Simple Markov-Perfect Industry Dynamics

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TWO RELEVANT PRIOR PAPERS

- Abbring and Campbell (Econometrica, forthcoming)
  - Stochastic demand
  - Sunk costs of entry
  - Irreversible exit
  - Homogeneous firms
  - Sequential consinuation decisions
  - Markov-perfect equilibrium
  - Last-In First-Out strategy
- Ericson and Pakes (ReStud, 1995)
  - Investment stochastically improves profitability.
  - Outside good improves exogenously.
  - Outside technology spills over to new entrants.
  - Successful firms eventually coast.

REQUIRED TOOLS

STATIC PRIMITIVES

Special case: Cournot Competition

- C consumers with demand: $q = 1 - p$.
- Up to two producers with marginal cost c_H or $c_L > c_H$.
- Static profit from each consumer:

$$\pi_H(H) = \left(\frac{1-c_H}{2}\right)^2 > \pi_H(HL) = \left(\frac{1+c_L-2c_H}{3}\right)^2 > \pi_H(HH) = \left(\frac{1-c_H}{3}\right)^2$$

$$\pi_L(L) = \left(\frac{1-c_L}{2}\right)^2 > \pi_L(LL) = \left(\frac{1-c_L}{3}\right)^2 > \pi_L(HL) = \left(\frac{1+c_H-2c_L}{3}\right)^2$$

General case: Producer's surplus $= C\pi_K(N)$.

- $K \in \{1, \ldots, \check{K}\}$ and $N = (n_1, n_2, \ldots, n_{\check{K}})$.
- $\pi_K(N) > \pi_{K-1}(N)$.
- Increasing any element of N strictly decreases $\pi_K(N)$.
- Increasing the l'th element of N by 1 and decreasing its $(l-1)$st element strictly decreases $\pi_K(N)$.

DYNAMIC PRIMITIVES

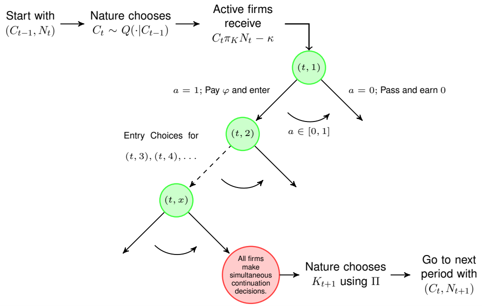
Sequential Entry

- Discrete time $t \in \{0, 1, \ldots\}$. Countably many firms.
- Firms have names in $T \times N$.
- In period t, $(t, 1)$ is the first firm to make entry decision, $(t, 2)$ is the second, . . .
- Sunk cost of entry, φ.
- All entrants have profitability type 1.
- Entry decisions stop when one firm decides to stay out.

Post Entry Evolution

- Irreversible exit. Payoff 0 outside the market.
- $\Pi_{kk'} = \Pr[K_{t+1} = k' | K_t = k]$, $\Pi_{kk'} = 0$ if $k' < k$. (No regress)
- Number of consumers, $C_{t+1} \in [\hat{C}, \check{C}]$, is first-order Markovian.
- Profit $C_{t+1}\pi_{K_{t+1}}(N_{t+1}) - \kappa$. Discount rate β.

TIMING OF ACTIONS



Payoff Relevant Histories

For Potential Entrant (t, j)'s Decision

$$H_E = \begin{cases} C & \text{Demand state} \\ N + j \times (1, 0, 0, \dots) & \text{Market structure after entry} \end{cases}$$
$$H_E \in \mathcal{H}_E \equiv \left[\hat{C}, \check{C}\right] \times \mathbb{Z}^{\check{K}}$$

For Continuation Decision After Entry of J Firms

$$H_S = \begin{cases} C & \text{Demand state} \\ N + J \times (1, 0, 0, \dots) & \text{Current market structure} \\ K & \text{Type} \end{cases}$$
$$H_S \in \mathcal{H}_S \equiv \left[\hat{C}, \check{C}\right] \times \mathbb{Z}^{\check{K}} \times \mathbb{K}$$

A *Markovian strategy* is a pair $(A_S(H_S), A_E(H_E))$ for each $H_S \in \mathcal{H}_S$, and $H_E \in \mathcal{H}_E$. The strategies are *probabilities* of survival or entry.

Symmetric Markov-Perfect Equilibrium

Definition

A symmetric Markov-perfect equilibrium is a subgame-perfect equilibrium in which all firms follow the same Markovian strategy.

An incumbent's payoff function is

$$v_S(H_S) = A_S(H_S)\beta\mathbb{E}\left[C'\pi_{K'}(N') - \kappa + v_S(H'_S) \,\middle|\, H_S\right]$$

A potential entrant's payoff function is

$$v_E(H_E) = A_E(H_E)\left(\beta\mathbb{E}\left[C'\pi_{K'}(N') - \kappa + v_S(H'_S) \,\middle|\, H_E\right] - \varphi\right)$$

$\mathbb{E}[\cdot|H_E]$ and $\mathbb{E}[\cdot|H_S]$ condition on participation next period and on all other firms' strategies.

A DUOPOLY EXAMPLE

- Firms are all identical.
- $\kappa > 0$; $\varphi > 0$; and $\pi(3) < \kappa$.
- In any symmetric equilibrium, no firm will enter a market with two firms already committed to produce next period.
  - I If a third firm does enter, it receives negative profit next period.
  - II Negative profits continue while two rival firms remain.
  - III If any firm exits, by symmetry all firms receive 0.
  - IV Therefore, a third firm's expected payoff from entry is negative.

DUOPOLY PAYOFF AND ENTRY

- Suppose $N = 2$.
- In any symmetric equilibrium:
  - I If one firm receives a positive payoff, both firms do.
  - II Any firm receiving a positive payoff continues.
  - III If a payoff is positive, it must equal

$$\beta\mathbb{E}[C'\pi(2) - \kappa + v(C', 2)|C].$$

- Assume (and later verify) that the payoff to continuing alone exceeds the payoff to continuing as a dupolist.

$$v(C,2) = \max\{0, \beta\mathbb{E}[C'\pi(2) - \kappa + v(C',2)|C]\}$$

- This defines a contraction mapping.
- Strategy of an entrant facing a monopolist

$$A_E(C,2) = \begin{cases} 1 & \text{if } v(C,2) > \varphi \\ 0 & \text{otherwise.} \end{cases}$$

Monopoly Payoff and Strategy

- Monopoly equilibrium payoff

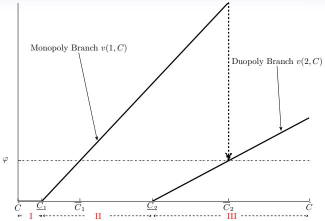
  $$v(C,1) = \max\{0, \beta\mathbb{E}[C'\pi(1) - \kappa + A_E(C',2)v(C',1+A_E(C',2))|C]$$

  This defines a contraction mapping given $A_E(\cdot,2)$ and $v(\cdot,2)$.
- Strategies of a monopolist and of an entrant facing an empty market

$$A_S(C,1) = \begin{cases} 1 & \text{if } v(C,1) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$A_E(C,1) = \begin{cases} 1 & \text{if } v(C,1) > \varphi \\ 0 & \text{otherwise.} \end{cases}$$

Duopoly Survival Strategy



$$A_S(C,2) = \begin{cases} 1 & \text{if } v(C,2) > 0 \\ \frac{v(C,1)}{v(C,1) - \beta\mathbb{E}[C'\pi(2) - \kappa + v(C',2)|C]} & \text{if } v(C,2) = 0, v(C,1) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

HOMOGENEOUS OLIGOPOLY EQUILIBRIUM ANALYSIS

DEFAULT TO INACTIVITY

Potential entrants and monopolists choose inactivity whenever it gives the same payoff as entry or continuation.

PROPOSITION

If firms are homogeneous ($\check{K} = 1$) then there exists a unique symmetric Markov-perfect equilibrium with a strategy that defaults to inactivity.

ALGORITHM FOR HOMOGENEOUS OLIGOPOLY

A Set $A_E(C, N) = 0$ for $N > \check{N}$.

B For $N = \check{N}, \ldots, 1$

B.I Calculate the fixed point of

$$v(C, N) = \max\{0, \beta\mathbb{E}[C'\pi(N) - \kappa + v(C', N + \sum_{j=N+1}^{\infty} A_E(C', j))]\}.$$

B.II Set $A_E(C, N) = I\{v(C, N) > \varphi\}$.

C For $N = \check{N}, \ldots, 1$

C.I For C with $V(C, N) = 0$ and $V(C, 1) > 0$, solve for $p(C)$

$$\sum_{j=0}^{N-1} \binom{N-1}{j} p(C)^j (1-p(C))^{N-1-j} \mathbb{E}[C'\pi(j+1) - \kappa + v(C', j+1)|C] = 0.$$

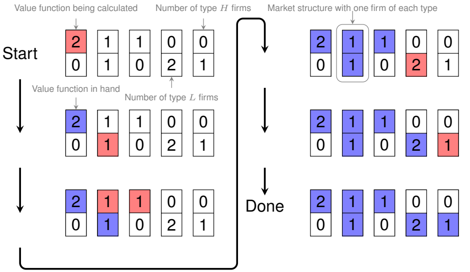
C.II Set

$$A_S(C, N) = \begin{cases} 1 & \text{if } v(C, N) > 0 \\ p(C) & \text{if } v(C, N) = 0 \text{ and } v(C, 1) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

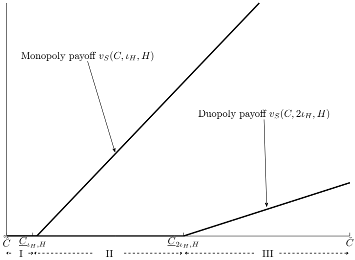
TWO TYPES OF FIRMS

- Two profitability types, Low $L = 1$, and High $H = 2$.
- $\Pi_{LH} = \delta, \Pi_{LL} = 1 - \delta, \Pi_{HH} = 1, \Pi_{HL} = 0$
- $N = (n_L, n_H)$
- Assume that a high profitability firm never exits the market when a low profitability rival still continues with positive probability.

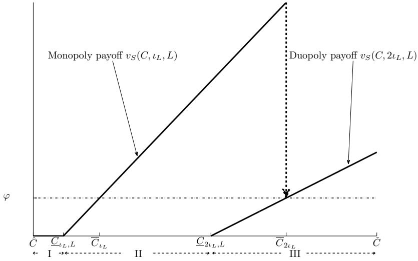
DUOPOLY EQUILIBRIUM WITH TWO TYPES



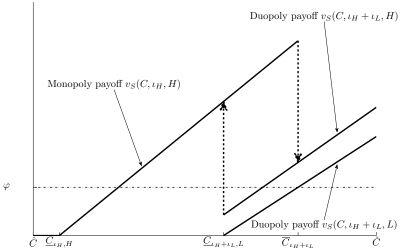
Exit Strategy: High Type Duopolists



Exit Strategy: Low Type Duopolists



Remaining Equilibrium Payoffs



Duopoly Equilibrium Analysis

Natural Equilibrium

No firm ever exits leaving behind a lower type rival. (Cabral, JET 1993, Pakes, Gowrisankaran and McGuire 1993)

Proposition

If $\pi(K, 3) < \kappa$ for $K = 1, \ldots, \check{K}$, then there exists a unique symmetric and natural Markov-perfect equilibrium with a strategy that defaults to inactivity.

Oriental Lexicographic Algorithm: Setup

A Order market structures lexicographically **reading right to left**.

$$\begin{aligned} N_1 &= (0,\ldots,0,2) \\ N_2 &= (0,\ldots,1,1) \\ &\vdots \\ N_{\binom{\check{K}+2}{2}} &= (1,0,\ldots,0) \equiv \iota \end{aligned}$$

B Define $s = (N, K)$

C Define $\underline{K}(N) = \min\{K | N(K) > 0\}$

D Define $\mathcal{S}_1 = \{(N_1, \check{K})\}$

E For $j = 2, \ldots, \binom{\check{K}+2}{2}$, define

$$\mathcal{S}_j = \mathcal{S}_{j-1} \bigcup \{(N_i, \underline{K}(N_j)) | i \leq j \ \ \& \ \ N_i(\underline{K}(N_j)) > 0\}$$

Oriental Lexicographic Algorithm: Implementation

A Calculate $v(C, N_1, \check{K})$ as the unique fixed-point to

$$v(C, N_1, \check{K}) = \max\{0, \beta\mathbb{E}[C'\pi_{\check{K}}(N_1) - \kappa + v(C', N_1, \check{K})]\}.$$

B For $j = 2, \ldots, \binom{\check{K}+2}{2}$, suppose that $v(C, s)$ is known for $s \in \mathcal{S}_{j-1}$.

B.I For any $s = (N, K) \in \mathcal{S}_j / \mathcal{S}_{j-1}$ and $K' < K$ with $N(K') > 0$, set

$$A_S(N, K', C) = I\{v(N, K', C) > 0\}.$$

B.II Combine the probabilities in Π with these strategies to calculate the transition probabilities $q(s'|s)$ for all $s' \in \mathcal{S}_j$.

B.III For any $s = (N, K) \in \mathcal{S}_j$, set

$$A_E(N + \iota, C) = I\{v(N + \iota, 1, C) > \varphi\}.$$

B.IV For all $s \in \mathcal{S}_j / \mathcal{S}_{j-1}$, calculate $v(C, s)$ as the unique fixed point to

$$v(C, s) = \max\{0, \beta\mathbb{E}[C'\pi_{K'}(N') - \kappa + v(C', N' + A_E(N' + \iota, C), K')]\}$$

C Calculate $A_S(C, N, K)$ appropriately.

Non-unique Natural Symmetric MPE

At most 3 active firms and 2 profitability types (L and H):

- $\kappa = 4, \varphi = 1, \beta = 0.5, \Pi_{LH} = 0.5$.
- $C_t \in \{C_1 = 0, C_2 = 1e^{-6}, C_3 = 5\}$. Deterministic growth.

π_L/π_H	$1H$	$2H$	$3H$
$0L$	/102	/100	/0.9
$1L$	99/101	0.89/1.57	
$2L$	1.56/1.58		

Start at C_1 with two type H firms in the market.
Both firms continuing deters further entry,

$$\beta\left((C_2\pi_H(2H) - \kappa) + v(C_2, 2H, H)\right) = 246.$$

A firm continuing alone will face two entrants,

$$\beta\left((C_2\pi_H(H) - \kappa) + v(C_2, 1H2L, H)\right) = -1.475.$$

Three equilibria, $A_S(C_1, 2H, H) = 1, 0$, and $5.96e^{-3}$.

General Oligopoly Equilibrium Analysis

Sequentially Pareto Superior

No coalition can form a **self-enforcing** agreement which changes their current choices and thereby strictly increases all their payoffs.

Proposition

There is a modified version of the OLA which always calculates a symmetric and natural Markov-perfect equilibrium that is sequentially Pareto superior.

Corollary

If the indifference condition for a mixed-strategy equilibrium always uniquely determines $A_S(C, N, K)$, then the calculated equilibrium is unique.

Computational Burden

Number of contraction mappings for different $\check{N}, \check{K}$

$\check{K}/\check{N}$	1	2	3	4	5	6	7
5	5	20	55	125	251	461	791
10	10	65	285	1000	3002	8007	19447
15	15	135	815	3875	15503	54263	170543
20	20	230	1770	10625	53129	230229	888029