

Optimization and Equations:
Connections Between Economics and Numerical Analysis

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Optimization is Fundamental in Economics Modeling

- What is economics:
 - Definition: The study of the allocation of scarce resources
 - Assumption: actors make choices that maximize an objective function
 - Hence, economics problems are constrained optimization problems: maximize objective subject to scarcity constraints
- Examples
 - Consumer choice
 - Social planning problems
 - Principal-agent problems
 - Life-cycle problems
 - Profit maximization
 - Portfolio choice

Equations are Fundamental in Economics Modeling

- Equilibrium: a collection of choices by economic actors that are consistent with scarcity and individual rationality
- Demand equals supply
 - Competitive equilibrium
 - Asset market equilibrium
 - Dynamic market equilibrium
- Nash-Cournot
 - Oligopoly theory
 - Games of incomplete information
 - Games of asymmetric information
 - Political games

All Economic Analysis Uses Optimization and Equations

- Analysis of economic data is an optimization problem
- Unknown parameters are chosen so as to maximize the compatibility between statistical model and data
 - Least squares methods
 - Method of moments
 - Maximum likelihood
- Unknown parameters are chosen to fit data and satisfy equilibrium conditions
 - Structural estimation
 - A constrained optimization problem

Numerical Analysis is Applied Economics

- Numerical analysis is the development of computational tools that best use scarce computational resources to accomplish a computational task
- Scarce resources
 - Computer time
 - Space
 - Programmer time
 - Programmer ability
- Objective
 - Accuracy
 - Speed
- Technologies
 - Memory
 - Processor
 - Communication links

Computation is About Using Computers

- You need to understand what computers do
 - Numbers - stored with infinite precision
 - Operations - executed with small errors
 - Storage methods and cache management
 - Interpreted versus compiled code
- As computer technologies change, the choice of algorithms changes
 - Single precision to double precision
 - Expensive memory to cheap memory
 - Serial to parallel processing (e.g., GPUs)

Hardware and Algorithms: Substitutes or Complements?

- Typical economist view: speed allows you to avoid learning about computation and methods
- Computational mathematicians' view
 - There are many possible methods, varying in fixed costs and marginal costs
 - Faster computers make it rational to invest in the fixed costs
 - * Development costs
 - * Use high fixed-cost methods that reduce marginal costs
 - Hardware speed and algorithm development are complements
- Historical pattern
 - Speed doubles every two years.
 - Algorithm efficiency grows at a similar, sometimes faster, rate

Objectives of ICE10

- Acquaint PhD students with the current state-of-the-art algorithms and software
- Teach the basic concepts behind algorithm development, now and in the future.