fixed initial capital stock and fiscal policies.

An intuitive feature of equilibria in this model is the tendency to converge to a steady state where consumption, labor supply, and output are constant if tax and spending policies are also constant. In particular, the economy returns to its original state after a temporary policy shock. My infinite-horizon approach can make the useful distinction between temporary, anticipated, and permanent shocks.

There are conditions under which this representative agent model also represents the equilibrium of a disaggregated model. In particular, if utility is isoelastic and labor is inelastically supplied (i.e., \[ u = c^{\gamma+1}/(\gamma + 1) \]), the individual Euler equations can be written in terms of consumption, yielding

\[ \dot{c} = c \left( p - (r(1 - \tau_K) + \delta \theta)/(1 - \theta) \right)/\gamma. \]

If all individuals face the same marginal tax rate and investment return these individual conditions aggregate to a "consumption function,"

\[ \dot{c} = C \left( p - (r(1 - \tau_K) + \delta \theta)/(1 - \theta) \right)/\gamma. \]

A similar aggregation with elastic labor supply is possible if \[ u = (c^\gamma + l^{\gamma+1})/(\gamma + 1). \] For these cases, my model can examine "class" effects of various policies.

Before discussing the applications of this model, it is useful to indicate its advantages. First, it is a simple model in which movements of economic aggregates can be examined. Second, equilibria are locally unique, a distinct advantage over overlapping-generations models. Third, it is easy to examine the dynamics around a steady state, an approach to dynamics which is more realistic than the alternative of two-period general equilibrium models wherein the second period is also the last. Fourth, only a model with many periods can minimize intertemporal aggregation problems and be believably parameterized with estimates of technology and tastes.

Fifth, a study of the economy's dynamics around a steady state is useful in thinking about the incentives and tradeoffs which exist in a rational expectations policy equilibrium. Rational expectations imply that in equilibrium the government will choose to do what is expected; however, the government's alternatives, the paths not chosen, are unexpected policy shocks. Examining such shocks gives information about the incentives facing the government. Finally, it is easy to expand the model to incorporate extra elements of realism. One example of this flexibility will be my discussion of uncertain fiscal policy.

There are important elements which this model ignores for reasons for tractability. The infinite-life assumption implicitly makes strong assumptions about the nature of bequest behavior. Other analyses, such as Alan Auerbach, Laurence Kotlikoff, and Jonathan Skinner (1983), make the alternative extreme assumption that there is no bequest motive. For issues related to intergenerational distribution, that approach surely dominates. However, for aggregate issues it is unclear which is preferred. Ultimately the choice rests on empirical determination of the nature of bequest motives. Until then it is valuable to examine a variety of paradigms since such a study will indicate which issues are robust to these specifications.

II. Fiscal Policy

I first discuss the critical positive features of the economy's response to current and anticipated tax changes. In all exercises below, it is assumed that the economy has converged to the steady state corresponding to a previous constant spending and tax policy, and that the policy shock eventually settles down. For example, suppose that \( \tau_K \) is currently .5, that individuals had expected it to be .5 forever, and that we are currently in the corresponding steady state. One possible shock would be the announcement that \( \tau_K \) will remain at .5 for two more years, be increased to .52 for one year, but then move back down to .5 permanently. Such a change would be a partially anticipated temporary
increase in capital income taxation. I focus on such one-period policy changes, since any policy change is a sum of such elementary changes.

When we speak of changing a policy, it is obvious that we cannot change just one policy variable. If a tax is increased then revenues will change, necessitating a change in some other tax or spending policy. In all of the exercises in this section I shall assume that lump sum taxes adjust to balance the government’s budget. This choice is the best for analyzing the impact of any single policy parameter.

Some propositions always hold in my model, while others depend on actual parameter values. One advantage of a multiperiod model is that one can use empirical estimates of the critical parameters, which themselves implicitly assume a multiperiod model. Below I will assert propositions which are true for most empirical estimates of the underlying parameters. My papers (1985a, 1987) provide a more complete accounting.

When one calculates the impact of such policy changes, important results are found, some obvious, some less so. First, announcements of future $\tau_K$ increases will reduce current investment. The assumed rebate of revenues implies that there will be no direct income effect. The increase in the price of future goods increases current consumption and reduces investment. A negative income effect arises since the efficiency of the economy is reduced, but it only dampens the increase in consumption.

Anticipations of increased future government consumption will increase current investment and usually increase current labor supply. This is due to a pure income effect since the increased government extraction of output will reduce the utility derived from private consumption. Since I assumed separability between $c$ and $g$, this income effect is the only direct effect. Immediate and temporary increases in $g$ will crowd out investment, however, due to consumption smoothing by consumers.

Anticipated future wage taxation will usually increase current investment and labor supply: workers work today when wages are relatively high. This substitution effect dominates if utility is additively separable in $c$ and $l$, and holds for most empirical estimates of utility functions. However, immediate wage tax increases reduce labor supply and investment.

Anticipated future increases in the ITC will reduce current investment because investors will wait for the subsidy. Conversely, immediate and temporary ITC increases have a substantial positive effect on current factor supply, since agents want to take advantage of the subsidy while it exists.

An important feature of the short-run responses to future policies is that different discount rates are used when calculating different aspects of equilibrium. When computing the impact on revenue, the net interest rate is the appropriate discount rate to apply to future policies. However, a higher discount rate is applied to the policy change when calculating the impact on current consumption or labor supply. (See my 1985a paper.) This fact will be important below when I examine balanced-budget exercises.

### III. The Efficiency Cost of Taxation

One important use of this kind of model is to evaluate the relative efficacy of various tax policy changes. This section discusses the efficiency cost of taxation where by efficiency cost I mean the wealth equivalent of the loss in utility due to using a distortionary tax instead of a lump sum tax to raise a dollar in revenue. The results are intuitive given the reactions described above. In discussing the quantitative importance of various effects, it will be convenient to refer to a basic example. For the purposes of illustration in this essay, I use the example of a Cobb-Douglas production function with capital share of .30 and steady-state depreciation equal to .12 of net output. Tastes will be $u(c, l) = \ln c + l^2$.

The time unit is chosen to be that period over which utility is discounted by .01. It will be assumed that $\tau_K = .4$ and $\tau_L = .3$.

In the short run, capital income taxation is essentially a lump sum tax on existing capital. However, future capital income taxation will be distortionary since it reduces investment. In general, the efficiency cost of a tax increase rises rapidly as it is more anticipated.
In my example, a \( \tau_K \) increase announced today to be implemented in four periods and last for one generates an efficiency cost of 24 cents per dollar of revenue whereas the efficiency cost of a one-period increase 20 periods hence is 84 cents. An immediate and permanent increase has an efficiency cost of 60 cents. From these figures we see that only immediate and temporary capital taxation is like lump sum taxation.

In the case of labor taxation, the result is usually reversed. An immediate labor tax increase generates only a price effect reducing current labor supply. However, we saw above that increases in future labor tax rates will increase current labor supply and investment, and increase efficiency in the presence of factor taxation. Therefore, the efficiency cost of a wage tax increase falls as it becomes more anticipated. In my example, the efficiency cost ranges from 12 cents for an immediate one-period increase to a loss of 6 cents for a perfectly anticipated increase, with a permanent increase losing 8 cents. Since the ratio between cost of unanticipated and anticipated wage taxation is 2:1, and since they both exceed 5 cents, the static estimate, I find that dynamic considerations are important even when evaluating wage taxation.

Raising revenues by reducing the ITC is very costly in this model. In fact, total revenues may even be reduced because of the substantial impact on capital formation. In my example, revenue falls for immediate ITC reductions that last for 15 periods or less and the welfare cost of a permanent ITC reduction is $4 per dollar of revenue. Intuitively, these results occur because the ITC is a more targeted form of tax incentive for investment than \( \tau_K \) reductions, being primarily a subsidy of capital formation instead of relief for old capital. It is particularly interesting that all classes may want increases in the ITC. In my example and many others (see my 1981 paper), even a permanent ITC increase financed by wage taxation is Pareto-improving, since the tax cost for workers is more than offset by increased worker productivity and wages.

While the exact efficiency cost of revenue in this model is very sensitive to parameter choices, the ranking of various policies is surprisingly robust. A permanent wage tax increase is less distortionary than a permanent capital income tax increase for all parameterizations suggested by the empirical literature and reducing the ITC is far more costly than raising either factor income tax. While this model cannot yield precise estimates as to the magnitude of efficiency gains from tax changes, it has strong implications as to the appropriate direction.

The major purpose of taxation is the financing of public consumption. When taxes are distortionary, the efficiency cost of the necessary extra revenue should be considered when evaluating a potential project. The usual argument is that the benefits should exceed the direct costs of a project, the excess representing the efficiency cost of taxation. However, the large excess burdens noted above do not imply that the critical benefit-cost ratios should substantially exceed unity. The reason is that an increase in future government consumption increases the current supply of both factors, alleviating the tax distortions. The result is that the premium which must be borne due to distortionary taxation is not nearly as large as the marginal efficiency cost of taxation. In my benchmark case, the appropriate benefit-to-cost ratio to use is unity if permanent wage taxation is used to finance permanent expenditure and 1.28 if permanent capital taxation is used. There is also a large reduction, up to 20 percent in my example, in the critical benefit-cost ratio if the expenditure is delayed. In some cases, the stimulus to current factor supply of future expenditure is so strong that if we are to finance a project requiring a constant expenditure by making a permanent increase in \( \tau_t \), then the critical ratio is less than one. These considerations argue that projects requiring large immediate expenditures would be held to a substantially higher standard than those that involve a steady or deferred stream of expenditures.

These same considerations also predict biases in government factor usage. For example, a capital-intensive, but immediate and temporary, project will face a tougher benefit-cost criterion than a labor-intensive one, since the latter will increase labor supply.
whereas the former will crowd out investment. On the other hand, labor-intensive long-term projects will face a tougher criterion than capital-intensive ones since anticipated demand for capital services stimulates factor supplies, whereas anticipated demand for labor services reduces them.

While the neglect of many aspects of reality, in particular, uncertainty and asset heterogeneity, make these results largely suggestive, they do point and the importance of dynamic considerations in even the most elementary policy problems. Furthermore, the approach taken in my paper (1987) can be used in more realistic models.

IV. Balanced Budget Changes

Many of the exercises above assumed that the government's budget is balanced by changes in lump sum taxation. I next discuss the impact of policy changes when government consumption and distortionary taxes are altered to balance the budget, a more common experience.

One of the most studied exercises of this type is the temporary reduction of taxes, causing increased reliance on debt in the short run, but ultimately leading to increased taxation. Standard Keynesian arguments, based on the finite life of agents or capital market imperfections, assert that such a policy shock will increase current consumption because individuals regard the new bonds as part of wealth. However, my paper (1985d) showed that consumption can fall for two reasons: reducing $\tau_k$ will cause future goods to be cheaper, and shifting taxation to future capital income generates a negative income effect since a lump sum tax is replaced by a distortionary one. Current output and investment increase, and interest rates fall. Furthermore, the magnitude of this effect is of the same order as the Keynesian effects. Labor tax changes will generate opposing results since its dynamic reactions differ. However, for the case of a uniform income tax, the $\tau_k$ effects dominate. Hence, introducing distortionary taxation into this debate shows that the net result is an empirical matter and argues for Ricardian equivalence as an appropriate benchmark.

An alternative way to finance a current cut in taxes is a cut in future spending. If this is the anticipation of private agents, then the price effect of lower interest taxation is countered by the income effect generated by less government extraction. As we push the spending cut into the future, both the debt and the future necessary spending cut will grow at the net interest rate. However, the impact of future spending changes on current consumption is discounted at a much higher rate. Hence the net effect of the budget-balancing future spending cut on current consumption will fall as the spending cut is delayed. While the net effect is theoretically ambiguous, the income effect dominates for most reasonable parameterizations of tastes and lags, implying an increased consumption and crowding out of investment in the short run, causing interest rates to rise and output to fall. When utility is separable between labor and consumption, a fall in labor supply will also result, aggravating these contradictory impacts. Even when there is no immediate contraction, some period of falling investment and output will precede the spending cut.

The major conclusion from these cases is that there is no sharp contemporaneous relation between deficits, spending, output, and interest rates. A critical determinant is the expectation of how the deficit will ultimately be financed.

V. Uncertain Future Fiscal Policy

An unrealistic assumption of my analysis so far is that agents know future policy with certainty. It is clear that there is much uncertainty in reality and private agents are often heard indicating that such uncertainty affects their current actions. The model I have examined is modified in my paper (1985c) to discuss the impact of uncertainty about future policy.

The first principle that arises is the existence of a magnification effect. More specifically, a mean-preserving increase in the uncertainty of a policy's timing will preserve the direction and increase the magnitude of the policy's effect on current private decisions whenever the uncertainty about the
magnitude of the change is small. For example, an announcement that in five years the capital income tax will be raised for one year will reduce current investment. An announcement that the temporary increase will, with equal probability, occur either four or six years from now will cause an even greater fall in current investment. This is somewhat surprising, since it contradicts conventional precautionary saving arguments.

This magnification effect has some interesting implications for various fiscal policies. We saw above that a temporary substitution of debt for capital income taxes will reduce consumption in the short run. Since an anticipated future $\tau_k$ rise increases current consumption, uncertainty in the date of a budget-balancing increase will reduce that initial fall in consumption. If the debt is to be financed by future spending cuts, then we saw that consumption increases. Since future spending cuts increase current consumption, uncertainty as to when the cut comes will make the rise in current consumption even greater. Therefore, uncertain timing reduces the net balanced-budget effect in the case of debt financed by future capital taxation, but magnifies the net effect if future spending cuts are expected.

One of the causes of uncertainty is that opposing political factions champion alternative policies. The outcome is often uncertain, particularly since it may hinge on non-economic developments. We can examine the impact of such political battles on resource allocation in my model. In particular, assume a tax cut sends the budget into deficit, and that one side wants to balance the budget by future spending cuts, whereas the other side prefers to increase $\tau_k$. Suppose that this debate continues until one side yields, and that there is a constant probability in each period that a side yields. In this model, the immediate response to the outbreak of this conflict depends on its expected duration. If a quick resolution is expected, then the outcome, whether a spending cut or tax increase, will act to increase current consumer spending and dominates the price effect of the short-lived lower $\tau_k$. On the other hand, if resolution is expected only in the distant future then the price effect of the $\tau_k$ cut dominates, increasing investment and output.

VI. Redistribution and Taxation

Factor taxation is determined not only by efficiency considerations, but also by judgments of who should bear the burden. It would appear that little can be said about what tax policies are desirable, since all will want someone else to pay the taxes which finance public spending. However, if we have only factor taxation, then it turns out that much can be said about long-run rational tax policy. In particular, in a large class of models there is unanimity that capital should bear no tax burden in the long run.

Suppose we have two classes that differ by tastes, wealth, and labor endowment. Suppose that we examine a richer class of utility functions allowing pure rates of time preference, $\rho$, to depend on past and current consumption. Such specifications generate a long-run saving supply function with finite elasticity, whereas our original specification had an infinite long-run savings elasticity. Furthermore, assume that for any constant interest tax rate, a steady state existed in which both groups hold finite wealth. My paper (1985b) showed that any convergent Pareto-efficient tax policy will put no tax on capital in the long run. The general principle is that there should be no taxation of stocks, only flows.

This is clearly not a reasonable prediction as to what tax policy will be implemented, because almost any Pareto-efficient program is dynamically inconsistent. Implementing a dynamically inconsistent policy is difficult, since policies are really chosen sequentially and promises of future policies are seldom enforceable. One way to measure the importance of this is to ask how high could $\tau_k$ be before all would agree that a permanent $\tau_k$ increase, with the revenues going to the workers, would be detrimental. Such a tax rate would be a Phelps-Pollak solution to the dynamic consistency problem. That tax rate is usually between .3 and .6, being .5 in our example. This indicates that the incentives to deviate from the optimum plan will be substantial and that the dynamic inconsistency
problem can be particularly costly in the context of factor taxation. It also points to the importance of extending this model in the strategic directions necessary to examine the effects of and solutions to dynamic consistency problems.

VII. Conclusions

This paper has reviewed some aspects of the theory of dynamic factor taxation in a rational expectations, representative agent model of dynamic general equilibrium. Many important extensions are desirable, particularly the incorporation of uncertainty in production and rigorous modeling of the determination of policy choices in the political arena. Also, explicit incorporation of the market imperfections implicit in conventional Keynesian macroeconomic would enrich the model. However, I have shown that this model can be profitably used to examine the impact of dynamic taxation policy, anticipated and unanticipated, certain and uncertain, on resource allocation decisions, efficiency, and the distribution of wealth.

REFERENCES


