Computing Equilibria of Repeated and Dynamic Games

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Introduction

- Repeated games have been used to model dynamic interactions in:
  - Industrial organization,
  - Principal-agent contracts,
  - Social insurance problems,
  - Political economy games,
  - Macroeconomic policy-making.
- These problems are difficult to analyze unless severe simplifying assumptions are made:
  - Equilibrium selection
  - Functional form (cost, technology, preferences)
  - Size of discounting

Goal

- The goal is to examine the *entire set* of (subgame perfect) equilibrium values in repeated and dynamic games with perfect monitoring
  - Propose a general algorithm for computation that can handle
    - large state spaces,
    - flexible functional forms,
    - any discounting,

Approach

- Computational method based on Abreu-Pearce-Stacchetti (APS) (1986,1990) set-valued techniques for repeated games.
- APS show that set of equilibrium payoffs is a fixed point of a *monotone* operator similar to Bellman operator in DP.
- APS method not directly implementable on a computer. Requires approximation of arbitrary sets.
- Need a computational procedure that
  - represents a set parsimoniously on a computer,
  - preserves the monotonicity of the underlying operator.

Contributions

- Develop a general algorithm that
  - computes equilibrium value sets of repeated and dynamic games
  - provides upper and lower bounds for equilibrium values and hence computational error bounds.
  - computes equilibrium strategies.
- Based on: Judd-Yeltekin-Conklin (2003), Sleet-Yeltekin(2003), Yeltekin-Judd (2009)

REPEATED GAMES

Stage Game

- A_i – player i's action space, $i = 1, \cdots, N$
- $A = \times_{i=1}^{N} A_i$ – action profiles
- $\Pi_i(a)$ – Player i payoff, $i = 1, \cdots, N$

Supergame

Supergame, S^{∞}:

- $\times_{i=1}^{\infty} A$ – action space
- player i's payoff.

$$(1-\delta)\,\Pi_i(a(1)) + \delta\left[(1-\delta)\sum_{t=2}^{\infty}\delta^{t-2}\Pi_i(a\,(t))\right].$$

Assumptions

- A1: A_i, $i = 1, \cdots, N$ is a compact subset of R^m for some m.
- A2: Π_i, $i = 1, \cdots, N$, is continuous.
- A3: The stage game has a pure strategy Nash equilibrium.

Define bounds on average discounted payoffs:

$$\underline{\Pi}_i \equiv \min_{a \in A} \Pi_i(a), \quad \bar{\Pi}_i \equiv \max \Pi_i(a)$$

Then

$$V \subset \mathcal{W} = \times_{i=1}^{N} [\underline{\Pi}_i , \overline{\Pi}_i]$$

where V is the entire set of SPE payoffs.

Example 1: Prisoner's Dilemma

- Static game: player 1 (2) chooses row (column)

4, 4	0, 6
6, 0	0, 0

- Static Nash equilibrium is (Down, Right) with payoff $(0, 0)$
- Suppose δ is close to 1
- S^{∞} includes (Up, Left) forever with payoff $(4, 4)$
  - This is rational if all believe that a deviation causes permanent reversion to (Down, Right)
  - This is just one of a continuum of equilibria.

Static Equilibrium

- Static game

$b_{11},\ c_{11}$	$b_{12},\ c_{12}$
$b_{21},\ c_{21}$	$b_{22},\ c_{22}$

  where b_{ij} (c_{ij}) is player 1's (2's) return if player 1 (2) plays i (j).

- Let V be the set of Nash equilibrium payoffs in the supergame, S^{∞}.

Supergame Equilibrium

In an equilibrium, each stage has the following form:

- $v(a)$: continuation value if a is equilibrium, $v : A \rightarrow V$
- a^*: the equilibrium action profile, is the equilibrium of the one shot game $(1-\delta)\pi(a) + \delta\, v(a)$.

Supergame Equilibrium: Recursive Formulation

Each stage of a subgame perfect equilibrium of S^∞ is a static equilibrium to some one-shot game which is A augmented by values from δV:

$\delta^* b_{11} + \delta u_{11},\ \delta^* c_{11} + \delta w_{11}$	$\delta^* b_{12} + \delta u_{12},\ \delta^* c_{12} + \delta w_{12}$
$\delta^* b_{21} + \delta u_{21},\ \delta^* c_{21} + \delta w_{21}$	$\delta^* b_{22} + \delta u_{22},\ \delta^* c_{22} + \delta w_{22}$

where $\delta^* = 1 - \delta$

Characterization of Equilibrium

- Key to finding V is construction of self-generating sets.
- The analysis focusses on the map B defined on convex W:

$$B^P(W) = \bigcup_{(a,w)\in A\times W} \{(1-\delta)\Pi(a) + \delta w \mid \forall i \in N(IC_i)\}$$
$$B(W) = co\left(B^P(W)\right)$$

  - $IC_i : (1-\delta)\Pi_i(a) + \delta w_i \geq (1-\delta)\Pi_i^*(a_{-i}) + \delta \underline{w}_i$
  - $\underline{w}_i \equiv \inf_{w\in W} \; w_i$
  - $co(\circ)$ is the convexification operator
- A set W is self-generating if $W \subseteq B^P(W)$.

Factorization

- A value b is in $B(W)$ iff
  - there is some action profile, a, and a random continuation payoff with expected value $w \in co(W)$, such that:
  - b is the value of playing a today and receiving an expected value w tomorrow
  - for each i, player i will choose to play a_i because to do otherwise will yield him the worst possible continuation payoff

Properties of B^P operator

- It can be shown that the B^P operator is
  - monotone
  - preserves compactness.
- We alter the supergame by including randomization. Use the modified operator B.

Fixed Point

Theorem

Let V be the set of all possible supergame payoffs. V satisfies

$$co(V) = B(co(V)) = \bigcup_{\substack{W \subset \mathcal{W} \\ co(W) \subset co(B(W))}} W = \bigcup_{\substack{W \subset \mathcal{W} \\ co(W) = co(B(W))}} W$$

Proof.

Cronshaw and Luenberger (1990). □

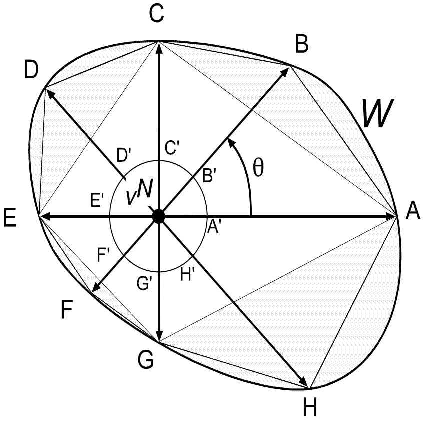
Computation

- V is a convex set
  - We need to approximate both V and the correspondence $B(W)$
  - We use different methods to accomplish different goals.

Piecewise-Linear **Inner** Approximation

- Suppose we have n points $Z = \{(x_1, y_1), ..., (x_n, y_n)\}$ on the boundary of a convex set W.
- The convex hull of Z, $co(Z)$, is contained in W and has a piecewise linear boundary.
- Since $co(Z) \subseteq W$, we will call $co(Z)$ the inner approximation to W generated by Z.

Inner approximation

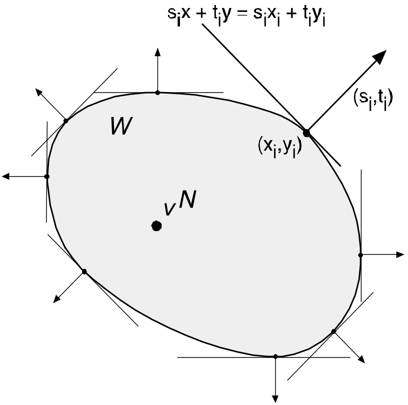


Inner approximations

Piecewise-Linear **Outer** Approximation

- Suppose we have
  - n points $Z = \{(x_1, y_1), ..., (x_n, y_n)\}$ on the boundary of W, and
  - corresponding set of subgradients, $R = \{(s_1, t_1), ..., (s_n, t_n)\}$;
- Therefore,
  - the plane $s_i x + t_i y = s_i x_i + t_i y_i$ is tangent to W at (x_i, y_i), and
  - the vector (s_i, t_i) with base at (x_i, y_i) points away from W.

Outer approximation



A convex set and supporting hyperplanes

Key Properties of Approximations

Definition

Let $B^I(W)$ be an inner approximation of $B(W)$ and $B^O(W)$ be an outer approximation of $B(W)$; that is $B^I(W) \subseteq B(W) \subseteq B^O(W)$.

Lemma

Next, for any $B^I(W)$ and $B^O(W)$, (i) $W \subseteq W'$ implies $B^I(W) \subseteq B^I(W')$, and (ii) $W \subseteq W'$ implies $B^O(W) \subseteq B^O(W')$.

Fixed Point

These results together with the monotonicity of the B operator, implies the following theorem.

Theorem

Let V be the equilibrium value set. Then (i) if $W_0 \supseteq V$ then $B^O(W_0) \supseteq B^O(B^O(W_0)) \supseteq \cdots \supseteq V$, and (ii) if $W_0 \subset B^I(W_0)$ then $B^I(W_0) \subset B^I(B^I(W_0)) \subseteq \cdots \subseteq V$. Furthermore, any fixed point of $B^\bullet$ is contained in the maximal fixed point of B, which in turn is contained in the maximal fixed point of B^O.

Sufficient Condition: Self-Generation

The following property of the B operator provides a way to verify that a set W contains equilibria.

Theorem

If $B^O(W) \supseteq W$ then $W \subseteq V$.

Monotone Inner Hyperplane Approximation

Input: Vertices $Z = \{z_1, \cdots, z_M\}$ such that $W = co(Z)$.

Step 1: Find extremal points of $B(W)$:

For each search subgradient $h_\ell \in H$, $\ell = 1, .., L$.

(1) For each $a \in A$, solve the linear program

$$
\begin{aligned}
c_\ell(a) \quad = \quad & \max_w \; h_\ell \cdot [(1-\delta)\Pi(a) + \delta w] \\
& \text{(i) } w \in W \\
& \text{(ii) } (1-\delta)\Pi^i(a) + \delta w_i \geq \\
& \qquad (1-\delta)\Pi_i^*(a_{-i}) + \delta \underline{w}_i, \;\; i = 1, .., N
\end{aligned} \tag{1}
$$

Let $w_\ell(a)$ be a w value which solves (1).

Monotone Inner Hyperplane Approximation cont'd

(2) Find best action profile $a \in A$ and continuation value:

$$\begin{aligned} a_\ell^* &= \arg\max \{c_\ell(a) | a \in A\} \\ z_\ell^+ &= (1-\delta)\Pi(a_\ell^*) + \delta w_\ell(a_\ell^*) \end{aligned}$$

Step 2: Collect set of vertices $Z^+ = \{z_\ell^+ | \ell = 1, ..., L\}$, and define $W^+ = co(Z^+)$.

The Outer Approximation, Hyperplane Algorithm

- Definition of a set:

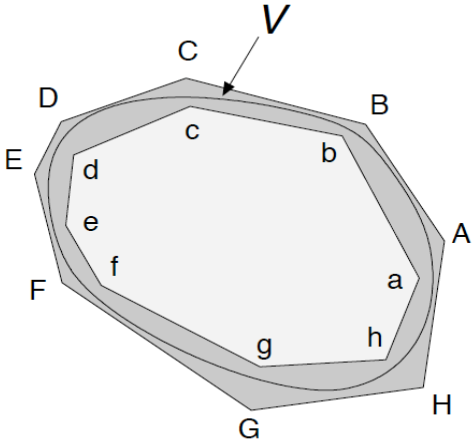
$$
\begin{aligned}
n \in N &= \text{state of normals} \\
v \in V &= \text{list of vertices} \\
W &= \{w \,|\, n \cdot w \leq n \cdot v, \forall n \in N\} \\
\underline{w}_i &= \min_{w \in W} w_i
\end{aligned}
$$

- Outer approximation: Same as inner approximation except record normals and continuation values z_ℓ^+

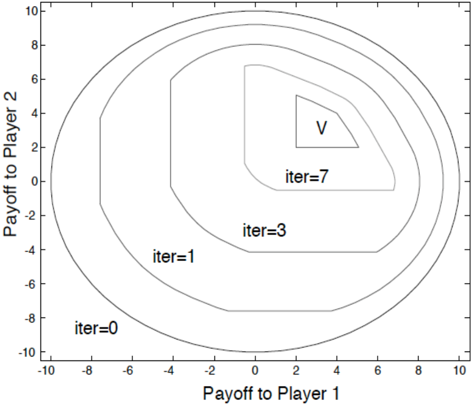
Outer vs. Inner Approximations

- Any equilibrium is in the inner approximation
  - Can construct an equilibrium strategy from V.
  - There exist multiple such strategies
- No point outside of outer approximation can be an equilibrium
  - Can demonstrate certain equilibrium payoffs and actions are not possible
  - E.g., can prove that joint profit maximization is not possible
- Difference between inner and outer approximations is approximation error
- Computations actually constitute a proof that something is in or out of equilibrium payoff set - not just an approximation.
- Difference is small in many examples.

Error Bounds



Convergence



DYNAMIC GAMES

Goal

- Provide an algorithm for computing all equilibrium payoffs and strategies for dynamic games.
- Method covers a large class of dynamic games in IO, macro, public finance
- Method provides:
  - two approximations that together provide error bounds,
  - equilibrium strategies.

A specific example: Dynamic Oligopoly

Oligopoly game with endogenous productive capacity.

- Study the nature of dynamic competition and its evolution.
- Study the nature of cooperation and competition.
- Specifically:
  - Is ability to collude affected by state variables?
  - Do investment decisions increase gains from cooperation?
  - Does investment present opportunities to deviate from collusive agreements?

Existing Literature in Dynamic Oligopoly

Existing literature in IO

- Two stage games
  - Firms choose capacities in stage one, prices in stage two
  - Kreps-Scheinkman (1983), Davidson-Deneckere (1986)
- Dynamic games
  - Firms choose capacities and prices
  - Benoit-Krishna (1987), Davidson-Deneckere (1990)

Goals revisited

- Limiting assumptions in previous work
  - Capacity chosen at t=0 , OR
  - No disinvestment, OR
  - Examine only equilibria supported by Nash reversion, OR
  - Restrictive functional forms for demand and cost functions
- **Our goal**: Examine full set of pure strategy Nash equilibria for dynamic games with arbitrary cost and demand functions.

Stage Game

- Action space for player i: A_i, $i = 1, ..., N$
- Action profiles: $A = A_1 \times A_2 \times \cdot \times A_N$
- State space: $X = \cup_{k=1}^{K}\{X_k\}$

Assumptions

Assumption 1: A_i, $i = 1, \cdot, N$,compact subset of $\Re^m$.

Assumption 2: $\Pi_i(., x)$, $i = 1, \cdot N$ is continuous.

Assumption 3: The game has a pure Nash equilibrium.

Supergame

- Strategy profile for supergame: $A^{\infty} \equiv \times_{t=1}^{\infty} A^t$
- Preferences:
$$\frac{1-\delta}{\delta} \Sigma_{t=1}^{\infty} \delta^t \Pi_i(a_t, x_t).$$
- Histories h^t:
$$h^t \equiv \{a_s, x_s\}_{s=0}^{t}$$
- Minimal and maximal payoffs:
$$\underline{\Pi}_i \equiv \min_{(a,x) \in AxX} \Pi_i(a, x)$$
$$\overline{\Pi}_i \equiv \max_{(a,x) \in AxX} \Pi_i(a, x)$$

Equilibrium

- In the dynamic case the object of interest is a correspondence that maps a physical state variable to sets of equilibrium payoffs.
- Subgame perfect equilibrium (SPE) payoffs:
  - Initial state x, strategy profile $\sigma \in A^\infty$, payoff $v(x,\sigma)$

$$v(x,\sigma) \in V_x \subset \mathcal{W}, \;\; x \in X$$

where $\mathcal{W} = \times_{i=1}^{N} [\underline{\Pi}_i, \overline{\Pi}_i]$

- Equilibrium Value Correspondence:

$$V \equiv \{V_{x_1}, ..., V_{x_K}\} \subseteq \mathcal{W}^K \subseteq \{\Re^N\}^K$$

Steps: Computing the Equilibrium Value Correspondence

1. Define an operator that maps today's equilibrium values to tomorrow's at each state.
2. Show that this operator is monotone and the equilibrium correspondence is its largest fixed point.
3. Define an appropriately chosen initial correspondence, apply the monotone operator until convergence.
4. Additional complexity:
   - Representing correspondence parsimoniously on computer
   - Preserving monotonicity of operator

Set Valued Dynamic Programming

D map:

- Let $W \subseteq \mathcal{W}^K$
- $D(W)_x$: set of possible payoffs consistent with Nash play in state x today and continuation values from W

$$D(W)_x = \cup_{(a,x',w)}\{(1-\delta)\Pi(a,x) + \delta w\}$$

subject to:

$$\begin{aligned} w &\in co(W_{x'}) \\ x' &= g(a,x) \end{aligned}$$

and for each $\forall i \in N$, $\forall \tilde{a}_i$

$$(1-\delta)\Pi_i(a,x) + \delta w_i \geq \Pi_i(\tilde{a}_i, a_{-i}, x) + \delta \tilde{w}_{i,g(\tilde{a}_i,a_{-i},x)}$$

where $\tilde{w}_{i,x} = \min_i W_x$.

Self-generation

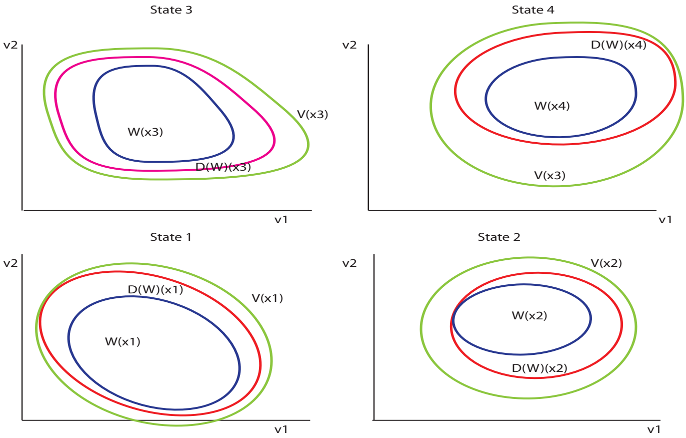
A correspondence W is self-generating if :

$$\text{Graph}\, W \subseteq \text{Graph}\, D(W).$$

An extension of the arguments in APS establishes the following:

- Graph of any self-generating correspondence is contained within $Graph(V)$,
- V itself is self-generating.
- V is a fixed point of operator D. It is the largest fixed point in $\mathcal{W}^K$.

Self-generation visually

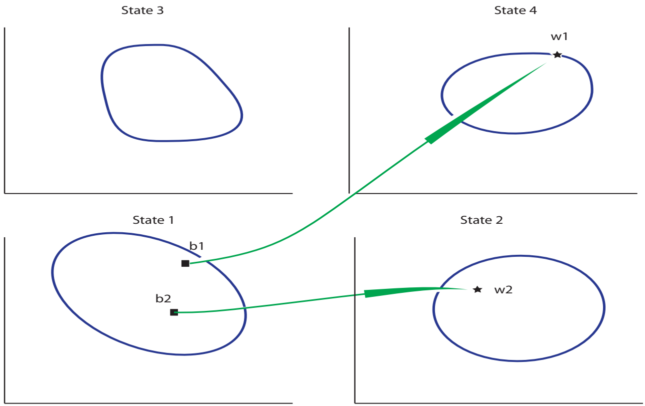


Factorization

$b \in D(W)_x$ if there is an action profile a and continuation payoff $w \in co(W_{x'})$, s.t

- b is value of playing a today in state x and receiving continuation value w ,
- for each i, player i will choose to play a_i
- $x\prime = g(a, x)$ if no defection
- $\tilde{x} = g(\tilde{a}_i, a_{-i}, x)$ if defection.
- punishment value drawn from set $W_{\tilde{x}}$.

Factorization



Fixed Point: Equilibrium Value Correspondence

Factorization and self-generation imply that:

1) V is the maximal fixed point of the mapping D;

2) V can be obtained by repeatedly applying D to any set that contains graph of V.

Dynamic Cournot Duopoly with Capacity Investment

- Classic Cournot duopoly game with endogenous capital.
- Firms can invest in capital to relax a capacity constraint.
- Two cases:
  - Reversible Investment: Market for resale.
  - Irreversible Investment: No market for resale.

Environment: Dynamic Cournot with Capacity

- Firm i has sales of $q_i \in Q_i(k_i)$, and unit cost c_i.
- MC= maintenance cost of machine
- SP= resale/scrap value of machine
- FC =cost of a new machine
- Cost of capital maintenance and investment:

$$C(k_i, k_i') = \begin{cases} MC * (k_i - 1) + FC * (k_i' - k_i) & \text{if } k_i' \geq k_i \\ MC * (k_i - 1) - SP * (k_i - k_i') & \text{if } k_i' \leq k_i \end{cases}$$

Profit: Dynamic Cournot with Capacity

- Firm i's current profits:

$$\Pi_i(q_1, q_2, k_i, k_i') = q_i(p(q_1, q_2) - c) - C(k_i, k_i')$$

- Linear demand curve:

$$p(q_1, q_2) \ = \ \max\left\{a - b(q_1 + q_2), 0\right\}.$$

Stage Game: Dynamic Cournot with Capacity

- Action Space:
  - sets of outputs
  - sets of capital stocks
- State Space:
  - set of feasible capital stocks
- $A_i = Q_i \times K_i$
- $X = K_1 \times K_2$

Dynamic Strategies and Payoffs

- Strategies: collection of functions that map from histories of outputs and capital stocks into current output and capital choices.
- Maximize average discounted profits.

$$\frac{(1-\delta)}{\delta}\sum_{t=0}^{t=\infty}\delta^t \Pi_{i,t}(q_1, q_2, k_i, k_i')$$

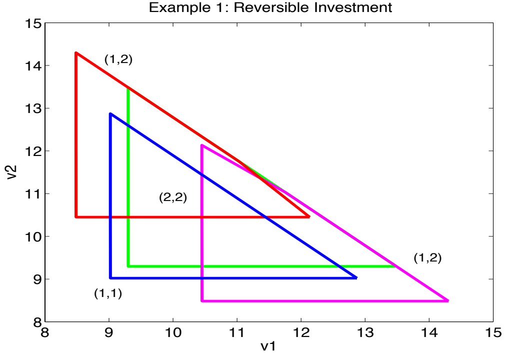
Dynamic Duopoly: Example 1

- Finite action version of the dynamic duopoly game.
- Discretize action space over q_i and k_i
- Full capacity: 16 actions from interval $[0, \bar{Q}]$
- Partial capacity: 8 actions from interval $[0, \bar{Q}/2]$
- Firms endowed with 1 machine each.
- 4 states: $(k_1, k_2) \in \{(1,1),(1,2),(2,1),(2,2)\}$
- 48 hyperplanes for the approximation.

Example 1: Reversible Investment

Parameters: MC =SP=1.5, FC =2.5, $\delta = 0.8$, $\bar{Q} = 6.0$ $c = 0.6$, $b = 0.3$, $a = 6.0$

$p(q_1, q_2) \; = \; \max\{a - b(q_1 + q_2), 0\}$.



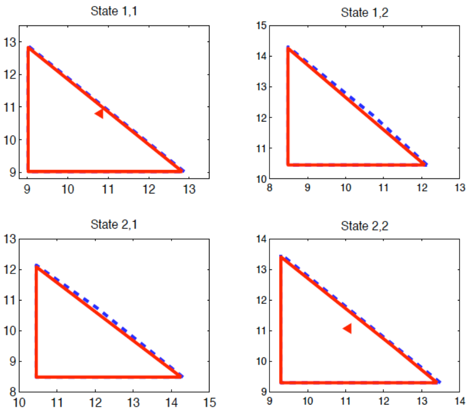
Outer and Inner Approximations, Error Bounds

- Outer approximation : Start with W s.t. $D(W) \subseteq W$
- Inner approximation: Start with W s.t. $W \subseteq D(W)$
- Any v in inner is an equilibrium value. Any v outside inner is NOT an equilibrium value.
- Error bound: Difference between inner and outer approximations.

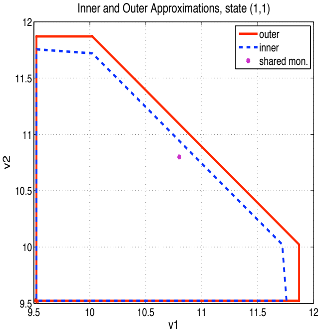
Example 1: Inner and Outer Approximations, N=48

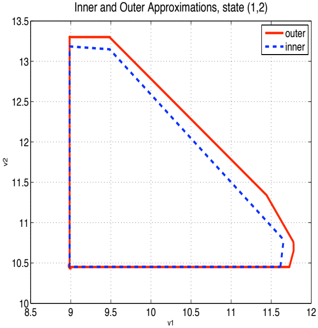
Parameters: MC =SP=1.5, FC =2.5, $\delta = 0.8$, $\bar{Q} = 6.0$ $c = 0.6$, $b = 0.3$, $a = 6.0$

$p(q_1, q_2) = \max\{a - b(q_1 + q_2), 0\}$.

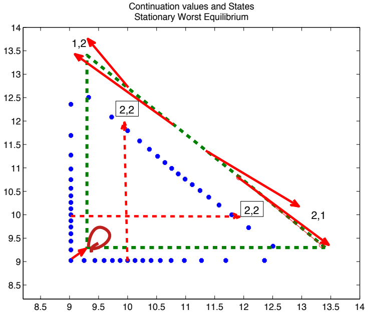


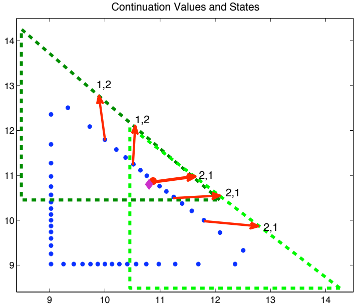
Example 1: Error Bounds, with N=24





Fluctuating Market Power

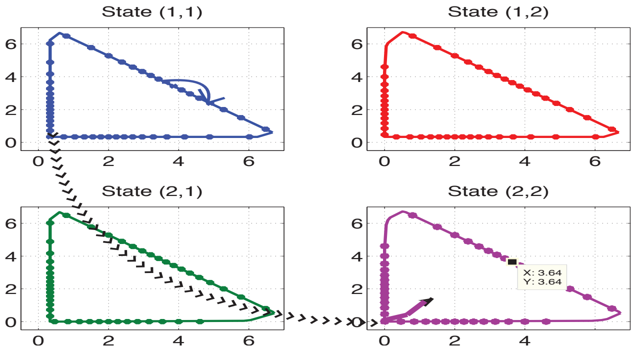




Strategies: Fluctuating Market Power

- Firms can do better than *symmetric* Nash collusion.
- Frontier of equilibrium value sets supported by
  - continuation play where firms alternate having market power.
- Worst equilibrium payoffs
  - firms produce at full capacity in current period
  - over-investment and over-production thereafter (symmetric cases).

Example 2: Striving for Cooperation



MC =SP=1.5, FC =2.5, $\delta = 0.8$, $\bar{Q} = 6.0$ $c = 0.6$, **b=1.0**, $a = 6.0$.

Parameters:

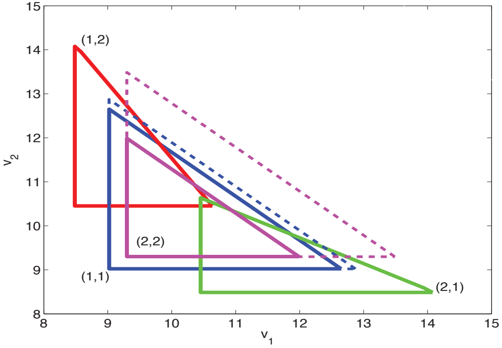
Striving for Cooperation

- *Symmetric* Nash collusion payoffs on the frontier.
- Frontier of equilibrium value set for all cases supported by
  - continuation play where firms each have 1 machine and produce below capacity.
- Worst equilibrium payoffs
  - firms over-produce in current period
  - over-investment and over-production for a limited period.
  - firms move towards Pareto-frontier after a punishment phase.

Example 3: Irreversibility of investment and over-investment

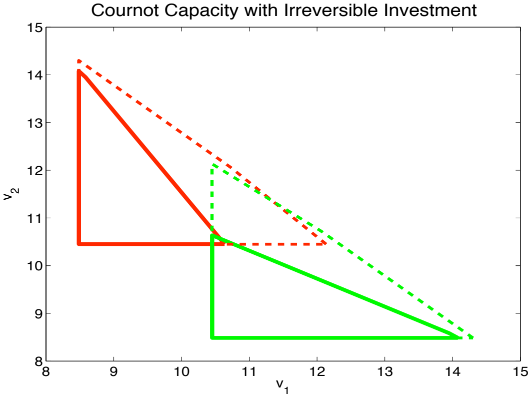
Parameters: MC =1.5, FC =2.5, $\delta = 0.8$, $\bar{Q} = 6.0$ $c = 0.6$, **b=1.0**, $a = 6.0$
$p(q_1, q_2) = \max\{a - b(q_1 + q_2), 0\}$.

Cournot Capacity with Irreversible Investment



Irreversibility of Investment and over-investment

Parameters: MC =1.5, FC =2.5, $\delta = 0.8$, $\bar{Q} = 6.0$ $c = 0.6$, **b=1.0**, $a = 6.0$
$p(q_1, q_2) \;=\; \max\{a - b(q_1 + q_2), 0\}$.



Irreversibility of investment and over-investment

- Worst equilibrium payoff at states $(1, 1)$ and $(2, 2)$
  - firms produce at full capacity in current period
  - over-investment and over-production thereafter.
- Worst equilibrium payoff at states $(1, 2)$ and $(2, 1)$
  - firms produce at full capacity in current period
  - over-investment and over-production thereafter.

Summary

- Computation of equilibrium value correspondence reveals
  - dynamic interaction and competition missed by simplifying assumptions
  - rich set of equilibrium outcomes that involve
    - fluctuating market power
    - over-investment and over-production when cooperation breaks down
    - phase of cooperation after a phase of uncooperative behavior
    - equilibria with current profit of leading firm less than smaller firm
    - under-utilization of capacity followed by phase of full capacity production

Extensions

- Method and algorithm suitable for
  - Larger state space
  - Flexible cost and demand functions
  - Any discounting
  - Multiple firms
  - Flexible informational assumptions

Extensions

- Strategy space can be expanded for other applications:
    - Multiproduct firms
    - Advertising
    - Learning curves
    - Spatial competition
- With this algorithm, we can quantitatively examine many important issues.
    - Determinants of the ability to cooperate
    - Impact of antitrust provisions
    - Effects of institutional arrangements
    - Importance of information asymmetry

Algorithm: Inputs

1. **Subgradients:** Set of subgradients (normals),

$$R_k^W = \{(s_{k,1}, t_{k,1}), \ldots, (s_{k,n}, t_{k,n})\}$$

2. **Levels:** Boundary points for each state k:

$$Z_k^0 = \{(x_{k,l}^0, y_{k,l}^0), \cdots, (x_{k,n}^0, y_{k,n}^0)\}.$$

3. **Hyperplanes:** Define $c_{k,l}^0 = s_{k,l}x_{k,l}^0 + t_{k,l}y_{k,l}^0$ and

$$W_k^0 = \cap_{l=1}^n \{(x_{k,l}, y_{k,l}) \mid s_{k,l}x_{k,l} + t_{k,l}y_{k,l} \leq c_{k,l}^0).$$

4. **Search subgradients:** $B_k^W = \{(r_{k,1}, p_{k,1}), \ldots, (r_{k,m}, p_{k,m})\}$

Algorithm: New Value-Set Vector

For each $k \in K$ and each $(r_k, p_k)) \in B_k^W$:

1. For each action profile $(a_i, a_j) \in A \times A$:

$$\hat{c}_{k,l}(a_i, a_j, k) = (r_l, p_l) \cdot [(1-\delta)\Pi(a_i, a_j, k) + \delta w]$$
$$(i)\ w \in co(W_{g(a,k)})$$
$$(ii) \forall i \in N, \forall \tilde{a}_i,\ (1-\delta)\Pi_i(a_i, a_j, k) + \delta w_i$$
$$\geq (1-\delta)\Pi_i(\tilde{a}_i, a_{-i}, k) + \delta \tilde{w}_{i,g(\tilde{a}_i, a_{-i}, k)}$$

2. Compute value of best action profile

$$c_{k,l}^{+} = \max_{a_i, a_j} \{c_{k,l}(a_i, a_j, k) | (a_i, a_j) \in A \times A,\ k \in K\}$$

Algorithm: New Value-Set Vector

3 New $\{W_k\}$ sets are

$$W_k^+ = \cap_{l=1}^{n}\{(x_{k,l}, y_{k,l}) \mid s_{k,l}x_{k,l} + t_{k,l}y_{k,l} \leq c_{k,l}^+) \quad \textbf{outer approx.}$$

Extra References

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