Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model

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This paper examines the short-run impact of current and future changes in fiscal policy on current investment in a simple representative-agent, perfect foresight model. We show that anticipated investment tax credits may depress current investment, as may an immediate income tax cut financed by future cuts in government expenditure. These impacts do result when we parameterize the model with current empirical estimates of the relevant parameters.

I. Introduction

Many recent papers have developed models to investigate the dynamic evolution of the economy in order to analyze dynamic effects of fiscal and monetary policy. Blinder and Solow (1973), Tobin and Buiter (1976), and Turnovsky (1977) studied dynamic versions of the Keynesian IS-LM model. The other major line of investigation has been the analysis of perfect foresight models (e.g., Hall 1971; Brock and Turnovsky 1981; Abel and Blanchard 1983). The major strength of the perfect foresight framework is its foundation in standard microeconomic principles and the ease of long-run analysis, whereas quantitative short-run analysis has been lacking in these models. While qualitative phase diagram analysis (e.g., as in Abel and Blanchard) is instructive, it is incapable of determining the short-run response to many intertemporally complex policy shocks of interest.

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This paper develops the quantitative short-run analysis of a perfect foresight model. In particular, I examine how an economy initially in a steady state responds to an unanticipated and arbitrarily complex change in current and future levels of taxation and spending. I show that short-run analysis can be accomplished with relative ease through the use of Laplace transforms, reducing the differential equations to linear algebraic equations and yielding a simple and intuitive formula for the short-run effects. This technical feature of the analysis is clearly of general interest and applicability. The major difference between this and most other linear models is that my coefficients are derived from basic parameters of taste and technology, allowing the examination of the quantitative significance of policy shocks and their sensitivity to these parameters. Also, I add a bond market, allowing examination of policy shocks that do not have continuous budget balance.

The formulas developed below indicate the initial impact on investment, consumption, and production due to balanced-budget changes in income taxation, investment tax credit changes, and government consumption. This analysis is then applied to two issues. Suppose that a permanent cut in the income tax rate is followed, after a lag, by a future spending cut large enough to satisfy the government's dynamic budget constraint. We find that this policy shock may initiate a phase of capital decumulation and output decline that continues until government consumption declines, after which capital accumulates until it reaches the new, higher steady-state level. This possibility is realized when the elasticity of substitution between capital and labor and the intertemporal elasticity of substitution among goods are assigned values considered representative of the U.S. economy. This is only one example of how short-run movements may differ in a quantitatively significant fashion from long-run movements, pointing out the need for tools in analyzing these short-run effects.

The second policy issue addressed is the stimulative powers of the investment tax credit. We find that while tax credits today will stimulate investment today, future tax credits may stimulate or depress investment today, depending on whether the sum of the pure rate of time preference and the rate of depreciation is less than or exceeds the positive eigenvalue of the linearized equilibrium equations. In more intuitive terms this means future tax credits depress current investment in fast-adjusting economies, while they encourage current investment in slow-adjusting economies.

The paper is organized as follows. Section II contains a description of the basic model. Section III discusses a graphic analysis of one particular fiscal policy. In Section IV, the basic short-run quantitative analysis of perfect foresight models is developed. Section V applies these results to a fiscal policy shock. Section VI summarizes the paper's main points.

II. The Model

Assume that we have an economy of a large fixed number of identical, infinitely lived individuals. The common utility functional is assumed to be additively separable in time with a constant pure rate of time preference, ρ:

$$U = \int_0^\infty e^{-\rho t} u[C(t)]dt,$$

where $C(t)$ is consumption of the single good at time t and u is the instantaneous utility function. Let $\beta(C) \equiv -u''(C)C/u'(C)$ denote the elasticity of marginal utility, also called the coefficient of relative risk aversion. One unit of labor is supplied inelastically at all times t by each agent, for which he receives a wage of $w(t)$. An inelastic labor supply is assumed, so that we may focus on the techniques used here to deal with the dynamic problems. The case of elastic labor supply introduces several complications and is left for a separate study (see Judd 1983).

There are two perfectly substitutable assets in this economy, government bonds and capital stock, each with the same net riskless rate of return. Let $F(k)$ be a standard neoclassical constant returns to scale production function giving output per capita in terms of the capital-labor ratio, k. Output can be used for consumption or investment. At $t = 0$, k_0 is the endowment of capital for each person. Capital depreciates at a constant rate of $\delta > 0$ and $f(k)$ shall denote the net national product, that is, gross output minus depreciation. Elasticity of substitution between capital and labor in the net production function is denoted by σ. To allow the use of differential techniques, we assume that $u(c)$ and $f(k)$ are C^2 functions. The value of outstanding debt in terms of consumption is denoted by b.

We shall keep the institutional structure simple. Think of each agent owning his own firm, hiring labor, and paying himself a rental of $r_E(t)$ per unit of capital at t, gross of taxes, credits, and depreciation. It is straightforward that the alternative assumption of value-maximizing firms would be equivalent (see Brock and Turnovsky [1981] or Abel and Blanchard [1983] for formal demonstrations of this). Since there will be no discussion of policies that are sensitive to institutional structure, we can use that fact and ignore the institutional detail that firms bring. The gross return on bonds at t will be denoted $r_B(t)$.

The government will play the usual role: at time t, it taxes capital income net of depreciation at a proportional rate $\tau_K(t)$, taxes labor income at a proportional rate of $\tau_L(t)$, assesses a lump-sum tax of $l(t)$ per capita, pays an investment tax credit on gross investment of $\theta(t)$ units of consumption per unit of investment, consumes $g(t)$ units of output, pays interest on outstanding debt, and floats $\dot{b}(t)$ new bonds. The bonds are assumed to be continuously rolled over, allowing us to ignore effects due to the term structure of debt. The adjustments for consols will be noted.

This model is consistent with two types of public consumption. First, the public consumption can be thought of as either public goods that do not affect the marginal rates of substitution among private goods or transfers to individuals who participate in neither the capital nor labor market. Both interpretations could be modeled formally by assuming that the private utility functional is additively separable in private and in such public expenditure. Therefore, while there may be value to each taxpayer from public consumption or transfers to the poor, the level and path of such transfers do not affect the demand functions of the agents for their private goods. A second class of public expenditures consistent with this model are publicly provided private goods that are perfect substitutes for private consumption. Being perfect substitutes, their provision is equivalent to lump-sum transfers to taxpayers. Therefore, our model includes both classes of public goods. Let g be the public spending for goods that are additively separable with respect to private consumption. Lump-sum transfers will represent those that are perfect substitutes for private consumption. With this formulation we can concentrate on purely fiscal policy issues while allowing two major classes of public expenditures.

The representative agent will choose his consumption path, $C(t)$, capital accumulation, $\dot{k}(t)$, and bond accumulation, $\dot{b}(t)$, subject to the instantaneous budget constraint, taking the wage, rental, and tax rates as given:

$$\max_{C(t),k(t)} \int_0^\infty e^{-\rho t}u[C(t)]dt$$

$$\begin{aligned} \text{s.t. } C + \dot{k} + \dot{b} &= w(1 - \tau_L) + [(r_E - \delta)k + r_B b](1 - \tau_K) \\ &\quad - l + \theta(\delta k + \dot{k}), \\ k(0) &= k_0. \end{aligned} \tag{1}$$

(Time arguments are suppressed when no ambiguity results.) We define

$$q(t) \equiv \int_t^\infty e^{\rho(t-s)}[(r_E - \delta)(1 - \tau_K) + \delta\theta]u'[C(s)]ds, \tag{2}$$

where $q(t)$ is the current marginal utility value of an extra unit of capital at time t. Along an optimum path, each individual is indifferent between an extra $1 - \theta(t)$ units of consumption and the extra future consumption that would result from an extra unit of investment:

$$[1 - \theta(t)]u'[C(t)] = q(t). \tag{3}$$

The arbitrage condition for investment in bonds is similar:

$$u'[C(t)] = \int_t^{\infty} e^{\rho(t-s)}u'[C(s)]r_B(s)[1 - \tau_K(s)]ds. \tag{4}$$

Since these equalities hold at all times t, we may conclude that

$$\begin{aligned} \rho - \frac{\dot{p}}{p} = r_B(1 - \tau_k) &= \frac{(r_E - \delta)(1 - \tau_K) + \delta\theta - \dot{\theta}}{1 - \theta}. \\ &= \rho - \frac{\dot{q}}{q} - \frac{\dot{\theta}}{1 - \theta}, \end{aligned} \tag{5}$$

where $p = u'(c)$. In what follows, r_B will be regarded as the function of r_E, θ, $\dot{\theta}$, and τ_K implied by (5).[1]

We assume that the transversality conditions at infinity hold for both assets in order to ensure that p, q, and k remain bounded as $t \rightarrow \infty$:

$$(TVC_\infty) \lim_{t\rightarrow\infty} q(t)k(t)e^{-\rho t} = 0, \quad \lim_{t\rightarrow\infty} p(t)b(t)e^{-\rho t} = 0. \tag{6}$$

This is a necessary condition for the agent's problem if $u(\cdot)$ is bounded, which is a harmless assumption here since the net production function is bounded (see Benveniste and Scheinkman 1982). In the case of bonds, the content of these conditions is most clear: the government is not allowed to play a Ponzi game with consumers; that is, it cannot succeed forever in paying off interest on old bonds by floating new bonds.

To describe equilibrium, impose the equilibrium conditions

$$r_E = F'(k), \tag{7a}$$

$$w = f(k) - kf'(k), \tag{7b}$$

$$\begin{aligned} \dot{b} = {} & g + \theta(\delta k + \dot{k}) - \tau_K kf'(k) + br_B(1 - \tau_K) \\ & - \tau_L[f(k) - kf'(k)] - l(t) \end{aligned} \tag{7c}$$

[1] Without any real loss of generality, we may assume θ to be a C^1 function of time. That is unnecessary if one interprets all the foregoing as generalized functions and uses the operational calculus.

on (2) and the budget constraint. This yields the equilibrium equations

$$\dot{q} = q\left[\rho - \frac{(1 - \tau_K)f'(k) + \delta\theta}{1 - \theta}\right], \tag{8a}$$

$$\dot{k} = f(k) - c\left(\frac{q}{1 - \theta}\right) - g, \tag{8b}$$

where $u'[c(p)] = p$ defines $c(p)$, which expresses consumption as a function of the marginal utility of consumption. These equations describe only the real activity of the economy, the path of bond holdings being determined as a residual obeying equation (7c). The transversality conditions ensure that

$$0 < \lim_{t\to\infty} q(t), \lim_{t\to\infty} k(t) < \infty. \tag{9}$$

The pair of equations (8) describe the equilibrium of our economy at any t such that q and k are differentiable. To determine the system's behavior at points where q or k may not be differentiable, I impose the equilibrium conditions on (2), yielding

$$q(t)e^{-\rho t} = \int_t^\infty \frac{e^{-\rho s}q(s)\{f'[k(s)][1 - \tau_K(s)] + \delta\theta(s)\}}{1 - \theta(s)}\, ds, \tag{10}$$

which shows that $q(t)$ is a continuous function of time. The system of relations given by equations (8) and (10) and the inequality (9) will describe the general equilibrium of our economy.

Since there are many alternative models available for studying short-run effects in perfect foresight models, some preferable on grounds of realism and/or tractability, we should note reasons for examining this one. While 2-period overlapping generations models (e.g., Diamond 1970) are good for understanding the qualitative features of perfect foresight analysis, they are far too rigid for meaningful quantitative short-run analysis. For purposes of application, a period in such a model would be on the order of 25–30 years, far longer than what would be realistically regarded as the short run. The Cass-Yaari (1967) model of continuous-time overlapping generations is not analytically tractable. Because of the inherent errors, numerical simulation of the Cass-Yaari model, as in Auerbach and Kotlikoff (1983), is limited to examination of large changes in the parameters, whereas the analytical approach used here is capable of computing marginal effects of changes in the parameters. These may be substantially different because of the nonlinearities of such models. Since legislative deliberations often concern relatively small changes, the ability to compute marginal effects is desirable. Also, this choice avoids the nonuniqueness problems that plague overlapping generations models and render comparative dynamic exercises invalid.

Although it is absurd to assume that any person has an infinite life, it is also an open question whether this is a bad approximation. The work of Kotlikoff and Summers (1981) indicates that substantial amounts of wealth are held for bequest purposes, in which case the true economic agent would consist of several generations of a family, having a life in excess of the roughly 50-year economic life span of an individual person.[2] The neoclassical growth model is not used because it models savings as a function of current rate of return on capital, rendering it incapable of analyzing anticipation effects, which are very important in our analysis and for many of the arguments made by policymakers and analysts. In summary, the choice of the infinite-life version was made because (i) meaningful short-run analysis is tractable, yielding simple and intuitive formulae, where sensitivity to basic parameters is easily determined, and (ii) empirical evidence indicates that it is not an absurd approximation.

These reasons are basically ones of theoretical soundness and realism, but not of demonstrated empirical validity. Nothing defensible on that issue will be said here. However, the analysis below will still be of interest to those who reject this full-employment approach to macroeconomic analysis since this model is close in spirit to the beliefs of some policymakers. We may test their arguments for logical consistency. For example, some policymakers argue that if taxes are cut immediately to be followed later by a spending cut, the tax cut will stimulate capital formation in spite of the temporary deficit. Can they believe in their perfectly competitive model and believe that there are no substantial short-run consequences of the resulting deficit for capital accumulation and production? Let us now move to a graphic analysis of this issue in our model. This will serve to illuminate the basic features of this model, illustrate the limitations of graphic analysis, and demonstrate how short-run effects may differ from long-run effects.

III. Graphic Analysis

One can partially analyze the impacts of policy changes on the equilibrium in a graphic fashion using phase diagrams.[3] In this section I analyze the short-run consequences of an income tax cut followed with a lag by a cut in government consumption large enough to bal-

[2] The relevant open analytical question is how long the economic life of an economic agent has to be before the Cass-Yaari model is approximated well by the infinite-life model.

[3] Other examples of such graphical analysis can be found in Abel and Blanchard (1983).

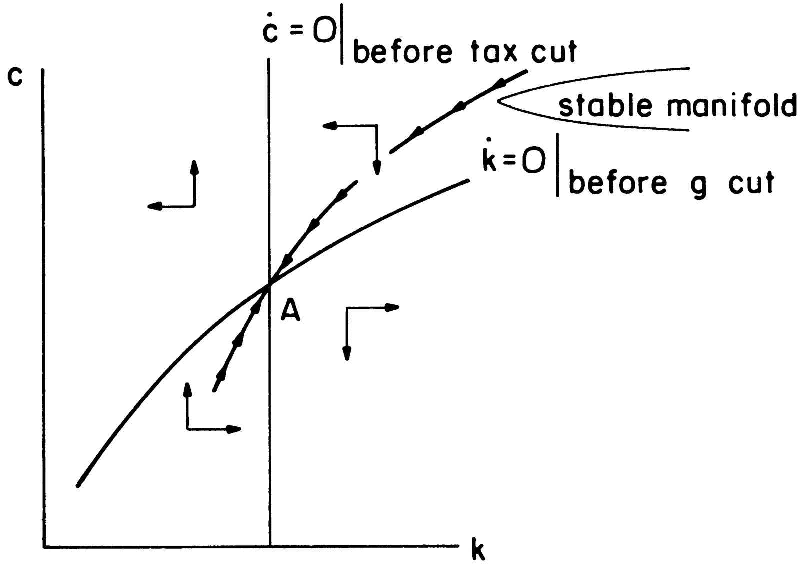

FIG. 1.—Local structure of a steady state

ance the government's dynamic budget. In particular, I determine whether the deficits incurred in the short run reduce capital formation. For the purpose of this example, I assume that there is no investment tax credit and that both capital and labor incomes are taxed at the rate τ, making the graphic analysis more transparent without losing the essential points. In this section I examine the more interesting case where all government expenditure is public-goods consumption, represented by g in the equilibrium equations above.

Equations (8) can be represented qualitatively by a phase diagram as in figure 1. Note that this phase diagram is in (c, k) space instead of (q, k) space. Since labor is inelastically supplied, this representation is equally simple and clearer. It is derived from equations (8) by means of the equality $q = u'(c)$, which holds since there is no investment tax credit. The vertical $\dot{c} = 0$ curve is the locus in (c, k) space where consumption is stationary and is derived from (8a); the upwardly sloped $\dot{k} = 0$ line represents the locus where investment is stationary, being derived from (8b). Within each of the four regions defined by these curves, the arrows indicate the general movement of the system described by equations (8). This system displays a saddle-point structure with a stable and an unstable manifold, the former being the set of points from which the system converges to the steady state, point A. Note that a change in τ will affect only the $\dot{c} = 0$ locus and that changes in g affect only the $\dot{k} = 0$ curve.

With these tools in hand, we can analyze the effects of a tax cut

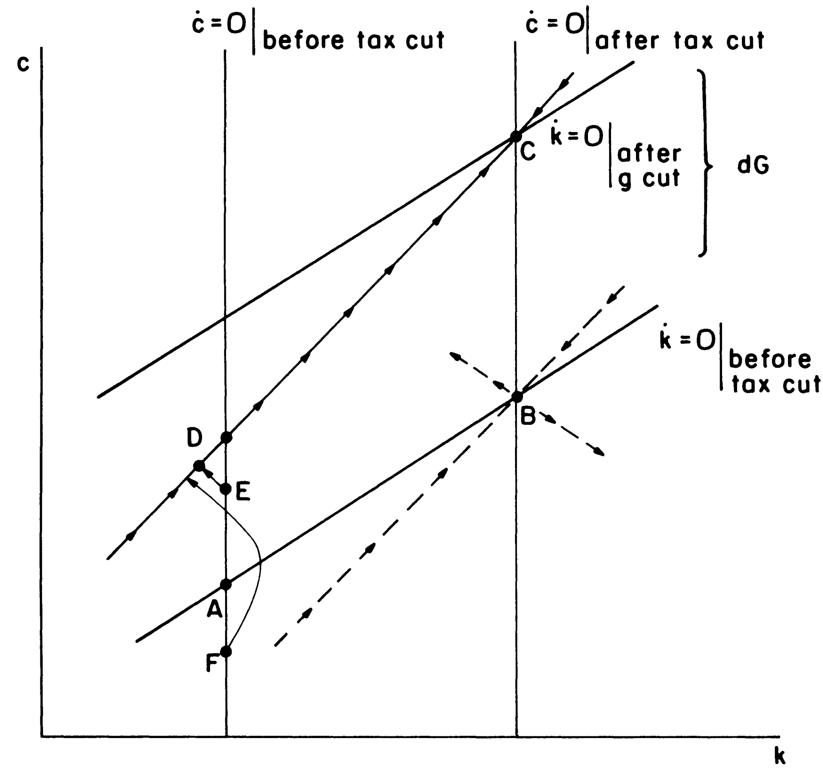

FIG. 2.—Analysis of a tax cut followed by an expenditure cut of dG

followed with a lag by an expenditure cut sufficient to balance the dynamic budget of the government. This is displayed in figure 2. In a high-τ and a high-g regime, the phase diagram is described by the two stationary loci intersecting at A, the corresponding steady state, whereas the low-τ and low-g regime has steady state C. If there were no lag between cuts in τ and g, stability implies that consumption would jump vertically to that point on the stable manifold of the system with steady state C. Suppose that point is D and that the new stable manifold is the curve through D and C. From D, the economy would converge to C along DC.

Now suppose that there is a lag between the cut in τ and the cut in g of T units of time. Then, in the time before the cut in g, the economy is governed by the AB-BC system with steady state at B: since τ is cut, the $\dot{c} = 0$ locus moves right, but the $\dot{k} = 0$ locus is unchanged since g is unchanged initially. If T is small, then continuity in T implies that the initial consumption level must be close to D, which is in the northwest sector of the AB-BC phase diagram where movement is northwesterly. Equation (10) implies that in equilibrium there are no jumps in c at $t = T$. Therefore, the system between $t = 0$ and $t = T$ must move from somewhere on the AD line segment to a point on DC. From this we may conclude that at $t = 0$, the economy jumps from A to a point between A and D, say E, and that it then moves northwesterly and hits a point on the line through DC at $t = T$. (The initial increase in consumption appears to be less, because of the positive lag. This is not necessarily the case, because if T were greater the necessary cut in spending would also be greater, pushing the $\dot{k} = 0$ locus upward.) For larger T, the economy may initially jump to a point such as F; but since it must be on DC at T, the economy must go through some phase of capital decumulation prior to the spending cuts since DC passes above A. The stable manifold around C may pass below A. In such an economy, the marginal propensity to save out of the tax cut would exceed unity if the tax and spending cuts were simultaneous, a feature generally considered implausible. However, we cannot rule it out, and in this phase diagram we cannot determine which case holds for plausible production and utility functions.

This example illustrates the basic principles of the model in a transparent graphic fashion but also shows that such graphic analysis is inconclusive even in a simple case. We shall return to this example in Section V below after developing the necessary analytical tools.

IV. Quantitative Analysis

While the graphic analysis above was instructive, it was inconclusive in determining qualitative features of the equilibrium and would always be incapable of answering questions concerning the quantitative importance of these effects. To answer such questions we must use analytical techniques. We will concentrate on analyzing a simple perturbation of a steady state, though the analysis can be easily adjusted when the initial condition is not the steady state. Suppose that the government has been taxing at constant rates τ_K and τ_L, granting an investment tax credit at a constant rate θ, and consuming goods at a constant rate $\bar{g}$, assessing a constant lump-sum tax of $\bar{l}$, and that the economy has reached the corresponding steady state, with bonds at that level consistent with budget balance. Next suppose that at $t = 0$ the government has announced that at $t \geqslant 0$, τ_K will be $\epsilon h_K(t)$ greater, τ_L will be $\epsilon h_L(t)$ greater, the lump-sum tax will be $\epsilon l(t)$ greater, the investment tax credit will be $\epsilon z(t)$ greater, and government consumption will be $\epsilon g(t)$ greater. To continue, it is necessary to make the following constancy assumption:

h_K, h_L, g, l, and z are all eventually constant functions of time.

This assumption is necessary to ensure the existence of a new steady state but is not an important limitation since the date of eventual constancy is arbitrarily distant.

For any fixed ϵ, equilibrium is the solution to the differential equations:

$$\dot{q} = q\left\{\rho - \frac{[1 - \tau_K - \epsilon h_K(t)]f'(k) + \delta[\theta + \epsilon z(t)]}{1 - \theta - \epsilon z(t)}\right\}, \tag{11a}$$

$$\dot{k} = f - c\left[\frac{q}{1 - \theta - \epsilon z(t)}\right] - [\bar{g} + \epsilon g(t)] \tag{11b}$$

with boundary conditions $|\lim_{t\to\infty} k(t)| < \infty$, $k(0) = k_0$. We shall denote the solutions $k(t, \epsilon)$ and $q(t, \epsilon)$, making explicit the dependence on ϵ. Since the economy is initially at the $\epsilon = 0$ steady state, the government announcement is essentially that ϵ has been increased. We would like to know the impact of this change in ϵ on the critical variables at future times; that is, we want to know the values of

$$\frac{\partial v}{\partial \epsilon}(t, 0) \equiv v_\epsilon(t), \ \frac{\partial}{\partial \epsilon}\left(\frac{\partial v}{\partial t}\right)(t, 0) \equiv \dot{v}_\epsilon(t), \quad v = k, q.$$

Since $u(c)$ and $f(k)$ are C^2, these derivatives exist (see Oniki 1973). Differentiation of the equilibrium system yields a linear differential equation in the variables k_ϵ, q_ϵ:

$$\begin{pmatrix} \dot{q}_\epsilon \\ \dot{k}_\epsilon \end{pmatrix} = \begin{bmatrix} 0 & \dfrac{-q(1 - \tau_K)}{1 - \theta} f'' \\ \dfrac{-c'}{1 - \theta} & f' \end{bmatrix} \begin{pmatrix} q_\epsilon \\ k_\epsilon \end{pmatrix} + \begin{Bmatrix} q\left[\dfrac{h_K f' - (\rho + \delta)z}{1 - \theta}\right] \\ \dfrac{-c'qz}{(1 - \theta)^2} - g(t) \end{Bmatrix}. \tag{12}$$

Since we are initially in a steady state, the matrix in (12) is actually a constant matrix, J, the Jacobian of the equilibrium differential equation. Therefore, the system in (12) is linear with constant coefficients and we can solve it with Laplace transforms. The Laplace transform of a function $f(t)$ defined for positive t is another function $F(s)$ defined for sufficiently large positive s, where $F(s) = \int_0^\infty e^{-st} f(t)dt$. Let $Q_\epsilon(s)$, $K_\epsilon(s)$, $H_K(s)$, $Z(s)$, and $G(s)$ be the Laplace transforms of $q_\epsilon(t)$, $k_\epsilon(t)$, h_K, z, and g, respectively. These Laplace transforms satisfy the Laplace transform of (12):

$$\begin{bmatrix} sQ_\epsilon(s) \\ sK_\epsilon(s) \end{bmatrix} = J \begin{bmatrix} Q_\epsilon(s) \\ K_\epsilon(s) \end{bmatrix} + \begin{Bmatrix} \dfrac{q}{1 - \theta}[H_K(s)f' - (\rho + \delta)Z(s)] + q_\epsilon(0) \\ -G(s) - \dfrac{c'qZ(s)}{(1 - \theta)^2} \end{Bmatrix}. \tag{13}$$

Solving for $Q_\epsilon(s)$ and $K_\epsilon(s)$ yields

$$\begin{bmatrix} Q_\epsilon(s) \\ K_\epsilon(s) \end{bmatrix} = (sI - J)^{-1} \begin{Bmatrix} \frac{q}{1-\theta}[H_K(s)f' - (\rho + \delta)Z(s)] + q_\epsilon(0) \\ -G(s) - \frac{c'qZ(s)}{(1-\theta)^2} \end{Bmatrix}. \quad (14)$$

We need to find the value of $q_\epsilon(0)$, the initial change in the marginal utility value of an extra unit of capital. This is tied down by invoking the stability condition. We know from stability that $q(t, \epsilon)$ and $k(t, \epsilon)$ are bounded in t for any fixed ϵ; we need to prove that $k_\epsilon(t, 0)$ and $q_\epsilon(t, 0)$ are also bounded. (The proof of lemma 1 is in the App.)

LEMMA 1. $k_\epsilon(t, 0)$ and $q_\epsilon(t, 0)$ are bounded in t.

Let μ, λ be the eigenvalues of J. They are given by the formula

$$\mu, \lambda = \frac{f'}{2}\left[1 \pm \sqrt{1 + \frac{4(1-\tau_K)\theta_L\theta_c}{\beta(1-\theta)\sigma\theta_K}}\right], \quad (15a)$$

where f' is the steady-state marginal product of capital, evaluated at the steady-state capital stock, k^{ss}, both defined by

$$f'(k^{ss}) = \frac{\rho(1-\theta) - \delta\theta}{1 - \tau_K}, \quad (15b)$$

and θ_c is the share of net output allocated to private consumption, c/f. Clearly, $\mu > 0 > \lambda$ if $\tau_K, \theta < 1$. If $(f' - \rho)/f'$, the net effective capital income tax rate, is positive, then $\mu > f' > \rho$. This fact will play a key role in understanding the short-run impacts of policy stocks. Lemma 1 implies that $K_\epsilon(s)$ is bounded for all $s > 0$. In particular, $K_\epsilon(\mu)$ is bounded, implying that the jump in the shadow value of capital at $t = 0$ is

$$\frac{q_\epsilon(0)}{q} = \frac{1}{1-\theta}[(\rho + \delta - \mu)Z(\mu) - H_K(\mu)f'] + \frac{\mu\beta}{c}G(\mu). \quad (16)$$

Combining (14) and (16), we have the solution for $K_\epsilon(s)$ and $Q_\epsilon(s)$. Having solved for the Laplace transforms of the adjustment paths of q and k, we can now use them to determine the impact of the shocks on economic variables and derive an expression for the government's dynamic budget constraint.

A. *Impact on Consumption and Investment at* t = 0

The solutions above determine the economy's response to a change in ϵ in terms of the Laplace transforms of the policy changes. However, it is possible to compute the values of k_ϵ and q_ϵ and their time derivatives at $t = 0$ without solving for the inverse Laplace transforms of K_ϵ and P_ϵ. The crucial fact about Laplace transforms is

$$f(0) = \lim_{s \to \infty} sF(s), \tag{17}$$

if $F(s)$ is the Laplace transform of $f(t)$.

THEOREM 1. The initial impact on investment of the announced changes is

$$\dot{k}_\epsilon(0) = \frac{c}{\beta(1 - \theta)}[z(0) + (\rho + \delta - \mu)Z(\mu) - f'H_K(\mu)] + \mu G(\mu) - g(0). \tag{18}$$

PROOF. Follows directly from (14), (16), (17), and L'Hospital's rule.

From the formula given in theorem 1 for the impact on investment, we can note several aspects of the relationship between fiscal policy and capital formation. First, an increase in government expenditure at $t = 0$, $g(0)$, causes a dollar for dollar decrease in capital formation. In a life-cycle model such as this one, a consumer endeavors to have a steady level of consumption; hence a momentary spurt in government consumption of $g(0)$ at $t = 0$ will be satisfied by less capital accumulation.

Second, the impact of future government consumption on capital formation is expressed in the term $\mu G(\mu)$, that is, discount the change in government spending at the rate μ and multiply the result by μ. To get some intuition for this, let us first examine a plausible but false procedure. One may have argued that the appropriate measure of future government consumption on investment would be $\rho G(\rho)$—take the discounted value of the expenditures, $G(\rho)$, as their capitalized value and note that a savings flow of $\rho G(\rho)$ would finance the expenditures at the existing real net rate of interest. This would be an individual's response if interest rates were unaffected. However, interest rates will respond to these policy changes. Equation (18) shows that this procedure is valid for general equilibrium calculations with the proper discount rate being μ, not ρ. This fact points out the importance of general equilibrium analysis versus partial equilibrium analysis, since the positive eigenvalue is generally much larger than the pure rate of time preference for realistic values of the crucial parameters. Since $\mu > \rho$, $\mu G(\mu)$ puts more weight on changes in government consumption in the near term relative to distant future changes than $\rho G(\rho)$ does; that is, the naive partial equilibrium approach overestimates the impact of government consumption in the distant future on investment today and underestimates the impact of such expenditures in the immediate future. In particular, we see that the anticipation effects of future policy changes decay rapidly relative to the utility discount rate as the date of the change becomes more distant.

One aspect of (18) may initially appear to be puzzling: an increase in future government consumption, with current government consumption held constant, encourages investment today. Since this term indicates the impact on investment today with the capital income tax rate held constant, the spending is implicitly being financed by lump-sum taxes. Because of the bond market, the timing of these lump-sum taxes is immaterial, but their existence is essential for the government to remain within its budget constraint. Therefore, with income taxes held constant, extra spending will cause $\mu G(\mu)$ to be positive, causing investment to increase because of the consumers' needs to finance the extra lump-sum taxes.

Third, the impact of future and present taxation on investment today is summed up in the first term. Again, note that the appropriate discount rate is μ, as expressed in $H_K(\mu)$. Again, since $\mu > \rho$, the anticipation effect of future taxes on current investment is much smaller than one may have expected. This expression has an interesting interpretation. $[\rho/(1 - \tau)]H_K(\mu)$ is the change in revenue discounted at μ if the capital stock does not change, expressed as a fraction of the capital stock. Hence the change in investment is this capitalization factor times consumption divided by the elasticity of marginal utility, yielding a decomposition of the change in investment into multiplicative factors representing consumption, curvature of utility, and the value of the tax change capitalized at μ. This expression for the impact on capital formation is useful for comparative dynamic analysis and highlights two important points. First, if β is large, the investment response to future tax changes is sluggish, since high curvature in the utility function indicates a desire for an even consumption stream and little taste for extreme changes in consumption to finance volatile investment plans. Second, investment today responds much more to tax changes today and in the near future than it does to more distant tax changes.

In examining the impact of the investment tax credit changes we see that the role of timing is more crucial, for $z(0)$, the extra tax credit today, plays an important role, as well as $Z(\mu)$. Clearly, as $z(0)$ increases, so does investment at $t = 0$. This is expected since $z(0)$ is the change in the initial subsidy to the initial investment. The impact of the rest of the tax credit on current investment is ambiguous. Future tax credit policy changes current investment by $(\rho + \delta - \mu)Z(\mu)c/\beta(1 - \theta)$. Even if $z(t) \geqslant 0$, the sign of this is ambiguous—positive for slow-adjusting economies, $\rho + \delta > \mu$, and negative for fast-adjusting economies, $\rho + \delta < \mu$. Fast-adjusting economies are associated with less concave utility functions. When faced with smaller future tax credits, such investors will invest more today to take advantage of the current short-lived tax credits, and when the tax credits are less generous in the future they will just as rapidly decumulate, treating today's tax credit as a subsidy to future consumption. For people with more concave utility functions, such fluctuations in consumption are disliked and future tax credits are an inducement for investment today, since more investment today leads to more depreciation in the future, the replacement of which is subsidized by future tax credits. This result differs from partial equilibrium analysis (e.g., Abel 1982), which argues that investment tax credits are generally stimulative whether they are permanent or temporary. These analyses do not take into account interest rate movements, ostensibly because the effects are trivial. Assuming that there would be no interest effects is odd in this context since investment tax credit policies are argued to have a macroeconomically significant impact on investment. We see that when we allow interest rate effects, the true general equilibrium result may be different from that indicated by the partial equilibrium analysis. Also reflected in (18) is the fact that whatever the impact of policy changes on investment today, that impact is magnified by the current investment tax credit.

B. *Balanced Budget Condition*

Next, we compute the relationship that must exist between the changes in taxation and expenditure due to the government's budget constraint. The differential equation governing bonds is

$$\begin{aligned}\dot{b} = \bar{g} + \epsilon g(t) + r_B(1 - \tau_K)b + [\theta + \epsilon z(t)](\delta k + \dot{k}) \\ - [\tau_K + \epsilon h_K(t)]kf'(k) - [\tau_L + \epsilon h_L(t)][f(k) - kf'(k)] - \bar{l} - l(t).\end{aligned} \tag{19}$$

The government's dynamic budget constraint requires the present value of its obligations and expenditures to equal the present value of its revenues, where the appropriate discount rate is the after-tax rate of return.[4] Differentiating that constraint with respect to ϵ, using the definition of r_B, equation (20), and the fact that $\dot{b}$ is zero in the initial steady state, we find theorem 2.

THEOREM 2. Budget balance implies the following constraint on the policy shocks:

$$\begin{aligned}0 = G(\rho) - \left[\frac{Q_\epsilon(\rho)}{q}\rho - \frac{q_\epsilon(0)}{q} + \rho Z(\rho) - z(0)\right]b \\ - \tau_K K_\epsilon(\rho)(f' + kf') - kf' H_K(\rho) + \tau_L kf'' K_\epsilon(\rho) \\ - H_L(\rho)(f - kf') - L(\rho) + \theta(\rho + \delta)K_\epsilon(\rho) + Z(\rho)\delta k,\end{aligned} \tag{20}$$

[4] This can be derived from the consumers' budget constraints and their transversality conditions, as in Brock and Turnovsky (1981).

where $H_L(s)$ and $L(s)$ are the Laplace transforms of h_L and l, respectively.

If b, the initial stock of bonds, is zero, (20) asserts extra revenue equals extra spending discounted at the rate ρ, the steady-state real net return. With $b > 0$, the real rate of interest that must be paid on bonds when they are rolled over changes, the net discounted value of the altered interest bill per unit of existing debt being the coefficient of b in (21). With a nontrivial term structure, this term would be different and would disappear if bonds were actually consols. In that case, the bearer may experience a capital gain or loss at $t = 0$.

V. Example: Cut Taxes, Then Spending

In this section we apply the quantitative techniques of Section IV to examine the impacts of the fiscal policy shock discussed in Section III. If taxes are cut immediately and spending cut later, the long-run effect is clear: increased capital formation and output. However, the short-run effects of this policy change on capital formation are not clear because the revenue losses are not matched by cuts in government expenditure. The resulting deficit must be financed by government bonds. Of course, in the long run the government's budget must be balanced, or more specifically, that must be the expectation if investors are to be willing to hold bonds today. That balancing can be accomplished by reducing government consumption, g, or decreasing lump-sum transfers to those who participate in the economy. To the extent that the budget will be balanced by reductions in transfers to workers and investors, the analysis is straightforward from the foregoing graphic analysis and equation (18): only the $\dot{c} = 0$ locus will be affected, and the economy will jump to the stable manifold associated with the new tax rate, converging monotonically to the new steady state where consumption, income, and the capital stock are all greater. Therefore, in this section we will initially address the case where the government's budget will be balanced by reductions in government consumption, g. The question we address is whether this unanticipated change in the financing and level of future government consumption will crowd out capital accumulation in the short run, contrary to the long-run increase in capital.

As in Section III, we assume that the taxes on both labor and capital incomes are equal to τ and the changes in these taxes are also identical. This is not meant to be a precise description of the U.S. economy or an exhaustive study of the short-run impact of this type of policy change. Such a study would need to include an elastic labor supply and costs of adjustment, at least. To do all this is beyond the scope and space of one paper. Our focus here is to illustrate how the analysis above can be applied to a particular issue and to demonstrate that these effects are not trivial in magnitude. The analysis will also indicate which parameters of taste and technology have a significant impact on the answers. The results of this section turn out to be largely unaffected by the initial level of bonds and the investment tax credit when they are assigned reasonable values, so both are set equal to zero.

The government decision to cut the tax rate immediately and reduce spending at some future date $T > 0$ can be modeled above by particular functional forms for g, h_K, and h_L:

$$h_K(t) = h_L(t) = -1 \tag{21a}$$

$$g(t) \equiv \begin{cases} 0, & t < T \\ -\gamma, & t \geq T \end{cases}, \tag{21b}$$

where γ, the magnitude of the future cut in g, is unknown a priori. The value of γ is determined by examining the balanced budget condition and is found to be

$$\gamma = f(k)e^{\rho T} \frac{1 - \dfrac{\tau}{1-\tau}\dfrac{\rho}{\rho-\lambda}\dfrac{\rho}{\beta\mu}}{1 - \dfrac{\tau}{1-\tau}\dfrac{\rho}{\rho-\lambda}\dfrac{\rho}{\rho-\mu}[e^{(\rho-\mu)T} - 1]} \equiv f(k)\tilde{\gamma}, \tag{22}$$

where $\tilde{\gamma}$ denotes the spending cut as a proportion of net national product.

One interesting index of this impact is the general equilibrium marginal propensity to save, that is, the portion of the extra disposable income at $t = 0$ saved by individuals in equilibrium, denoted by MPS. (This is to be distinguished from the individual marginal propensity to save out of current income.) It is equal to

$$MPS = \frac{\rho}{\beta\mu} - \tilde{\gamma}e^{-\mu T} + 1. \tag{23}$$

If $T > 0$, capital accumulation begins at $t = 0$ if and only if MPS exceeds unity, since only then are there any extra savings left after the deficit is financed. Standard differentiation exercises for MPS are tedious and inconclusive; furthermore, we really do not care about derivatives at all parameter values, just at reasonable ones, and we want some idea of the magnitudes involved. Therefore, table 1 lists values of MPS over a wide range of values for β, σ, T, and τ. The value of ρ is normalized to be 0.01, indicating that one period of time is that duration over which utility is discounted by 1 percent. To those who believe that the annual rate of discount is 4 percent, T equals the number of quarters between the tax cut and the spending cut. Casual

TABLE 1

VALUES OF *MPS*

		$\|\beta\|$					
σ	τ	.2	.5	1.0	2.0	5.0	10.0
.4	.2	.77	.47	.31	.21	.11	.07
		.87	.53	.36	.24	.13	.08
		1.19	.75	.52	.35	.20	.12
		1.43	.96	.68	.47	.27	.17
		1.74	1.44	1.29	1.19	1.08	.98
		(6.8)	(4.5)	(3.4)	(2.7)	(2.0)	(1.7)
	.4	.69	.42	.29	.20	.11	.07
		.81	.50	.34	.23	.13	.08
		1.16	.75	.53	.37	.21	.13
		1.39	.97	.72	.51	.31	.20
		1.63	1.37	1.24	1.16	1.08	1.03
		(8.0)	(5.4)	(4.1)	(3.2)	(2.5)	(2.1)
.7	.2	.99	.59	.39	.25	.13	.08
		1.07	.64	.43	.28	.15	.08
		1.32	.81	.54	.36	.19	.11
		1.54	.98	.67	.45	.24	.14
		1.94	1.55	1.36	1.22	1.05	.89
		(5.3)	(3.6)	(2.8)	(2.2)	(1.7)	(1.5)
	.4	.90	.55	.37	.28	.13	.08
		.98	.60	.41	.30	.15	.09
		1.26	.80	.55	.38	.20	.12
		1.48	.98	.70	.47	.27	.16
		1.80	1.46	1.30	1.20	1.08	1.00
		(6.2)	(4.3)	(3.4)	(2.5)	(2.2)	(2.0)
1.0	.2	1.16	.69	.45	.29	.15	.08
		1.23	.72	.48	.30	.15	.09
		1.44	.87	.57	.37	.19	.11
		1.64	1.01	.68	.44	.23	.13
		2.10	1.63	1.40	1.23	1.01	.82
		(4.5)	(3.2)	(2.5)	(2.0)	(1.6)	(1.5)
	.4	1.06	.64	.43	.28	.14	.08
		1.13	.69	.46	.30	.15	.09
		1.36	.84	.57	.38	.20	.11
		1.56	1.00	.70	.47	.25	.15
		1.93	1.53	1.33	1.20	1.07	.98
		(5.4)	(3.8)	(3.0)	(2.5)	(2.1)	(1.9)
1.3	.2	1.31	.77	.50	.31	.15	.09
		1.36	.80	.52	.32	.16	.09
		1.55	.92	.60	.38	.19	.11
		1.72	1.05	.69	.44	.22	.13
		2.23	1.70	1.43	1.23	.97	.74
		(4.1)	(2.9)	(2.3)	(1.9)	(1.5)	(1.4)
	.4	1.20	.72	.47	.31	.15	.09
		1.26	.76	.50	.32	.16	.09
		1.46	.89	.60	.38	.20	.11
		1.64	1.03	.70	.46	.24	.14
		2.03	1.58	1.36	1.21	1.06	.95
		(4.8)	(3.5)	(2.8)	(2.3)	(2.0)	(1.8)

NOTE.—Each column corresponding to a σ-τ-β entry is the *MPS* when taxes are cut immediately and spending cut with a lag of $T = 0, 2, 10, 20, 400$, followed by μ/ρ in parentheses.

examination of national income accounts suggests that we take capital share to be 0.25[5] and government consumption to be 0.2 of net production. These are reasonable values, especially since *MPS* is insensitive to reasonable changes in these parameters compared to its sensitivity to σ and β.

The elasticity of substitution, σ, has been estimated often with mixed results. We allow o to range between 0.4 and 1.3. This range includes some of the low estimates from time-series analysis, the higher cross-sectional estimates, and the reconciled estimates of Berndt (see Berndt [1976] for a general discussion; also Nerlove [1967]; Lucas [1969]).

The other major parameter is β. Two types of empirical analysis can be used to guide us in choosing an appropriate range. First, we may use the macroeconomic literature that argues for β between 0.5 and 6 (see Weber 1970, 1975; Grossman and Shiller 1981; Hansen and Singleton 1982, 1983). Second, the more disaggregated estimation of demand by Phlips (1978) also (ignoring the nonsensical result for "other services") implies a range of β from 0.5 to 6. We allow β to range between 0.2 and 10.0 in order to include at least part of the confidence intervals.

From table 1, we may conclude several things. First, the magnitudes of *MPS* indicate that the effects on savings at $t = 0$ of this policy shock are neither negligible nor unrealistic. They also indicate that for most values of the parameters, capital will begin to decumulate at $t = 0$ if there is a lag between tax cuts and spending cuts. Second, *MPS* increases as T increases. This has an intuitive explanation: as the spending cuts are pushed further into the future, their income effect on today's consumption decreases, resulting in less consumption and more savings today. Equations (22) and (23) also show this since $\tilde{\gamma}$ grows at the rate ρ as T increases but is discounted at the rate μ in the expression for *MPS*. Third, as β is less, that is, the utility function is less concave, *MPS* increases. This, too, is easily explained: a more linear utility function cares more about total consumption than about the smoothness of the consumption path; therefore, the price effect of the cheaper future goods dominates, depressing current consumption and increasing savings. Fourth, as the elasticity of substitution increases, savings out of the tax cut increases. This is because if σ is large, the marginal product of capital does not drop as rapidly during the accumulation of capital, resulting in a rate of interest that declines less rapidly. The impact of the initial tax rate is ambiguous but also not large. Finally, note that μ/ρ is substantially larger than one. There-

[5] This implies that our k excludes consumer durables, an appropriate assumption here since their services are not taxed.

fore, future tax and spending changes are discounted heavily in the computation of the initial impact on capital formation, equation (18).

Before ending the analysis of this policy shock we should discuss the case where the budget is eventually balanced by cutting consumption of public goods that are perfect substitutes for private goods. It is straightforward from (18), (22), (23), and the fact that $\mu > \rho$ that this case is equivalent to a tax cut with either no change in g or a change in g in the infinite future. For the parameter values we are examining, 400 periods is practically infinity since the positive eigenvalue substantially exceeds the pure rate of time preference. Therefore, table 1 tells us that when β exceeds 0.5, the *MPS* out of a dollar in tax cuts, financed eventually by increases in lump-sum taxes, is at most 1.5 and more likely about 1.2. Such balanced-budget changes in taxation and government expenditures therefore lead to capital accumulation immediately. However, note that the stimulus to capital formation due to these tax cuts, about 20–50 cents per dollar of tax cuts, is generally smaller than the capital decumulation from a dollar in tax cuts that will be balanced by a cut in g, especially if the cut in g is expected to occur in the near future. Hence, if β is not at the low end of the range, tax cuts financed by roughly equal increases in lump-sum taxes (or cuts in rebates), and cuts in government consumption, g, will depress investment since the capital decumulation induced by the latter will likely be the stronger influence on current investment.

VI. Conclusions

The primary accomplishment of this paper was the development of analytical tools for determining short-run consequences of fiscal policy in a perfect foresight model. These tools were applied to basic macroeconomic questions with strong results. We have seen that it is possible that a program of tax cuts today followed later by cuts in government consumption will initiate a period of nontrivial capital decumulation ending only when the spending cuts are initiated. We also found that it is unclear how future investment tax credits affect investment today, as they are stimulative for slow-adjusting economies and depressing to current investment in fast-adjusting economies.

The techniques used here are applicable to a wide variety of issues. For example, Judd (1981, 1983) uses them to analyze the excess burden of factor taxation in extensions of this model.

The major conclusion that follows from this analysis is that the long-run forces acting on an economy do matter in the short run in a quantitatively significant fashion. While conventional macroeconomics may be correct in arguing that other forces are important in the short run because of various rigidities, the results here show that the underlying long-run real forces cannot be ignored in short-run analysis. Just as significant, the analytical determination of these effects, taking into account the dynamic adjustment process, is a tractable exercise.

Appendix

Proof of Lemma 1

This is seen in two parts. Since the system is autonomous after T, then for all ϵ, $k(t, \epsilon)$ must be on the stable manifold for $t > T$. Theorem 5 of Otani (1982) (which applies here since our equilibrium solves some optimization problem; see Abel and Blanchard [1983]) shows that $k_\epsilon(t, 0)$ is bounded for $t > T$ for any finite value of $k_\epsilon(T, 0)$. Since k and q must be on the stable manifold of the asymptotic autonomous system at $t = T$, this stable manifold is the terminal surface when we view the problem for $t \in [0, T]$. Let $\tilde{k}(t, \epsilon; q_0)$ and $\tilde{q}(t, \epsilon; q_0)$ denote the solutions to (9) with $k(0, \epsilon; q_0) = k_0$ and $\tilde{q}(0, \epsilon; q_0) = q_0$. Then $(\partial\tilde{k}/\partial q_0(T, 0; q_0) > 0$ around the steady state under examination because of the local saddle-point nature of the flows. However, k decreases with q along any stable manifold. Hence, for small ϵ, there is a unique q_0^ϵ that causes $[\tilde{k}(T, \epsilon; q_0^\epsilon), \tilde{q}(T, \epsilon; q_0^\epsilon)]$ to be on the stable manifold of the system after T. Furthermore, due to the C^2 nature of the differential system, the dependence of q_0^ϵ and $\tilde{k}$ on ϵ is differentiable for all $t \in [0, T]$ (see Coddington and Levinson 1955). Therefore,

$$k_\epsilon(T, 0) = \frac{\partial \tilde{k}}{\partial \epsilon}(T, 0, q_0^0) + \frac{\partial k}{\partial q_0}(T, 0, q_0^0)\frac{\partial q_0^\epsilon}{\partial \epsilon}$$

is finite and $k_\epsilon(t, 0)$ is uniformly bounded in t.

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