DICE-CJL and DSICE — Integrated Assessment Models

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Climate Change Analysis

Question: What can and should be the response to rising CO2 concentrations?

- Analytical tools in the literature: IAMs (Integrated Assessment Models)
  - Two components: economic model and climate model
  - Interaction is often limited: Economy emits CO2 which affects world average temperature which affects economic productivity.
- Existing IAMs cannot study dynamic decisionmaking in an evolving and uncertain world
  - Most are deterministic; economic actors know with certainty the consequences of their actions and the alternatives
  - Most are myopic; standard reason is computational feasibility

Today's Presentation

- DICE: Simple Example of IAMs
- DICE-CJL (continuous-time reformulation of DICE)
  - Explicit finite difference
  - Trapezoidal finite difference
- DSICE with uncertainty
  - Economic shocks
  - Tipping point in climate system
    - additional preventative carbon tax has no ramp!
    - Two i.i.d. tipping points
    - Multi-stage tipping process

Nordhaus' DICE: The Prototypical Model

- DICE2007 was the only dynamic economic model used by the US Interagency Working Group on the Cost of Carbon
- Economic system
  - gross output: $Y_t \equiv f(k_t, t) = A_t k_t^{\alpha} l_t^{1-\alpha}$
  - damage factor: $\Omega_t \equiv \left(1 + \pi_1 T_t^{\mathrm{AT}} + \pi_2 (T_t^{\mathrm{AT}})^2\right)^{-1}$
  - emission control cost: $\Lambda_t \equiv \psi_t^{1-\theta_2} \theta_{1,t} \mu_t^{\theta_2}$, where μ_t is policy choice
  - output net of damages and emission control: $\Omega_t (1 - \Lambda_t) Y_t$
- Climate system
  - Carbon mass: $\mathbf{M}_t = (M_t^{\mathrm{AT}}, M_t^{\mathrm{UP}}, M_t^{\mathrm{LO}})^\top$
  - Temperature: $\mathbf{T}_t = (T_t^{\mathrm{AT}}, T_t^{\mathrm{LO}})^\top$
  - Carbon emission: $E_t = \sigma_t (1 - \mu_t) Y_t + E_t^{\mathrm{Land}}$
  - Radiative forcing: $F_t = \eta \log_2 \left(\left(M_t^{\mathrm{AT}} + M_{t+1}^{\mathrm{AT}} \right) / \left(2 M_0^{\mathrm{AT}} \right) \right) + F_t^{\mathrm{EX}}$

- Social planner's problem

$$\max_{c_t, \mu_t} \sum_{t=0}^{\infty} \beta^t u_t(c_t)$$

$$\begin{aligned} \text{s.t.} \quad k_{t+1} &= (1-\delta)k_t + \Omega_t(1-\Lambda_t)Y_t - c_t \\ \mathbf{M}_{t+1} &= \Phi^{\mathrm{M}}\mathbf{M}_t + (E_t, 0, 0)^\top \\ \mathbf{T}_{t+1} &= \Phi^{\mathrm{T}}\mathbf{T}_t + (\xi_1 F_t, 0)^\top \end{aligned}$$

- 6 continuous state variables

Limitations of DICE

- 10 year time periods
  - Economic life happens at far shorter time scales
  - Climate system is a continuous-time system
  - Decadal models cannot examine, for example, price caps in cap-and-trade systems
- Both the economic and climate systems are known perfectly to all actors

DICE-CJL (Cai-Judd-Lontzek)

DICE-CJL is a continuous-time reformulation of DICE.

$$
\begin{aligned}
\max_{c,\mu} \quad & \int_0^\infty e^{-\rho t} u(c, l(t)) dt \\
\text{s.t.} \quad & \dot{k} = \mathcal{Y}\left(k, T^{\mathrm{AT}}, \mu, t\right) - c - \delta k, \\
& \dot{\mathbf{M}} = \Phi^{\mathrm{M}} \mathbf{M} + \left(\mathcal{E}\left(k, \mu, t\right), 0, 0\right)^\top, \\
& \dot{\mathbf{T}} = \Phi^{\mathrm{T}} \mathbf{T} + \left(\xi_1 \mathcal{F}\left(M^{\mathrm{AT}}, t\right), 0\right)^\top.
\end{aligned}
$$

- Production function:
  $\mathcal{Y}(k, T^{\mathrm{AT}}, \mu, t) = \frac{1-\psi(t)^{1-\theta_2}\theta_1(t)\mu^{\theta_2}}{1+\pi_1 T^{\mathrm{AT}}+\pi_2(T^{\mathrm{AT}})^2} A(t) k^\alpha l(t)^{1-\alpha}$
- Carbon emission function:
  $\mathcal{E}(k, \mu, t) = \sigma(t)(1-\mu)A(t)k^\alpha l(t)^{1-\alpha} + E^{\mathrm{Land}}(t)$
- Radiative forcing function:
  $\mathcal{F}\left(M^{\mathrm{AT}}, t\right) = \eta \log_2\left(M^{\mathrm{AT}}/M_0^{\mathrm{AT}}\right) + F^{\mathrm{EX}}(t)$

Finite Difference Methods

To solve an ODE: $\frac{dx(t)}{dt} = f(x(t), t)$

- Explicit Euler method:

$$x_{n+1} = x_n + hf(x_n, t_n)$$

- Implicit method:

$$x_{n+1} = x_n + hf(x_{n+1}, t_{n+1})$$

- Crank-Nicolson (trapezoidal) method:

$$x_{n+1} = x_n + \frac{h}{2}\left(f\left(x_n, t_n\right) + f\left(x_{n+1}, t_{n+1}\right)\right)$$

Calibrating Parameters

Calibrating unknown parameters a in the function $f(x, t; a)$:

$$\begin{aligned} \min_{a} \quad & \sum_{n=0}^{N} \|x_n - x_n^*\|, \\ \text{s.t.} \quad & x_{n+1} = x_n + \frac{h}{2}\left(f\left(x_n, t_n; a\right) + f\left(x_{n+1}, t_{n+1}; a\right)\right), \end{aligned}$$

Explicit DICE-CJL

Use the explicit Euler method for the ODE in DICE-CJL model:

$$
\begin{aligned}
\max_{c_n,\mu_n} \quad & \sum_{n=0}^{N-1} e^{-\rho n h} u(c_n, l_n) h + e^{-\rho h N} \hat{V}\left(k_N, \mathbf{M}_N, \mathbf{T}_N\right), \\
\text{s.t.} \quad & k_{n+1} = k_n + \left(\mathcal{Y}\left(k_n, T_n^{\mathrm{AT}}, \mu_n, nh\right) - c_n - \delta k_n\right) h, \\
& \mathbf{M}_{n+1} = \mathbf{M}_n + \left[\Phi^{\mathrm{M}} \mathbf{M}_n + \left(\mathcal{E}\left(k_n, \mu_n, nh\right), 0, 0\right)^{\top}\right] h, \\
& \mathbf{T}_{n+1} = \mathbf{T}_n + \left[\Phi^{\mathrm{T}} \mathbf{T}_n + \left(\xi_1 \mathcal{F}\left(M_n^{\mathrm{AT}}, nh\right), 0\right)^{\top}\right] h.
\end{aligned}
$$

Trapezoidal DICE-CJL

Use the trapezoidal method for the ODE in DICE-CJL model:

- Utility: $\sum_{n=0}^{N} e^{-\rho nh} w_n u(c_n, l_n) h$ (trapezoidal formula for integration)
- Capital:

$$
\begin{aligned}
k_{n+1} \quad = \quad & k_n + \left[\mathcal{Y}\left(k_{n+1}, T_{n+1}^{\mathrm{AT}}, \mu_{n+1}, (n+1)h\right) - c_{n+1} - \delta k_{n+1}\right] \frac{h}{2} + \\
& \left[\mathcal{Y}\left(k_n, T_n^{\mathrm{AT}}, \mu_n, nh\right) - c_n - \delta k_n\right] \frac{h}{2},
\end{aligned}
$$

- Climate system:

$$
\begin{aligned}
\mathbf{M}_{n+1} \quad = \quad & \mathbf{M}_n + \left[\Phi^{\mathrm{M}} \mathbf{M}_{n+1} + \left(\mathcal{E}\left(k_{n+1}, \mu_{n+1}, (n+1)h\right), 0, 0\right)^{\top}\right] \frac{h}{2} + \\
& \left[\Phi^{\mathrm{M}} \mathbf{M}_n + \left(\mathcal{E}\left(k_n, \mu_n, nh\right), 0, 0\right)^{\top}\right] \frac{h}{2}, \\
\mathbf{T}_{n+1} \quad = \quad & \mathbf{T}_n + \left[\Phi^{\mathrm{T}} \mathbf{T}_{n+1} + \left(\xi_1 \mathcal{F}\left(M_{n+1}^{\mathrm{AT}}, (n+1)h\right), 0\right)^{\top}\right] \frac{h}{2} + \\
& \left[\Phi^{\mathrm{T}} \mathbf{T}_n + \left(\xi_1 \mathcal{F}\left(M_n^{\mathrm{AT}}, nh\right), 0\right)^{\top}\right] \frac{h}{2}.
\end{aligned}
$$

Running Times

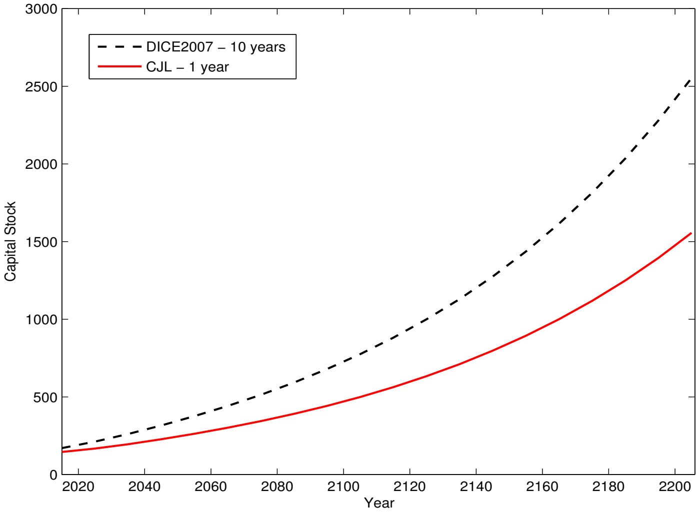
Table: Running Times of DICE-CJL with Finite Difference Methods

Step Size h	#Stages	Explicit DICE-CJL	Trapezoidal DICE-CJL
10 years	60	-	1.1 seconds
5 years	120	-	6.3 seconds
2 years	300	6.8 seconds	22 seconds
1 year	600	0.9 seconds	3.1 seconds
6 months	1200	4.6 seconds	13 seconds
3 months	2400	14 seconds	58 seconds
1 month	7200	133 seconds	241 seconds
2 weeks	14400	334 seconds	-
1 week	28800	1979 seconds	-

- Starting Point Strategy: For step sizes with 1 year or less, we use the linear interpolation of solutions with larger step size as the initial guess.

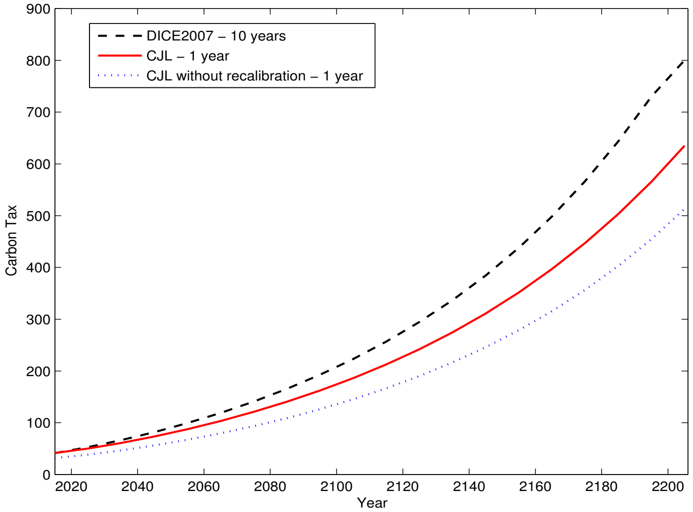
Capital Stock

Figure: Capital Stock



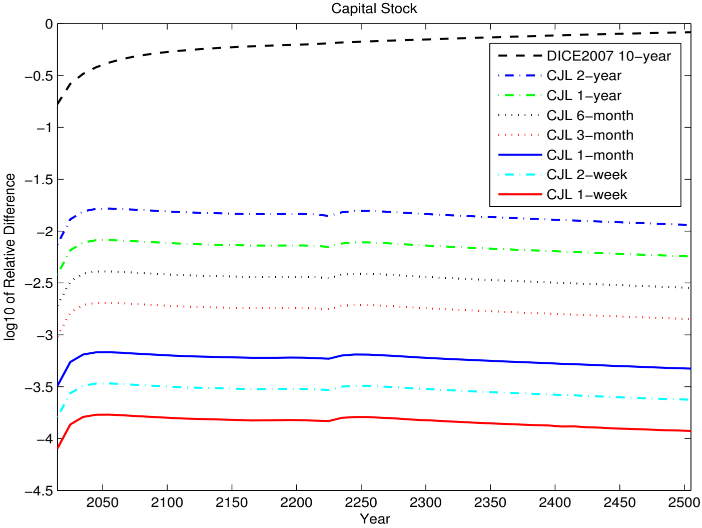
Carbon Tax

Figure: Carbon Tax



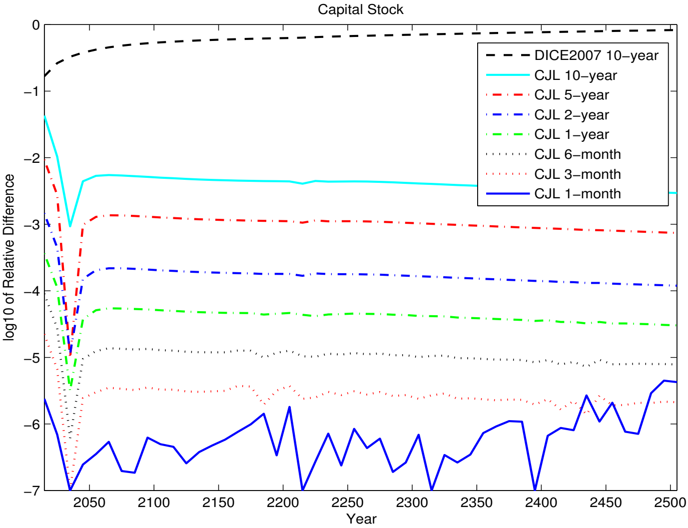
Errors of Explicit DICE-CJL

Figure: Relative Errors of Capital of Explicit DICE-CJL



Errors of Trapezoidal DICE-CJL

Figure: Relative Errors of Capital of Trapezoidal DICE-CJL



Uncertainty and Risk

All agree that uncertainty needs to be a central part of any IAM analysis
Multiple forms of uncertainty

- Risk: productivity shocks, taste shocks, uncertain technological advances, weather shocks
- Parameter uncertainty: policymakers do not know parameters that characterize the economic and/or climate systems
- Model uncertainty: policymakers do not know the proper model or the stochastic processes

Abrupt, Stochastic, and Irreversible Climate Change

Question: What is the optimal carbon tax when faced with abrupt and irreversible climate change?

- Common assumption in IAMs: damages depend only on contemporaneous temperature
- Our criticism: this cannot analyze the permanent and irreversible damages from tipping points
- We show that
  - Abrupt climate change can be modeled stochastically
  - The policy response to the threat of tipping points is very different from the policy response to standard damage representations.

Cai-Judd-Lontzek DSICE Model

DSICE (**D**ynamic **S**tochastic **I**ntegrated Model of **C**limate and **E**conomy)

$$\begin{aligned} DSICE &= DICE2007 \\ &+ \text{stochastic damage factor} \\ &+ \text{stochastic production function} \\ &+ \text{flexible period length} \end{aligned}$$

DSICE: new features

- Economic system: $Y_t \equiv f(k_t, \zeta_t, t) = \zeta_t A_t k_t^{\alpha} l_t^{1-\alpha}$ where $\zeta_{t+1} = g^{\zeta}(\zeta_t, \omega_t^{\zeta})$ is an AR(1) process for the productivity state ζ
- Climate system: $\Omega_t \equiv J_t \left(1 + \pi_1 T_t^{AT} + \pi_2 (T_t^{AT})^2\right)^{-1}$ where $J_{t+1} = g^J(J_t, \omega_t^J)$ is a Markov process for the damage factor state J

- DSICE model

$$\max_{c_t,\mu_t} \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t u_t(c_t)\right\}$$

$$\begin{array}{lrcl}
\text{s.t.} & k_{t+1} & = & (1-\delta)k_t + \Omega_t(1-\Lambda_t)Y_t - c_t, \\
 & \mathbf{M}_{t+1} & = & \Phi^M \mathbf{M}_t + (E_t, 0, 0)^\top, \\
 & \mathbf{T}_{t+1} & = & \Phi^T \mathbf{T}_t + (\xi_1 F_t, 0)^\top, \\
 & \zeta_{t+1} & = & g^\zeta(\zeta_t, \omega_t^\zeta), \\
 & J_{t+1} & = & g^J(J_t, \omega_t^J)
\end{array}$$

- 8 state variables (7 continuous, 2 random)

- DP model for DSICE

$$
\begin{aligned}
V_t(k, \mathbf{M}, \mathbf{T}, \zeta, J) &= \max_{c,\mu} \quad u_t(c) + \beta \mathbb{E}[V_{t+1}(k^+, \mathbf{M}^+, \mathbf{T}^+, \zeta^+, J^+)] \\
\text{s.t.} \quad k^+ &= (1-\delta)k + \Omega_t(1-\Lambda_t)Y_t - c, \\
\mathbf{M}^+ &= \Phi^M \mathbf{M} + (E_t, 0, 0)^\top, \\
\mathbf{T}^+ &= \Phi^T \mathbf{T} + (\xi_1 F_t, 0)^\top, \\
\zeta^+ &= g^\zeta(\zeta, \omega^\zeta), \\
J^+ &= g^J(J, \omega^J)
\end{aligned}
$$

- One year (or shorter) time steps over 600 years

Computational Challenges

Solving large-scale dynamic programming problems will require the use of existing numerical methods and development of new ones

- Approximate multidimensional value functions
- Approximate multidimensional integration methods
- Maintain stability of value function approximations

Accuracy Test

Table: Relative Errors and Running Time of the Numerical DP Algorithm

degree	k	M^{AT}	T^{AT}	c	μ	Time
4	6.4(−4)	5.7(−5)	7.2(−5)	2.0(−4)	8.5(−5)	8.5 minutes
6	6.6(−6)	6.2(−7)	4.5(−7)	1.7(−5)	2.0(−6)	2.7 hours

Note: $a(-n)$ means $a \times 10^{-n}$.

Economic Shock

- Economic shock is assumed to be a continuous random variable with the mean-reverting property.
- AR(1) mean-reverting process:

$$y_{t+1} = (1-\lambda)y_t + \sqrt{2\lambda - \lambda^2} z_t,$$

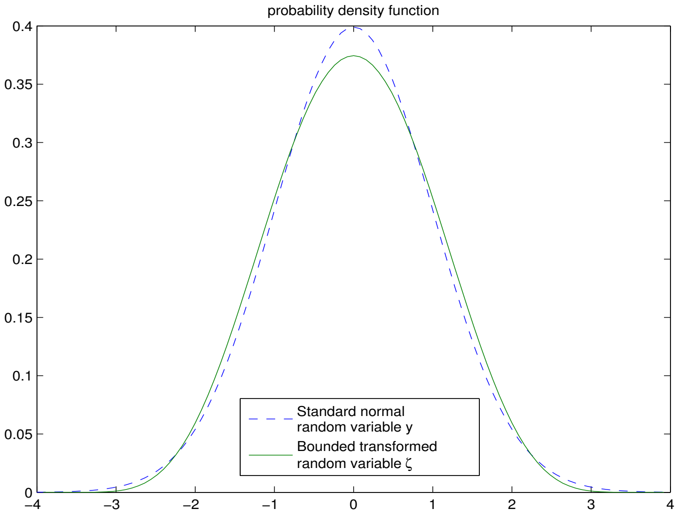
- Transform to be bounded:

$$\zeta = \frac{1 - e^{-\kappa y}}{1 + e^{-\kappa y}} \nu,$$

  - mean zero, unit variance
  - one example: $\nu = 4 \text{ and } \kappa = 0.532708$.
- General transformation:

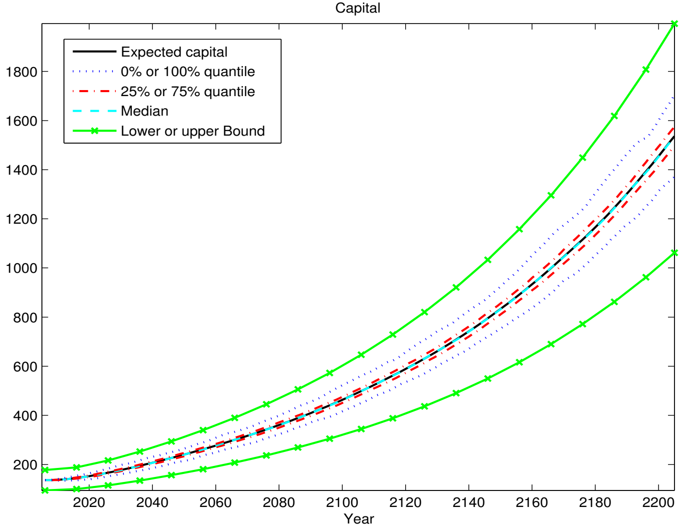
$$\zeta_t = \bar{\zeta} + \frac{1 - e^{-\kappa y}}{1 + e^{-\kappa y}} \nu\sigma$$

Figure: Probability density functions of standard normal random variable y and its transformed bounded random variable ζ



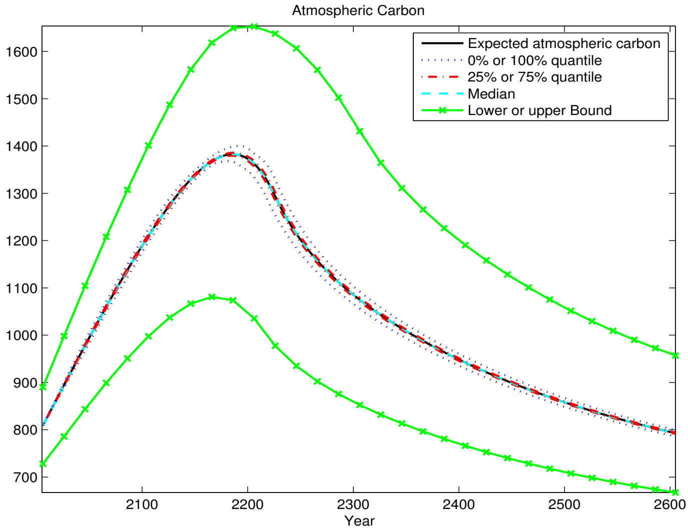
Capital Path with an Economic Shock

Figure: Capital in DSICE with an Economic Shock



Atmospheric Carbon with an Economic Shock

Figure: Atmospheric Carbon in DSICE with an Economic Shock



Surface Temperature with an Economic Shock

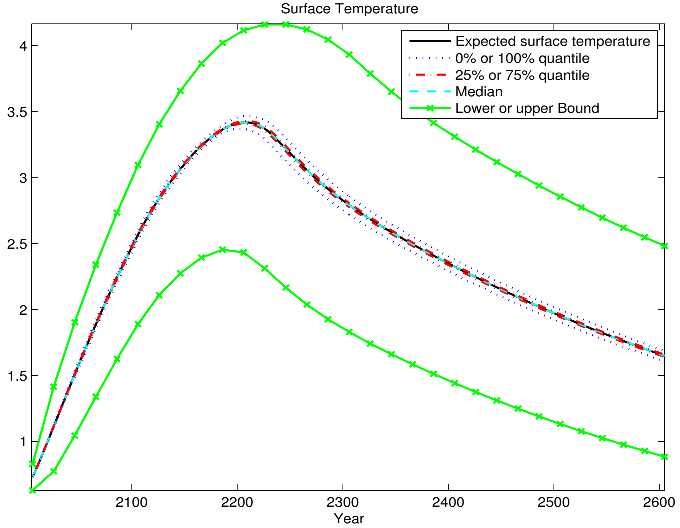
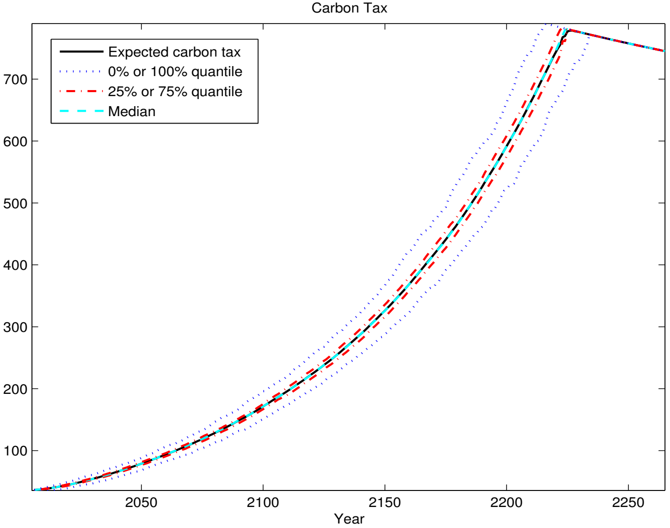


Figure: Surface Temperature in DSICE with an Economic Shock

Carbon Tax with an Economic Shock

Figure: Carbon Tax in DSICE with an Economic Shock



Tipping point

- A tipping point is where temperature causes a big event with permanent damage
- The time of tipping is a Poisson process
- Prob. of a tipping point occurring at t equals the hazard rate $h_t(T_t^{\mathrm{AT}})$
- Hazard rate
  - Simple models (e.g. DICE2007) are not well suited to describe the complex climate system.
  - A deterministic climate system is a highly complex system, and best modeled by a stochastic process
- Short time periods
  - Nature will not confine her choices for tipping times to be the beginning of each 10-year period. We have to go down to 1-year (or shorter) time periods to study tipping points.

Tipping Elements

- Lenton et al. (PNAS, 2008) characterize some major tipping elements in the earth's climate system:

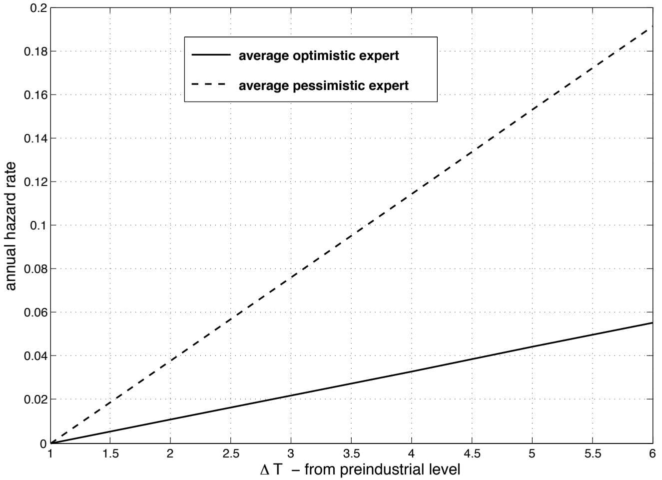
Tipping Element	Key Impacts
Thermohaline circulation collapse	sea level rise (1m), cool North Atl, warm south ocean
West Antarctic ice sheet melting	sea level rise (up to 5m)
changes in El Niño Southern Oscillation	Drought (e.g., SE Asia) + El Niño frequency and persistence
Permafrost melting	enhanced global warming due to CH_4 and CO_2 release

Calibration

- Kriegler et al. (PNAS, 2009) conduct an extensive expert elicitation on some major tipping elements and their likelihood of abrupt change.
  - THC collapse
  - Greenland ice sheet melting
  - WestAntarctic ice sheet melting
  - Amazon rainforest dieback
  - El Niño/Southern Oscillation
- They compute conservative lower bounds for the probability of triggering at least 1 of those events
  - 0.16 for medium ($2 - 4^\circ C$) global mean temperature change
  - 0.56 for high (above $4^\circ C$) global mean temperature change

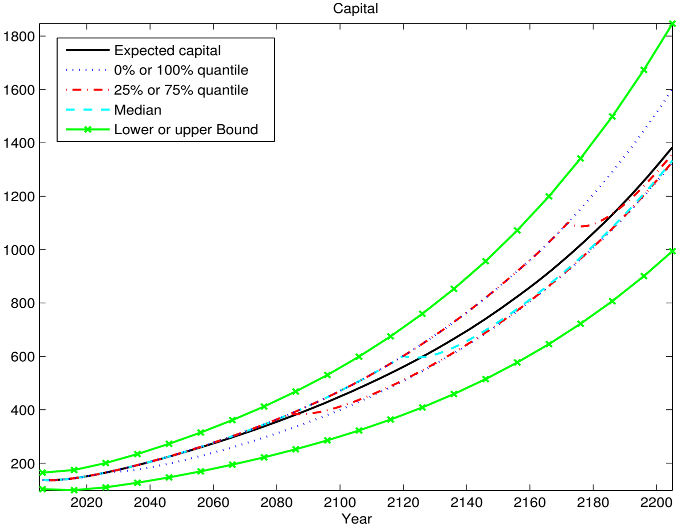
Hazard Rate

We "reverse engineer" the annual hazard rate of THC collapse as a function of global mean temperature rise based on Zickfeld et al. (2007, Climatic Change)



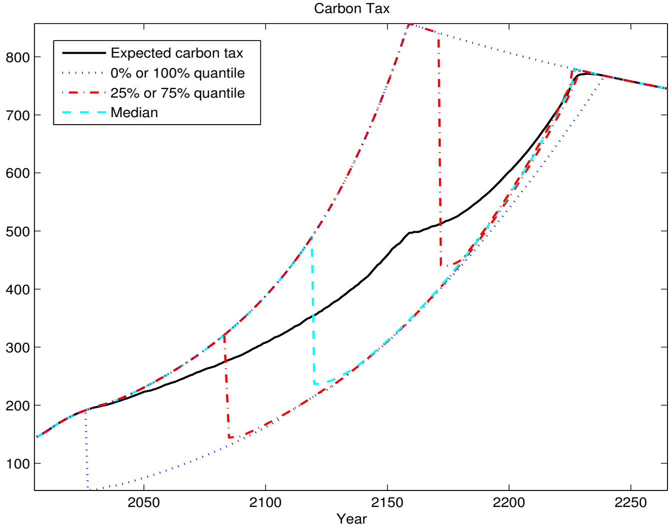
Capital with a Tipping Point

Figure: Capital in DSICE with a Tipping Point

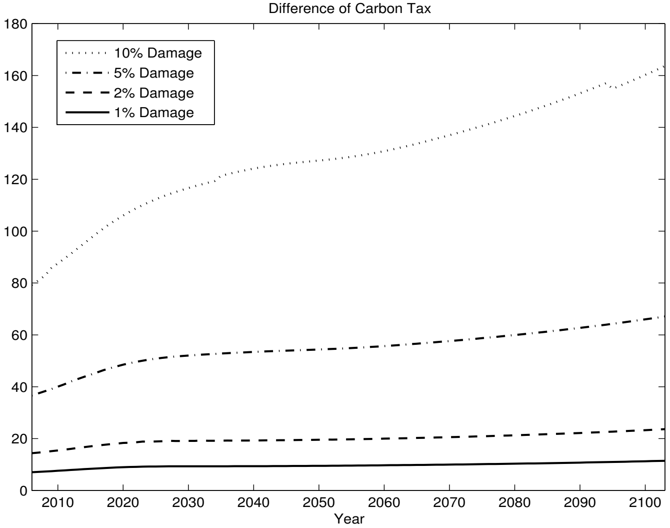


Carbon Tax with a Tipping Point

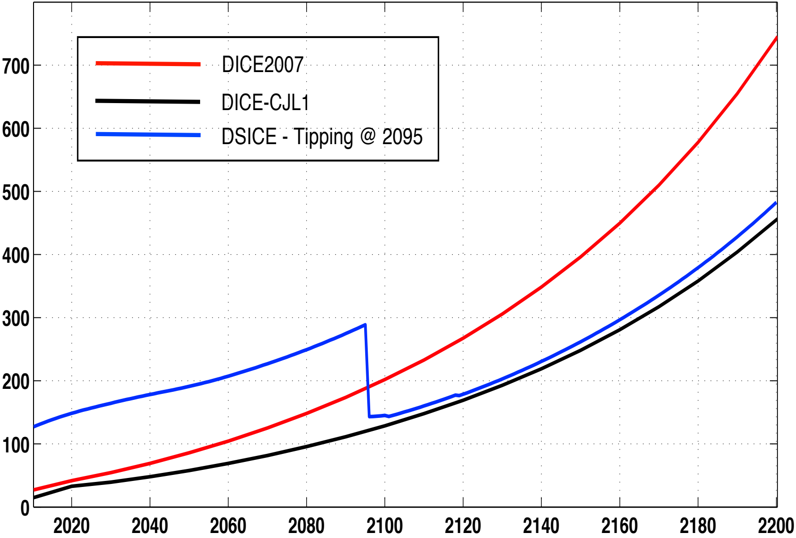
Figure: Carbon Tax in DSICE with a Tipping Point



Additional Preventive Carbon Tax

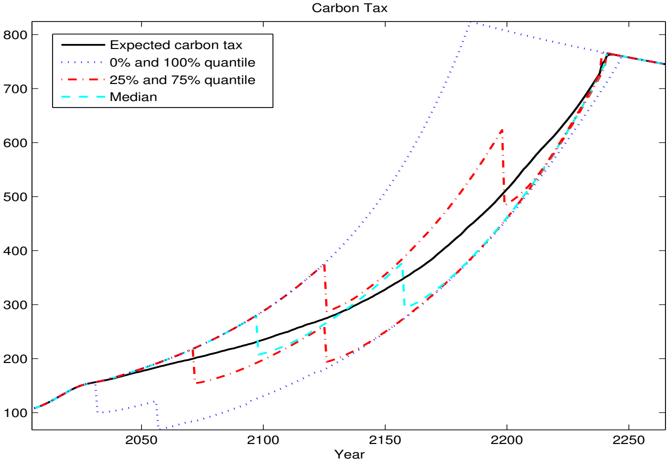


Carbon tax comparison



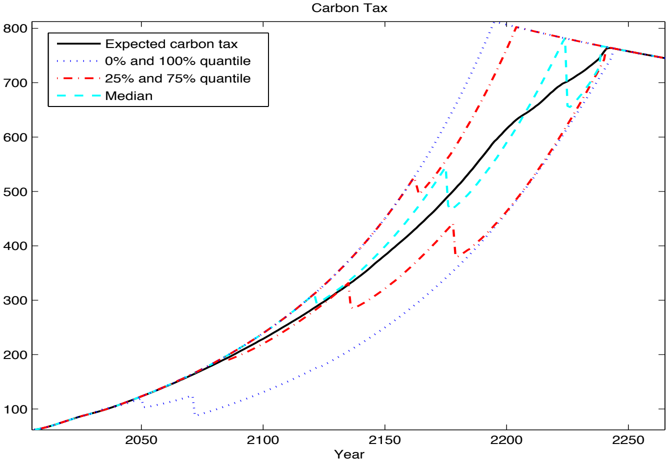
Two I.I.D. Tipping Points

Figure: Two i.i.d. tipping points, J_A and J_B, where $J_A J_B = J = 0.9$



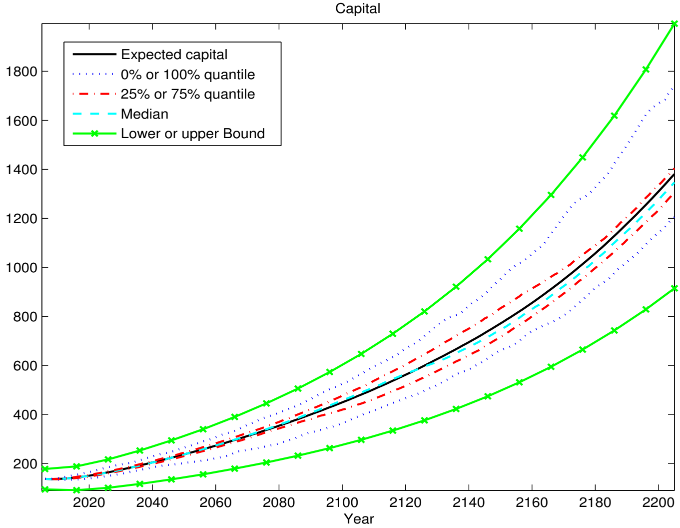
Multi-stage Tipping Process

Figure: A 3-stage tipping process with $J_1 = 0.97$, $J_2 = 0.935$, $J_3 = 0.9$



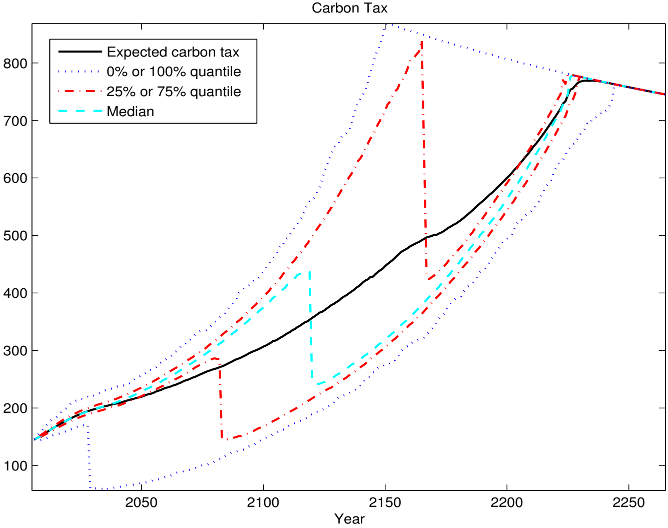
Capital Path with an Economic Shock and a Tipping Point

Figure: Capital in DSICE with an Economic Shock and a Tipping Point



Carbon Tax with an Economic Shock and a Tipping Point

Figure: Carbon Tax in DSICE with an Economic Shock and a Tipping Point



Running Times of DP for DSICE

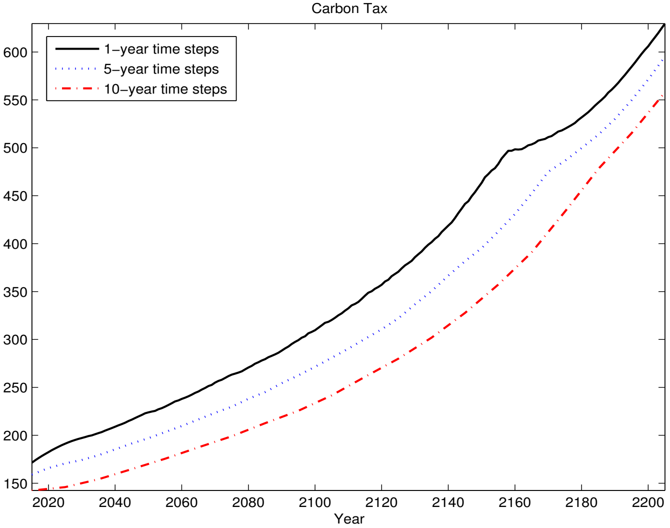
Table: Running Time of the Numerical DP Algorithm for DSICE

	Time
DSICE with an Economic Shock	2.0 hours
DSICE with a Tipping Point	17.5 minutes
DSICE with an Economic Shock and a Tipping Point	4.1 hours

Note: running on a single-core laptop.

Carbon Tax with Different Time Steps

Figure: Carbon Tax of DSICE with Different Time Steps



Summary

- Time step size with one-year or shorter is important
- Good finite difference method can improve a lot
- Dynamic stochastic IAM analysis is necessary for a coherent and reliable evaluation of policy alternatives to deal with uncertainty like Tipping Points
- Including Business-cycle Shock and/or Tipping Points into IAM substantially affects policy results
- DSICE implies an additional constant preventive carbon tax to delay a "low probability- low damage" catastrophe, despite the rising prob. of crossing a tipping point and higher expected damage
- Numerical DP is fast and reliable in solving high-dimensional problems with many stages