Exploring Dynamic Equilibrium Behavior with Interacting Drivers of Behavior: The Case of Learning-by-Doing and Organizational Forgetting

Based on David Besanko, Ulrich Doraszelski, Yaroslav Kryukov, and Mark A. Satterthwaite, "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics," *Econometrica*, 2010

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Learning-by-Doing

- Learning-by-Doing (LBD)
  The out-of-pocket cost for Big Boat Ship Building to construct a new, series X barge is:

Barge #	1	2	3	4	5	6
Cost	20	18	16	14	12	10.5

- Big Boat has signed, fixed price contracts to deliver two barges in 2008 and two in 2010. A buyer approaches the yard and offers $15 for delivery of a barge in 2009.
- Should Big Boat accept?
- Correct calculation of marginal cost says yes.
- Increasing, increasing dominance in equilibrium if discount factor is near 1. Both firms, however, inevitably progress down the learning curve.
- Spence (*Bell Journal of Econ. & Management Sci.*, 1981), Cabral & Riordan (*Econometrica, 1994)*

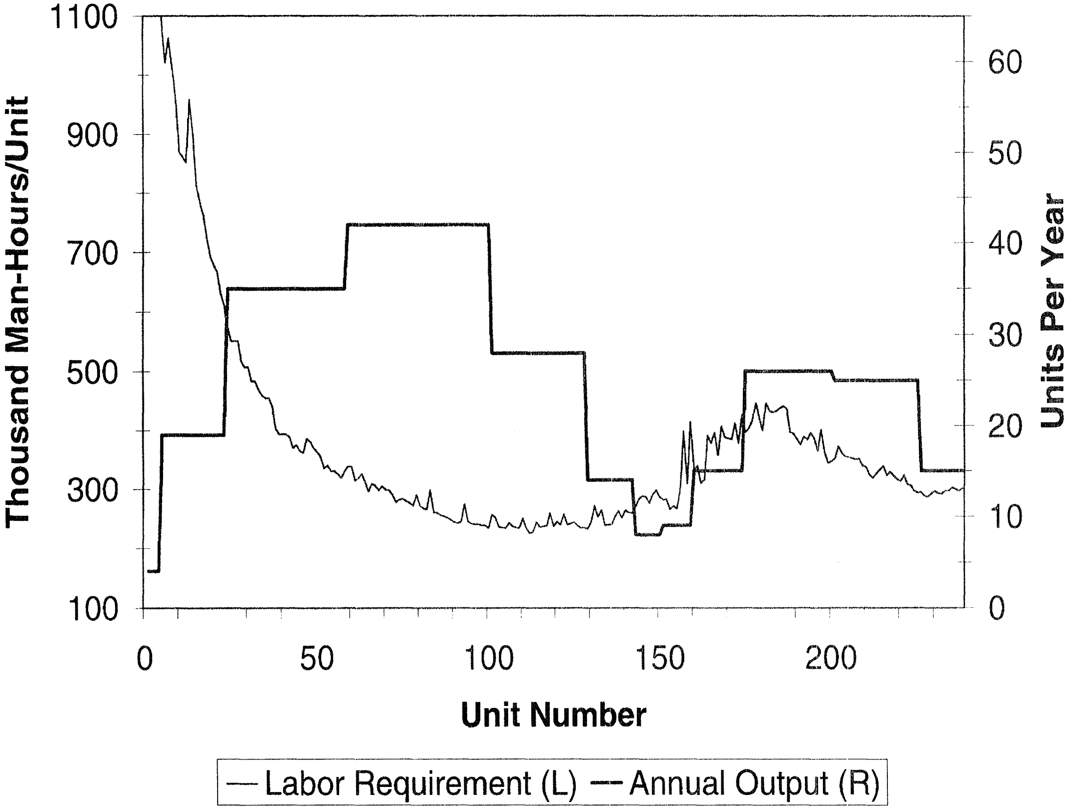


FIGURE 1. L-1011 PRODUCTION: DIRECT LABOR REQUIREMENT AND YEARLY OUTPUT

Organizational Forgetting

- The cost history of the Lockheed L1011 widebody passenger jet
- Aircraft manufacturing is assembly intensive. Workers learn clever, more efficeint strategies for putting planes together with practice.
- Worker forget these strategies when:
  - design changes occur at behest of individual airlines,
  - fluctuations of orders cause workers to be laid off and rehire, if at all, after a period of time,
  - union rules allow worker to change roles on the basis of seniority when vacancies occur.
- Argote, Beckman, & Epple (*Management Science*,1990), Benkard (*AER*, 2000), Benkard (*REStud*, 2004)

Plan of Talk

- The issue: Learning-by doing and organizational forgetting appear to interact. How can we understand how the two *together affect* equilibrium behavior?
- Model of dynamic competition with learning and forgetting
- Representation of Results
    - Description of equilibria
    - Seeing multiplicity
- Logic of the different equilibria
- Discovering multiplicity
    - Failure of Pakes-McGuire algorithm when multiplicity exists
    - Homotopy approach to finding equilibria
    - Tracing out the full equilibrium manifold
- Summary

Setup and Timing

- Discrete time, infinite horizon.
- Two firms with potentially different stocks of know-how $(e_1, e_2) \in \{1, \ldots, M\}^2$.
- In each period, the timing is as follows:
  - Firms choose prices.
  - One buyer enters the market and makes a purchase.
  - Learning-by-doing and organizational forgetting occur and the firms' stocks of know-how change accordingly.
- Law of motion:

$$e'_n = e_n + q_n - f_n,$$

  where

  - $q_n \in \{0, 1\}$ indicates whether firm n makes a sale;
  - $f_n \in \{0, 1\}$ represents organizational forgetting.

Learning-by-Doing

- Marginal cost of production:

$$c(e_n) = \begin{cases} \kappa e_n^{\eta} & \text{if} \quad 1 \leq e_n < m, \\ \kappa m^{\eta} & \text{if} \quad m \leq e_n \leq M, \end{cases}$$

  where

  - $\eta = \log_2 \rho$ for a progress ratio of $\rho \in (0, 1]$;
  - κ is marginal cost at top of learning curve;
  - m is bottom of learning curve.

- Marginal cost decreases by $1 - \rho$ percent as the stock of know-how doubles.

Organizational Forgetting

- Probability of losing a unit of know-how:

$$\Pr(f_n = 1) = \Delta(e_n) = 1 - (1-\delta)^{e_n},$$

  where $\delta \in [0,1]$ is the forgetting rate.
- $\Delta(e_n)$ is increasing in e_n in line
  - with experimental evidence in management literature;
  - Jost's second law in psychology literature;
  - capital-stock models.
- Cabral & Riordan (1994) analyze the special case of $\delta = 0$.

Demand

- In each period, one buyer enters the market and makes a purchase.
- A buyer's idiosyncratic preferences are unobservable to firms.
- Demand is logit. Thus probability of making a sale is:

$$\Pr(q_n = 1) = D_n(p_1, p_2) = \frac{1}{1 + \exp\left(\frac{p_n - p_{-n}}{\sigma}\right)},$$

where σ is degree of horizontal product differentiation.

Bellman Equation

- $V_n(\mathbf{e})$ is the expected NPV to firm n of being in the industry given that the industry is in state $\mathbf{e} = (e_1, e_2)$.
- Bellman equation:

$$V_n(\mathbf{e}) = \max_{p_n} D_n(p_n, p_{-n}(\mathbf{e}))(p_n - c(e_n)) + \beta \sum_{k=1}^{2} D_k(p_n, p_{-n}(\mathbf{e}))\overline{V}_{nk}(\mathbf{e}),$$

  where

  - $p_{-n}(\mathbf{e})$ is the price charged by the other firm;
  - $\beta \in (0, 1)$ is the discount factor;
  - $\overline{V}_{nk}(\mathbf{e})$ is the expectation of firm n's value function conditional on buyer purchasing from firm $k \in \{1, 2\}$.
- $p_n(\mathbf{e})$ is uniquely determined by FOC.

Equilibrium

- Symmetric Markov perfect equilibrium (MPE):
  - Value function $V_1^*(\mathbf{e}) = V^*(\mathbf{e})$ and $V_2^*(\mathbf{e}) = V^*(\mathbf{e}^{[2]})$ where $\mathbf{e}^{[2]}$ denotes the vector (e_2, e_1) constructed by interchanging the stocks of know-how of firms 1 and 2.
  - Policy function $p_1^*(\mathbf{e}) = p^*(\mathbf{e})$ and $p_2^*(\mathbf{e}) = p^*(\mathbf{e}^{[2]})$.
- The Bellman equation and FOC for state $\mathbf{e}$ are

$$V^*(\mathbf{e}) = D_1^*(\mathbf{e})\left(p^*(\mathbf{e}) - c(e_1)\right) + \beta \sum_{k=1}^{2} D_k^*(\mathbf{e})\overline{V}_k^*(\mathbf{e}),$$

$$0 = \sigma - \left(1 - D_1^*(\mathbf{e})\right)\left(p^*(\mathbf{e}) - c(e_1)\right) - \beta \overline{V}_1^*(\mathbf{e})$$
$$+\beta \sum_{k=1}^{2} D_k^*(\mathbf{e})\overline{V}_k^*(\mathbf{e}).$$

  This system of $2M^2$ nonlinear equations, two for each state $\mathbf{e} \in \{1, \ldots, M\}^2$, defines a symmetric equilibrium.
- Existence in pure strategies is guaranteed, uniqueness is not.

Parameterization

- We explore equilibria for the full range of progress ratios $\rho \in (0, 1]$ and forgetting rates $\delta \in [0, 1]$.
- Empirical estimates: $\rho \in [0.7, 0.95]$ and $\delta < 0.1$.
- Remaining parameters:

parameter	M	m	κ	σ	β
value	30	15	10	1	$\frac{1}{1.05}$

- If $\rho = 0.85$, then $c(1) = 10$, $c(2) = 8.50$, and $c(15) = \ldots = c(30) = 5.30$.
- If $\rho = 0.85$, then in the static Nash equilibrium:
  - the own-price elasticity of demand is -8.86 in state $(1, 15)$ and -2.13 in state $(15, 1)$, and
  - the cross-price elasticity of firm 1's demand with respect to firm 2's price is 2.41 in state $(15, 1)$ and 7.84 in state $(1, 15)$.

Four Typical Equilibria

- Flat:
  - Firms price near their long run marginal cost from the beginning
  - Firms do not seek to preempt each other.
- Well:
  - Preemption battles fought by firms at the top of their learning curves.
  - Serve to build a competitive advantage $\longrightarrow$ transitory advantage.
- Diagonal trench:
  - Price wars fought by fairly symmetric firms.
  - Serve to build and defend a competitive advantage $\longrightarrow$ permanent advantage.
- Sideways trenches:
  - Price wars fought by fairly asymmetric firms.
  - Serve to build and defend a competitive advantage $\longrightarrow$ permanent advantage.
- *Note that representing in an understandable way the pricing policies of the firms is greatly facilitated by the industry being a duopoly.*

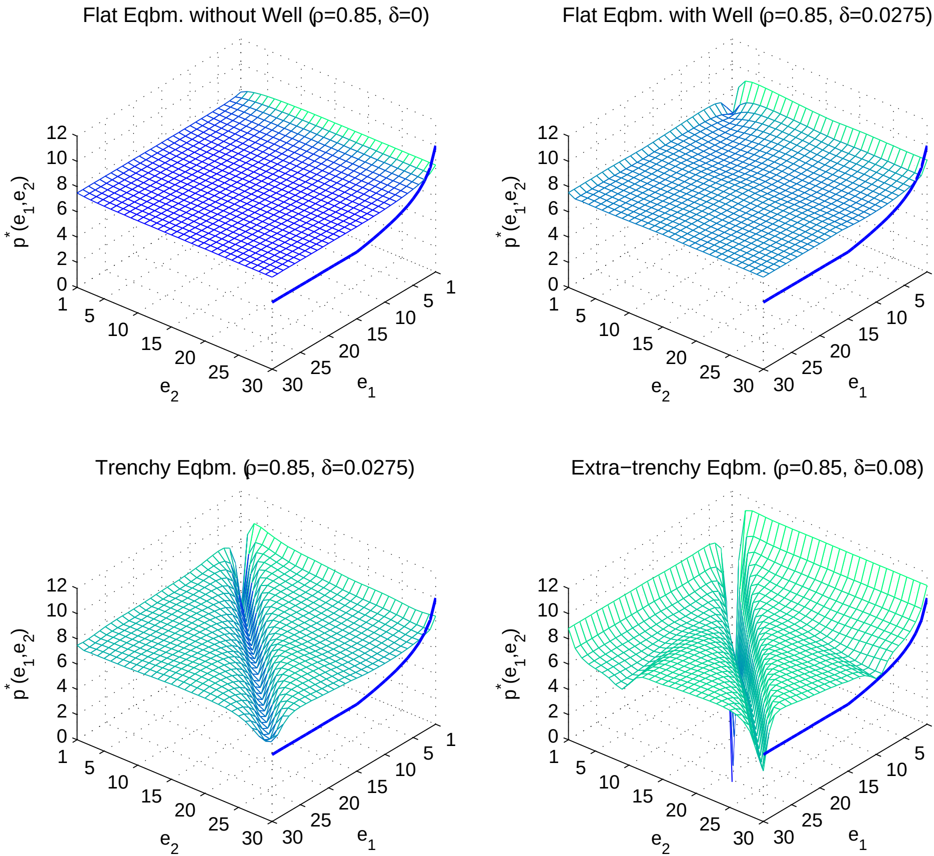


Figure 4: Policy function $p^*(e_1, e_2)$. Marginal cost $c(e_1)$ (solid line in $e_2 = 30$-plane).

Industry Dynamics

- Use tools from stochastic process theory to analyze the Markov process of industry dynamics.
- Construct the probability distribution over next period's state $\mathbf{e}'$ given this period's state $\mathbf{e}$.
- Compute the distribution over states:
  - $\mu^t(\cdot)$ is the transient distribution over states in period t starting from state $(1,1)$.
  - $\mu^\infty(\cdot)$ is the limiting (or ergodic) distribution over states.

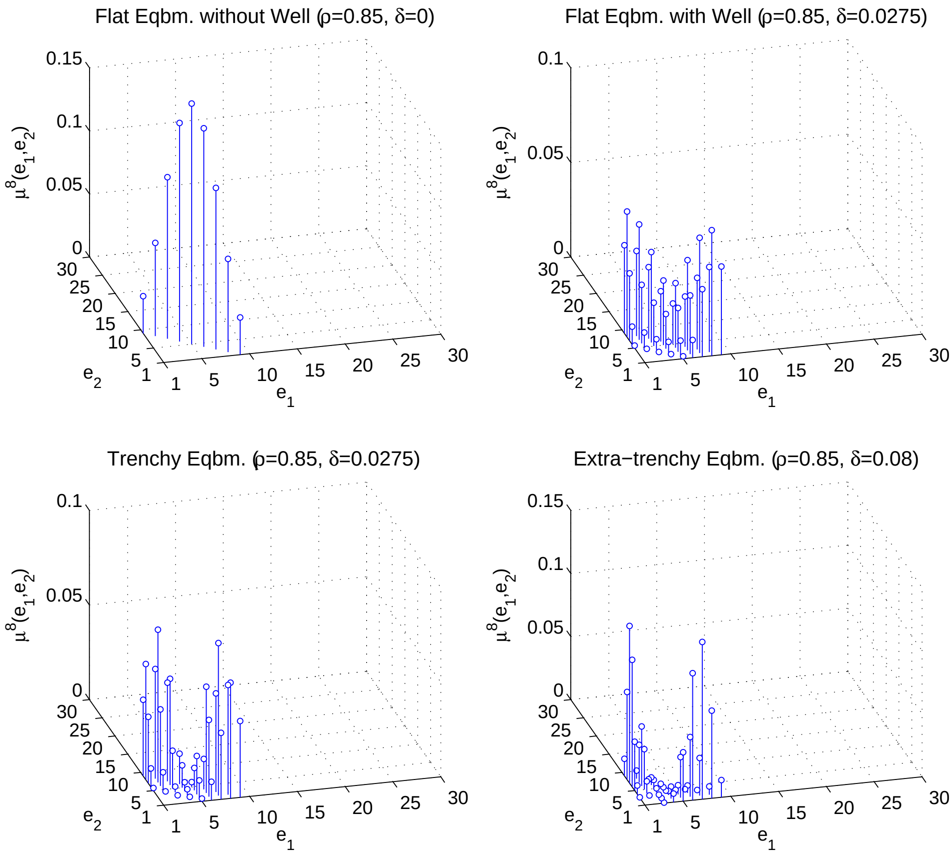


Figure 5: Transient distribution over states in period 8 given initial state $(1, 1)$.

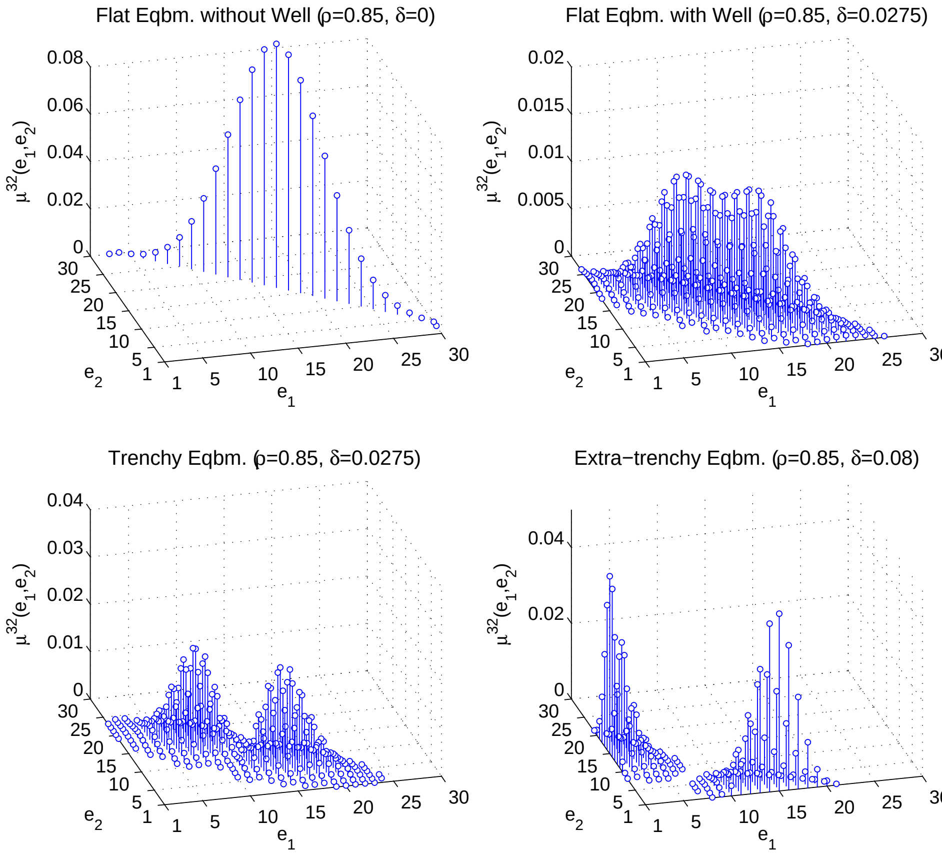


Figure 6: Transient distribution over states in period 32 given initial state $(1, 1)$.

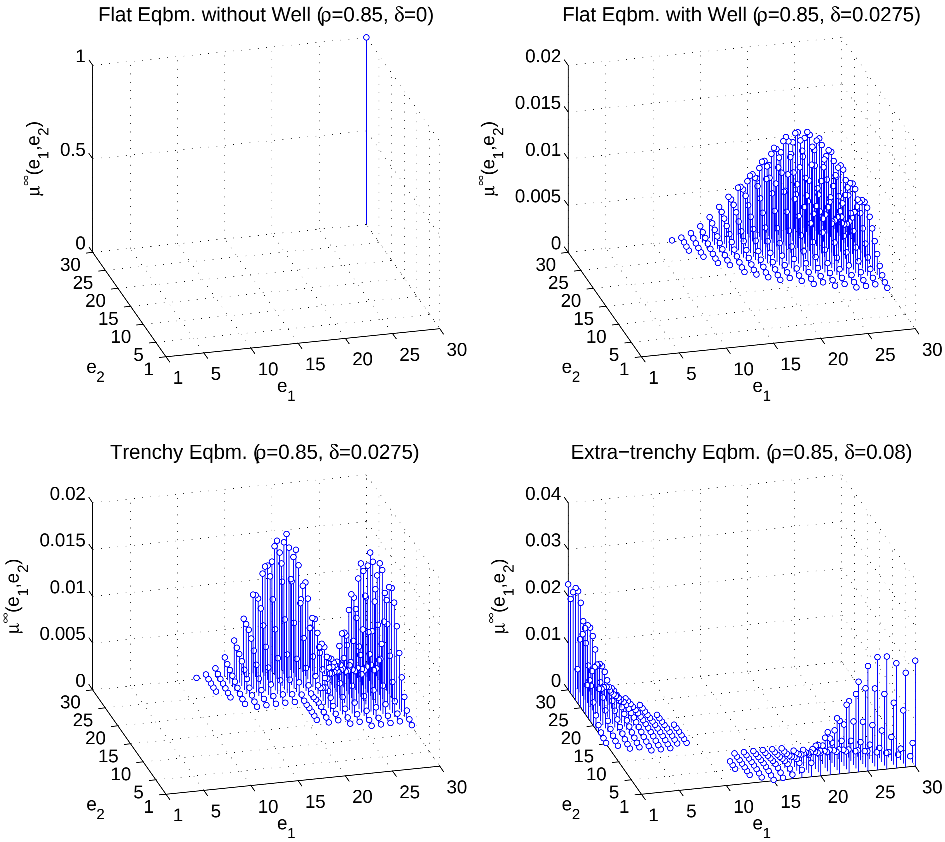


Figure 7: Limiting distribution over states.

Understanding Equilibria with Diagonal Trench

- Trench sustains leadership.
  - Suppose trench exists and leadership has value.
  - Follower does not contest the leadership because price war is too expensive and uncertain.
- Leadership generates value.
  - Suppose trench exists, both firms have reached the bottom of their learning curve, and both price identically.
  - If follower starts catching up, it backs off by raising price. Increasing price does not hurt his profits but improves the leaders profits.
- Value of leadership induces the trench.
  - Suppose the two firms are tied on the diagonal and that leadership has value.
  - Both bid aggressively—i.e., price very low—to seize leadership. This creates the trench.

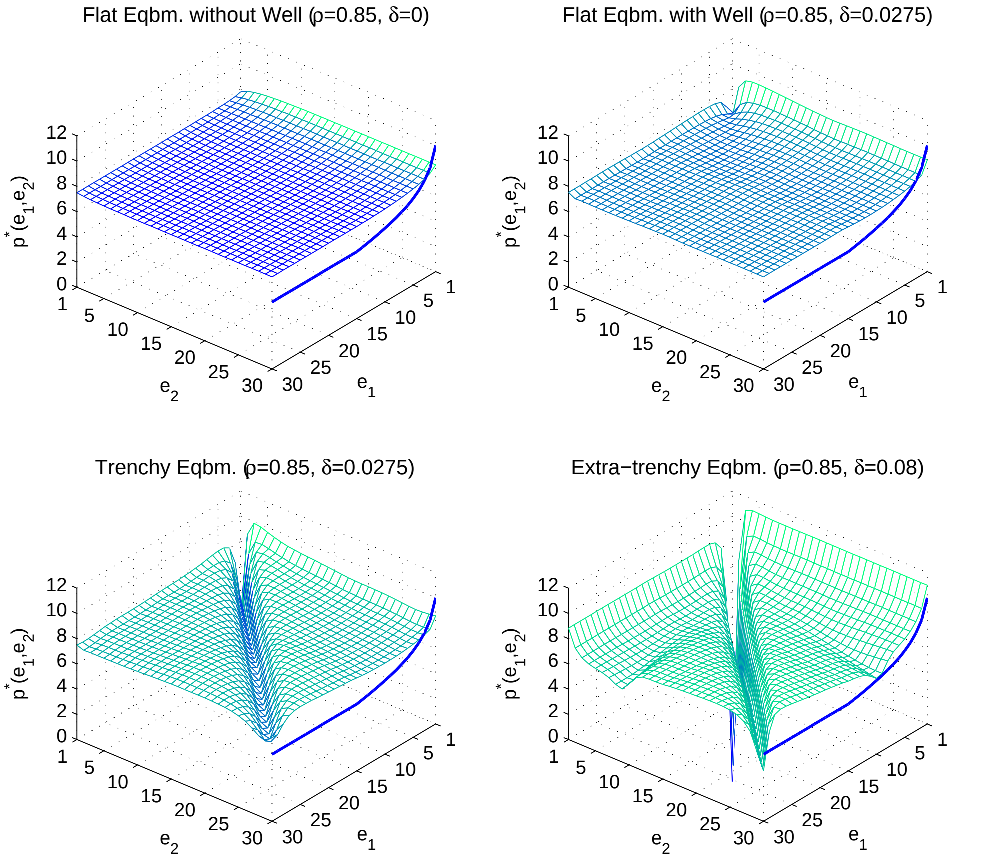


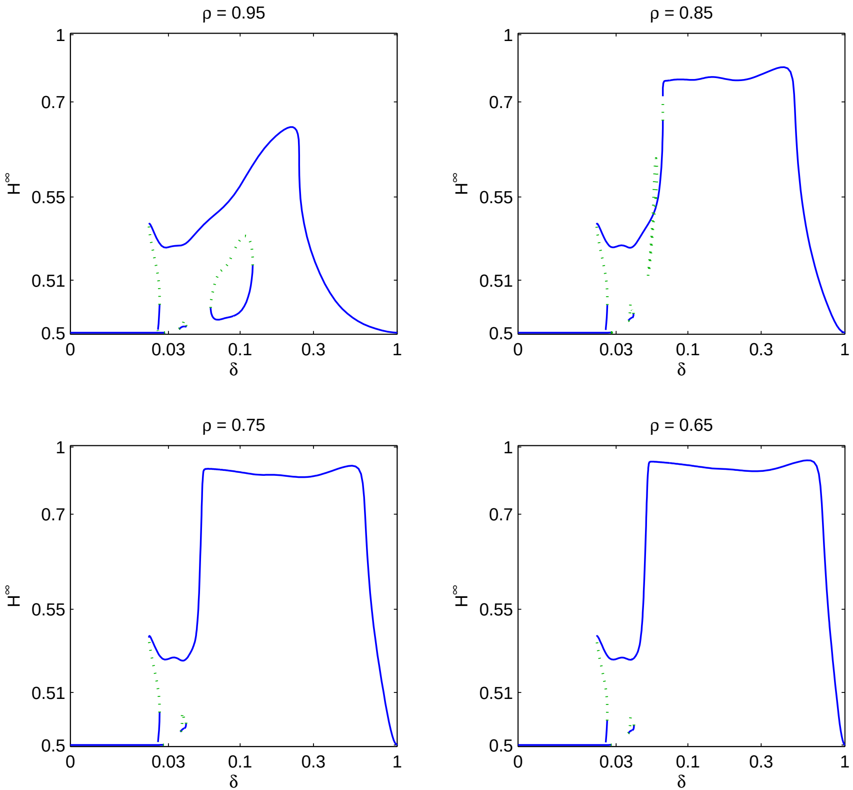
Figure 4: Policy function $p^*(e_1, e_2)$. Marginal cost $c(e_1)$ (solid line in $e_2 = 30$-plane).

Seeing Multiplicity

Multiple equilibria may exist. We can represent this multiplicity by plotting, for each ρ, the expected limiting Herfindahl index, H^∞, as a function of δ. Formally,

$$H^\infty = \sum_{\mathbf{e}} \left(D_1^*(\mathbf{e})^2 + D_2^*(\mathbf{e})^2 \right) \mu^\infty(\mathbf{e}).$$

The next slide shows H^∞ as a function δ for several values of ρ.



Limiting expected Herfindahl index H^∞.

Limitation of Pakes-McGuire (1994) Algorithm

- Combines value function iteration with best reply dynamics (akin to Cournot adjustment).
- Executes the iteration

$$\mathbf{x}^{l+1} = \mathbf{G}(\mathbf{x}^{l}), \quad l = 0, 1, 2, \ldots,$$

  where, for each state $\mathbf{e} \in \{1, \ldots, M\}^2$, old guesses for the value and policy of firm 1 are mapped into new guesses.
- In between two equilibria that can be computed by the P-M algorithm, there is one equilibrium that cannot:

Proposition

Let $(\mathbf{x}(s), \delta(s)) \in \mathbf{F}^{-1}$. *If* $\delta'(s) \leq 0$, *then* $\rho\left(\left.\frac{\partial \mathbf{G}(\mathbf{x}(s))}{\partial \mathbf{x}}\right|_{\delta=\delta(s)}\right) \geq 1$ *where here* ρ *is the spectral radius of the Jacobian.*

- The implication is that, whenever there is multiplicity, the P-M algorithm can at most find 2/3 of the equilibria.
- Darned important to use state-of-the-art algorithms.

Homotopy Technique

- Write the system of $2M^2$ nonlinear equations (Bellman equations and FOCs) as

$$\mathbf{F}(\mathbf{x}, \delta) = 0,$$

where

$$\mathbf{x} = \left(V^*(1,1), V^*(2,1), \ldots, V^*(M,M), p^*(1,1), \ldots, p^*(M,M)\right).$$

- The object of interest is the equilibrium graph

$$\mathbf{F}^{-1} = \left\{(\mathbf{x}, \delta) | \mathbf{F}(\mathbf{x}, \delta) = 0\right\}.$$

- The algorithm follows a path from the unique equilibrium at $\delta = 0$ to the unique equilibrium at $\delta = 1$.
- Polynomial example graphed on next slide:

$$\begin{aligned} f(x, \delta) &= -15.280 - \frac{\delta}{1+\delta^4} + 67.5x - 96.923x^2 + 46.154x^3 \\ &= 0. \end{aligned}$$

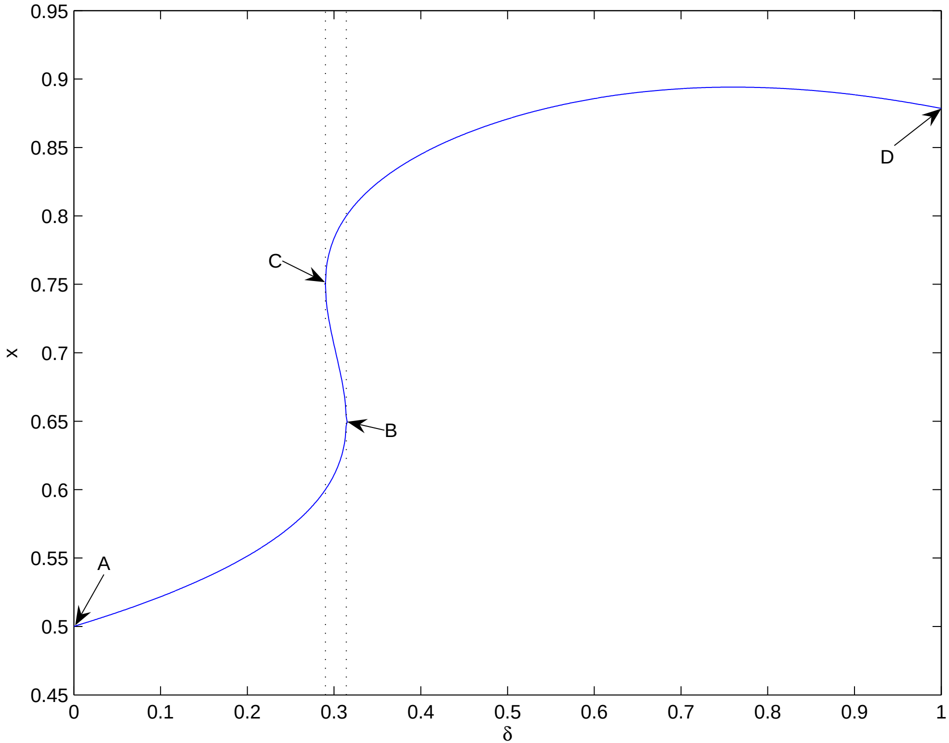


Figure 1: Homotopy example.

Graphing the Example

- Define a parametric path to be a set of functions $(\mathbf{x}(s), \delta(s))$ such that $(\mathbf{x}(s), \delta(s)) \in \mathbf{F}^{-1}$.
- The conditions that are required to remain "on path" are found by differentiating $f(x(s), \delta(s)) = 0$ with respect to s to obtain

$$\frac{\partial f\left(x\left(s\right),\delta\left(s\right)\right)}{\partial x}x'\left(s\right) + \frac{\partial f\left(x\left(s\right),\delta\left(s\right)\right)}{\partial \delta}\delta'\left(s\right) = 0.$$

- Solving for the ratio does not work at points C and D:

$$\frac{x'\left(s\right)}{\delta'\left(s\right)} = -\frac{\partial f\left(x\left(s\right),\delta\left(s\right)\right)}{\partial \delta} \div \frac{\partial f\left(x\left(s\right),\delta\left(s\right)\right)}{\partial x}.$$

- But starting at point A and solving the system of differential equations

$$x'\left(s\right) = -\frac{\partial f\left(x\left(s\right),\delta\left(s\right)\right)}{\partial \delta}, \delta'\left(s\right) = \frac{\partial f\left(x\left(s\right),\delta\left(s\right)}{\partial x}$$

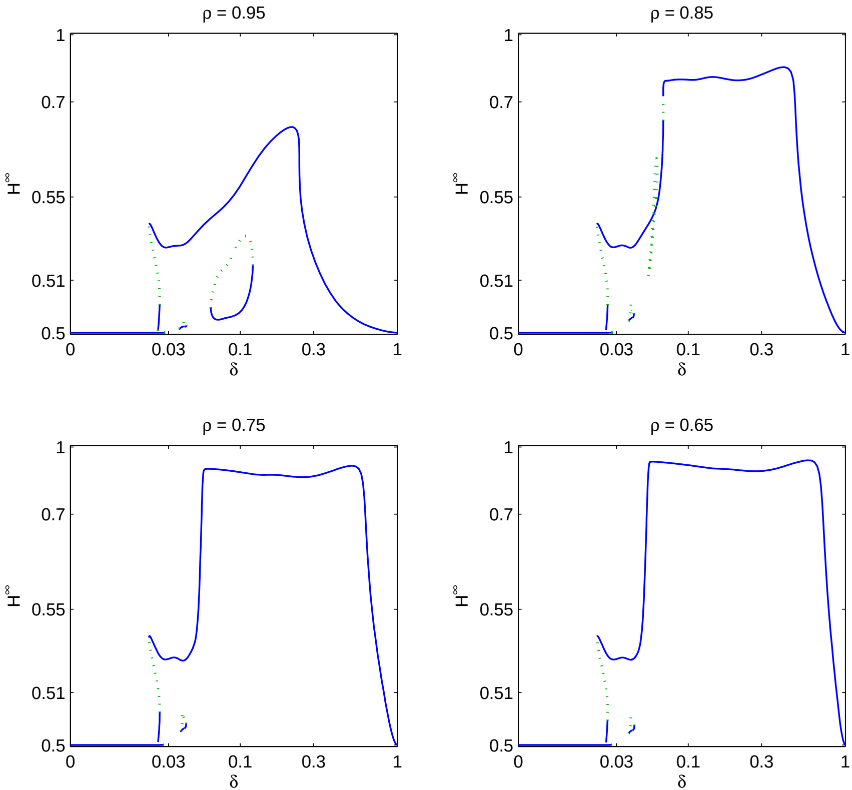
does work. Check it with $x^2 + \delta^2 = 1$ to get $x = \sin\left(s\right)$ and $y = -\cos\left(s\right)$.

Equilibrium Graph: Paths and Loops

Result 2 The equilibrium correspondence $\mathbf{F}^{-1}$ contains a unique path that connects the equilibrium at $\delta = 0$ with the equilibrium at $\delta = 1$. In addition, $\mathbf{F}^{-1}$ may contain (one or more) loops that are disjoint from the above path and from each other.

This result was derived by running homotopies on δ from zero to one on a 0.05 grid of ρ values. We display the results of the homotopy by plotting, for each ρ, the expected limiting Herfindahl index, H^{∞}, as a function of δ. Formally,

$$H^{\infty} = \sum_{\mathbf{e}} \left(D_1^*(\mathbf{e})^2 + D_2^*(\mathbf{e})^2 \right) \mu^{\infty}(\mathbf{e}).$$



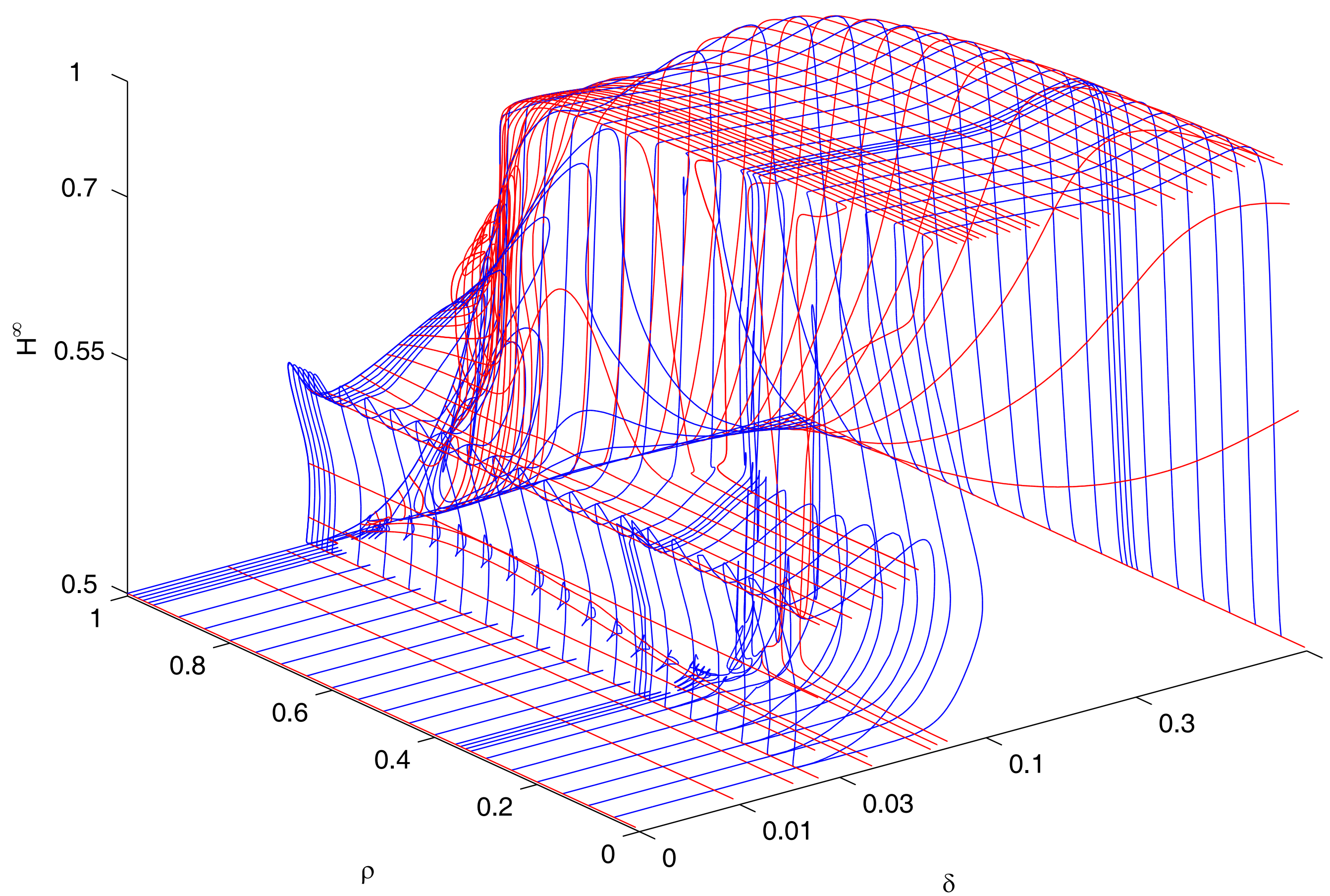
Limiting expected Herfindahl index H^∞.

Did we find "most" equilibria?

Observation. Except for small systems of polynomials it is currently infeasible to prove that one has found all solutions.

Conjecture. If the equilibrium correspondence is regular and connected, then all equilibria can be identified by running homotopies along a grid of local coordinates on the manifold.

Delta [blue] and rho [red] homotopies partially trace out equilibrium correspondence.



What we have discussed:

- Goal: Discover the variety of equilibria that can occur in a duopoly in the presence of interactions between LBD and OF.
- Representation of results:
  - Behavior: graph of policy function over states
  - Dynamics: graphs of transient distributions and limiting distribution over states
  - Equilibria should make sense
  - Multiplicity: graph of H^{∞} as a function of δ and ρ.
  - More than two firms pose difficulties
- Discovering multiplicity
  - Pakes-McGuire algorithm can only identify a fraction of multiple equilibria
  - Trace out equilibrium graph using homotopy technique
  - Dynamic stochastic games *may* have a wealth of multiplicity