Risk, Diversification and Growth
ICE08 Project

Martí Mestieri
Tarik Ocaktan
Thom Sampson
David Wiczer

August 7, 2008

Outline

The Model

- Acemoğlu and Zilibotti (JPE 1997)
- OLG economy: each generation lives for two periods.
- No population growth.
- Production sector consists of two sectors.

The Model

Production Sector

Production sector consists of two sectors:

- *Final goods sector* with Cobb-Douglas production function

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha} \tag{1}$$

  with full capital depreciation $\delta = 1$.

- *Intermediate sector* transforms savings $s(t-1)$ into capital $k(t)$ to be used for production at time t. Sector consists of a continuum $[0, 1]$ of intermediates, and stochastic elements only affect this sector. There is a riskless asset $X(t)$.

The Model

Production Sector

Intermediate sector:

- Possible states of nature are also within the unit interval. Intermediate sector $j \in [0, 1]$ pays a positive return only in state j and nothing in any other state.
- Each sector has a minimum size requirement $M(j)$ such that there are positive returns only if aggregate investment, $I(j, t)$, in sector exceeds $M(j)$

$$M(j) = \max\left\{0, \frac{D}{1-\gamma}(j-\gamma)\right\} \tag{2}$$

- Intermediate sectors $j \leq \gamma$ have no minimum size requirement.

The Model

Production Sector

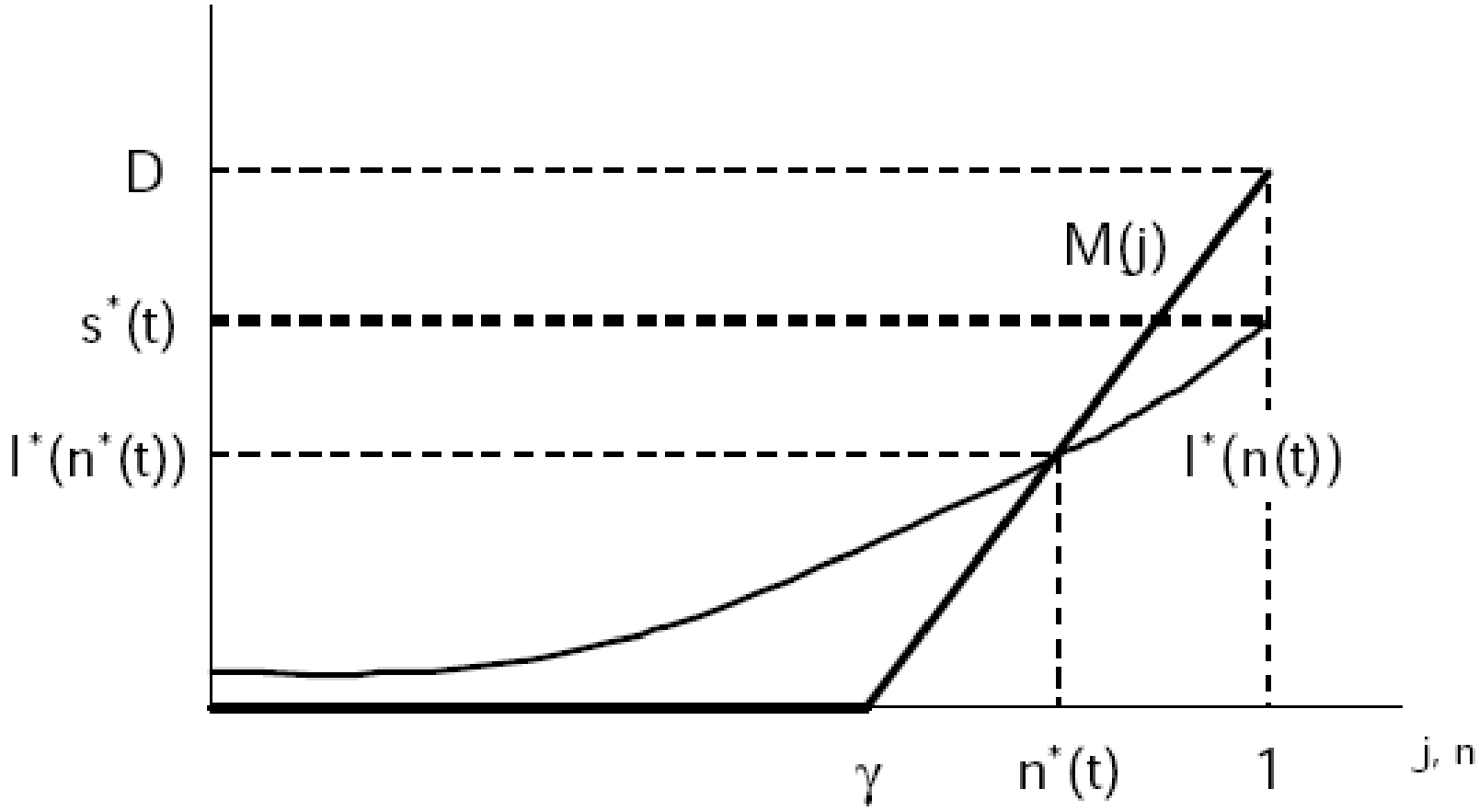


Figure: Minimum size requirements, $M(j)$, of different sectors and demand for assets, $I^*(n)$

The Model

Household Sector

- Preference of household from a generation born at time t

$$E_t U\big(c_1(t), c_2(t+1)\big) = U\big(c_1(t)\big) + \beta \int_0^1 U\big(c_2(j, t+1)\big) dj$$

- Each household has 1 unit of labor when young and no labor endowment when old

The Model

Timing of Events

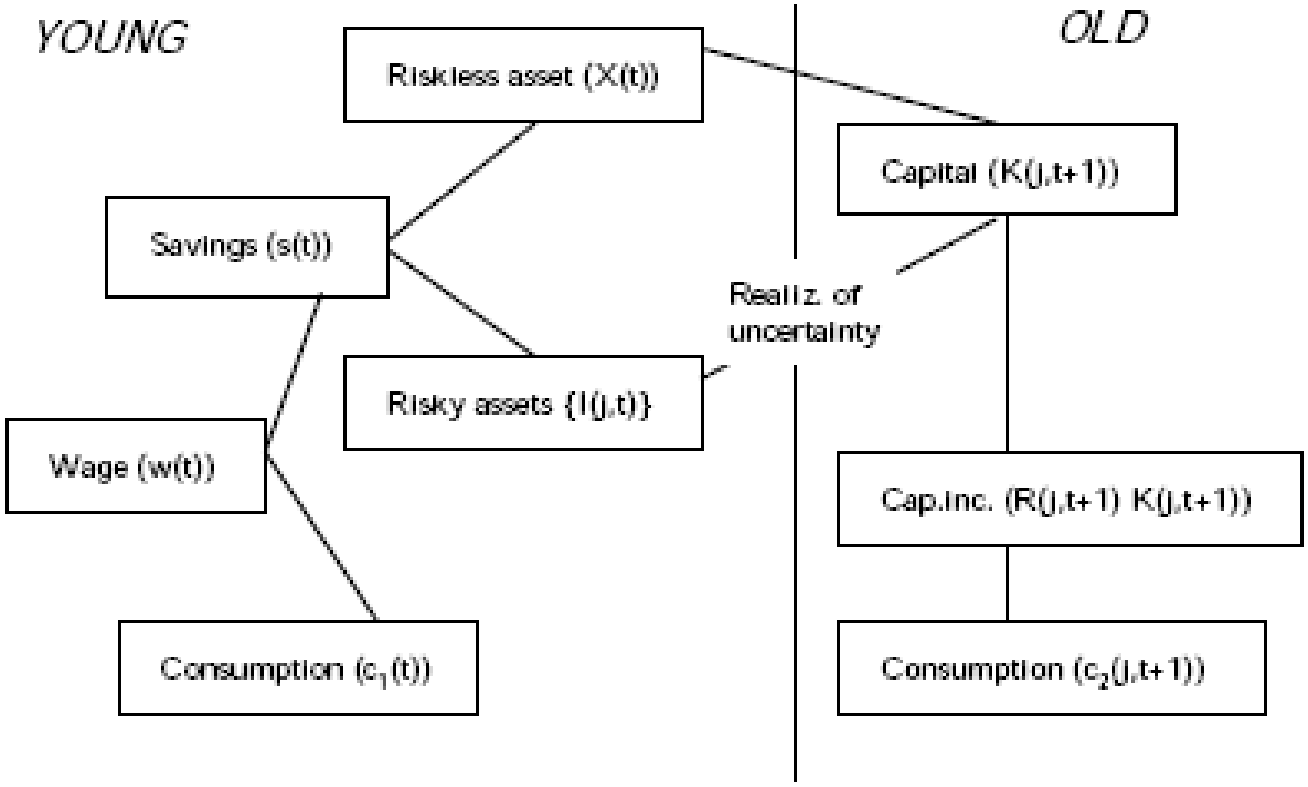


Figure: Life cycle of a typical household

The Model

Capital Stock

- In state j, the aggregate stock of capital is

$$K(j, t+1) = \int_{h \in \mathcal{H}_t} \left(qX^h(t) + QI^h(j,t) \right) dh$$

- $I^h(j,t)$: amount of savings invested by young agent $h \in \mathcal{H}_t$ in sector j at time t
- $X^h(t)$: amount invested in the safe intermediate sector

The Model

Equilibrium Factor Prices

- Wage equation

$$\begin{aligned} w(j,t+1) &= (1-\alpha)K(j,t+1)^{\alpha} \\ &= (1-\alpha)\left(\int_{h\in\mathcal{H}_t}\left(qX^h(t)+QI^h(j,t)\right)dh\right)^{\alpha} \end{aligned}$$

- Return to investment

$$\begin{aligned} R(j,t+1) &= \alpha K(j,t+1)^{\alpha-1} \\ &= \alpha\left(\int_{h\in\mathcal{H}_t}\left(qX^h(t)+QI^h(j,t)\right)dh\right)^{\alpha-1} \end{aligned}$$

The Model

Representative Household Problem

Households take prices and the set of available securities at time t as give. The problem of the representative household $h \in \mathcal{H}_t$ is given by

$$\max_{s(t),X(t),[I(t)]_{0\leq j\leq 1}} \left\{ U\big(c(t)\big) + \beta \int_0^1 U\big(c(j,t+1)\big)dj \right\}$$

$$s.t.$$

$$X(t) + \int_0^1 I(j,t)dj = s(t)$$

$$c(j,t+1) = R(j,t+1)\big(qX(t) + QI(j,t)\big)$$

$$I(j,t) = 0, \forall j \notin J(t)$$

$$c(t) + s(t) \leq w(t)$$

Outline

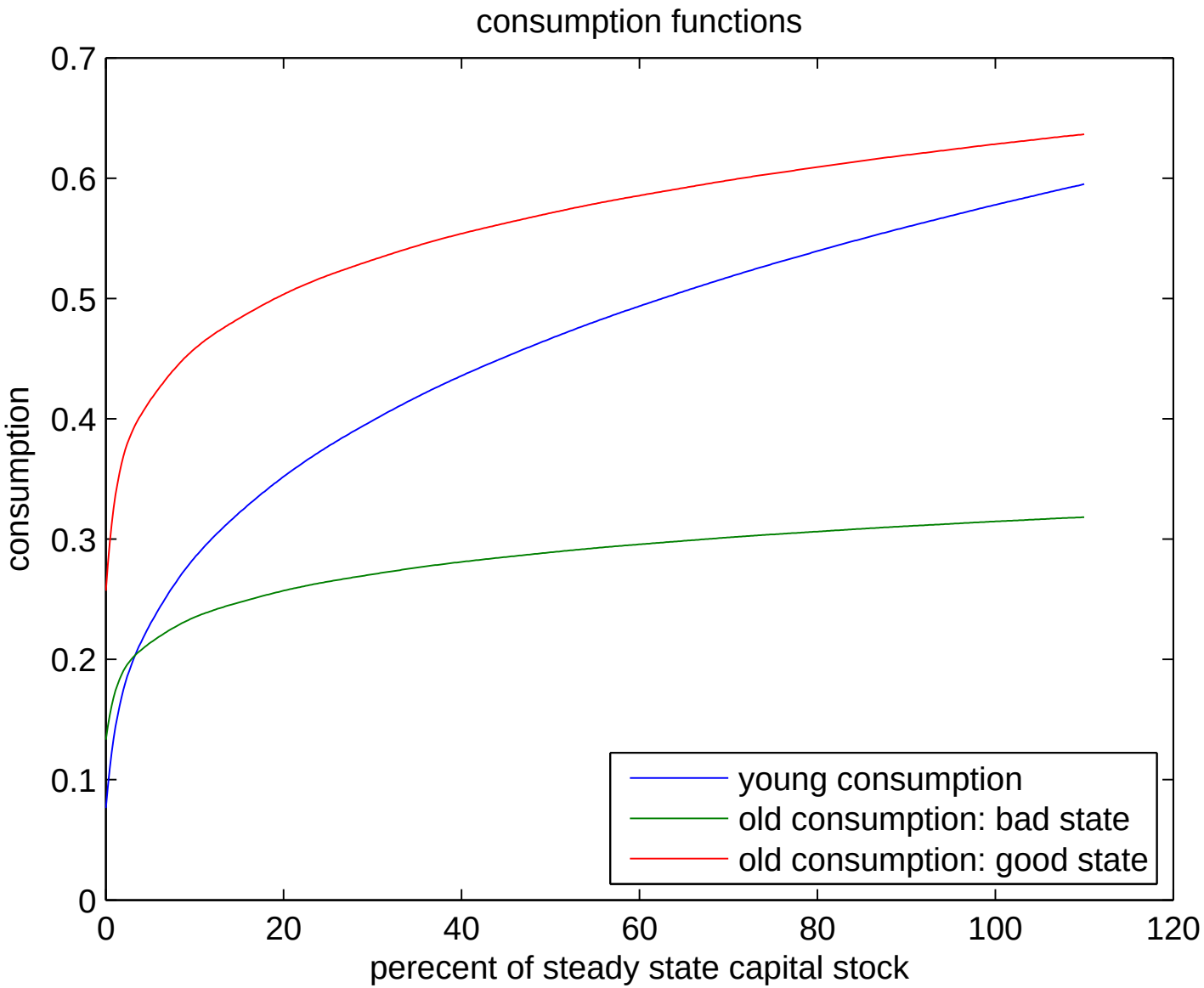
Method

Different Approaches to Solve the Problem

- Maximization problem by stating value function and market clearing constraints.
- First order condition: useful to characterize interior solutions.
- Complementarity conditions: to try complete characterization but PATHAMPL lacks appropriate algorithm.

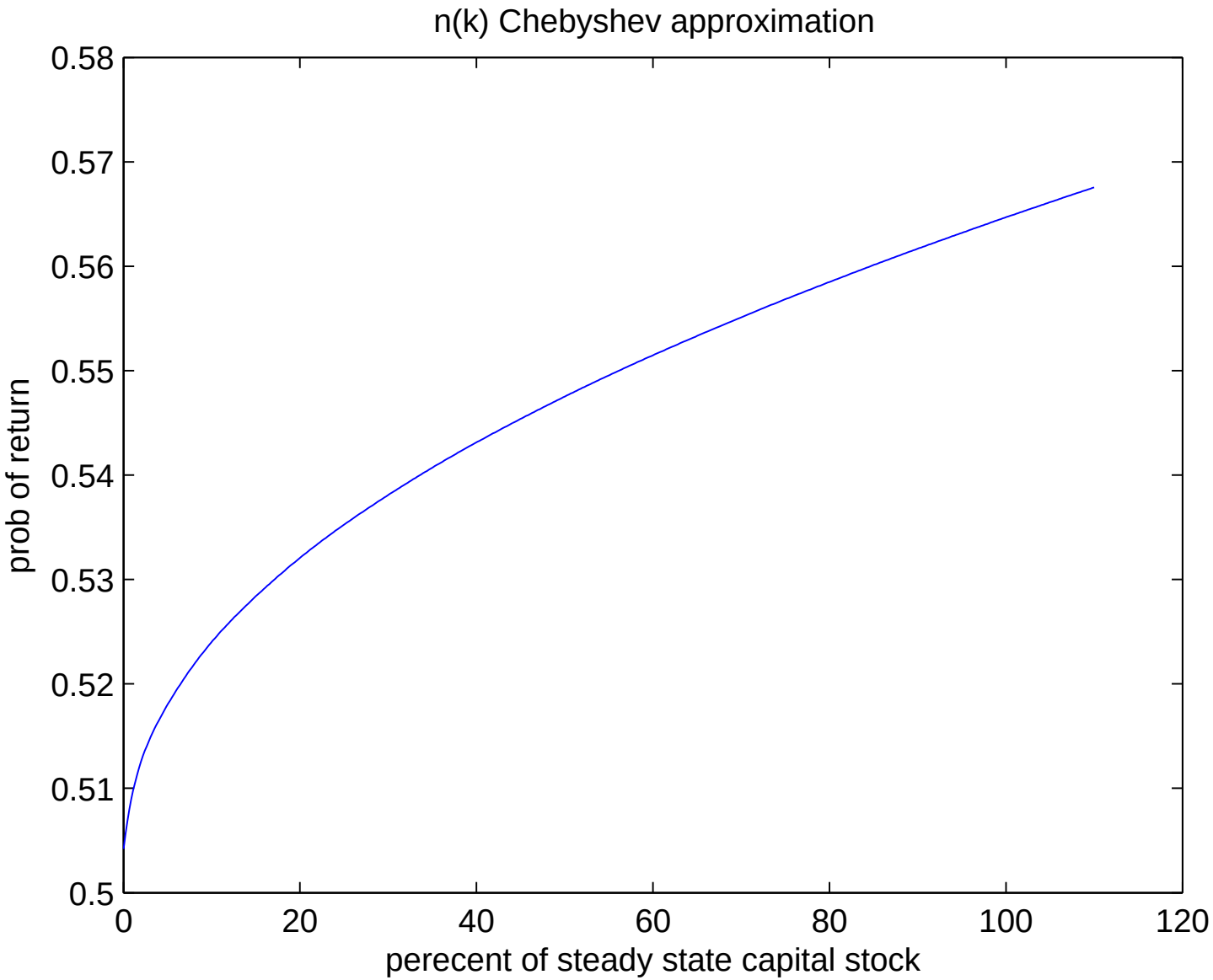
Results

Policy Functions



Results

Policy Functions



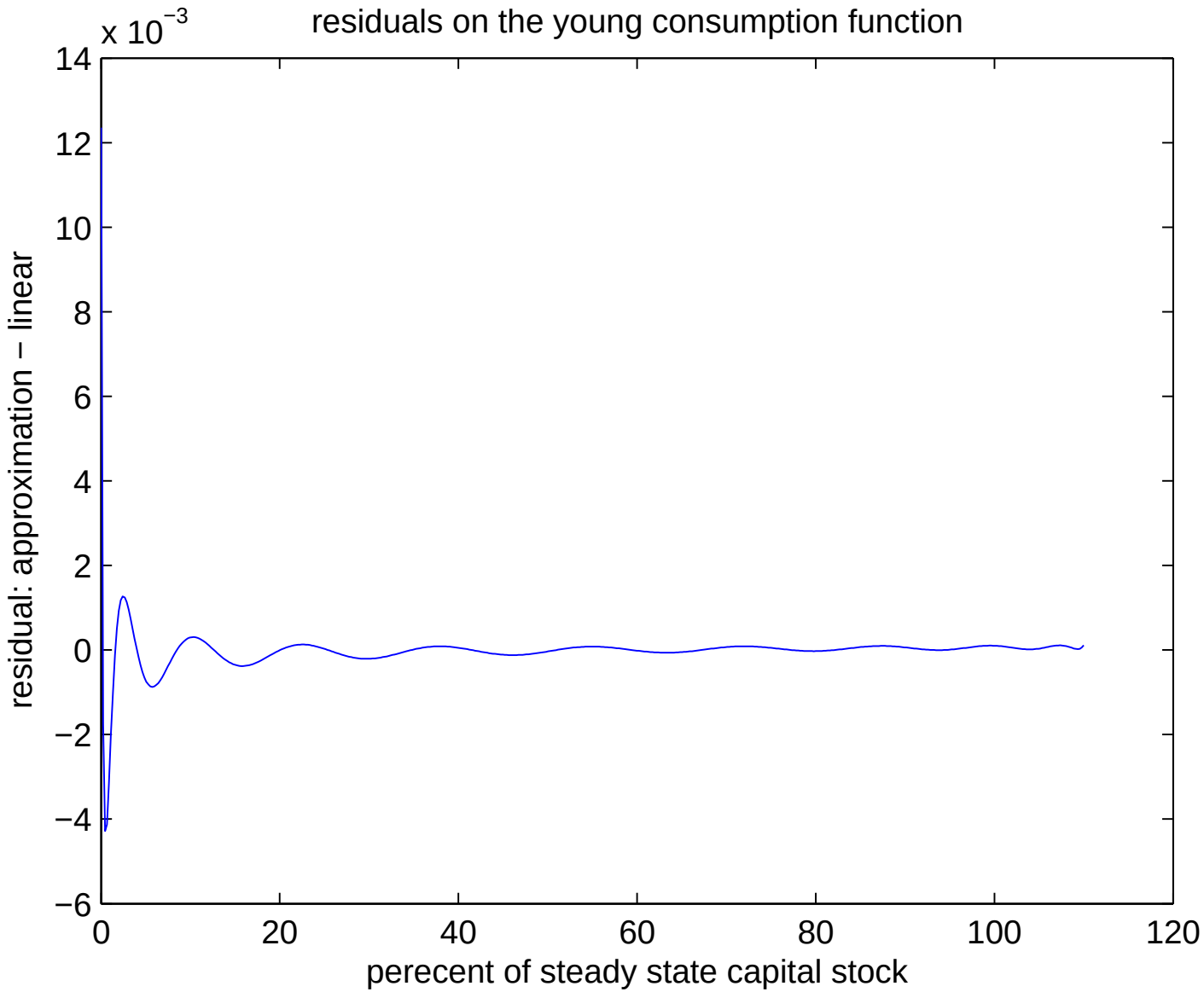
Results

Chebyshev Coefficients

1	0.54481
2	0.02682
3	-0.00581
4	0.00266
5	-0.00152
6	0.00098
7	-0.00068
8	0.00049
9	-0.00037
10	0.00028
11	-0.00022
12	0.00017
13	-0.00014
14	0.00011
15	-0.00009
16	0.00007
17	-0.00005
18	0.00003
19	-0.00002
20	0.00001

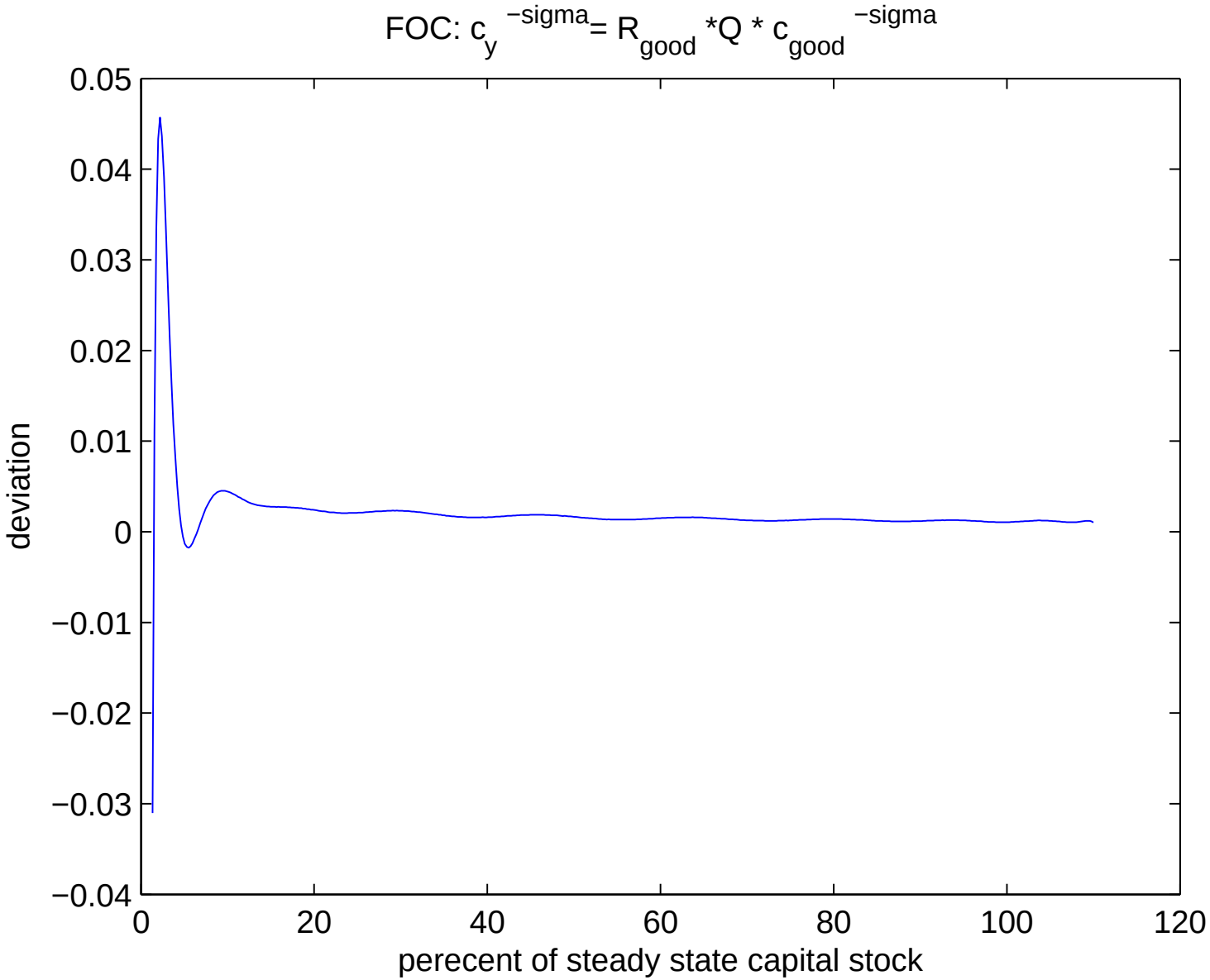
Results

Chebyshev Approximation Residuals



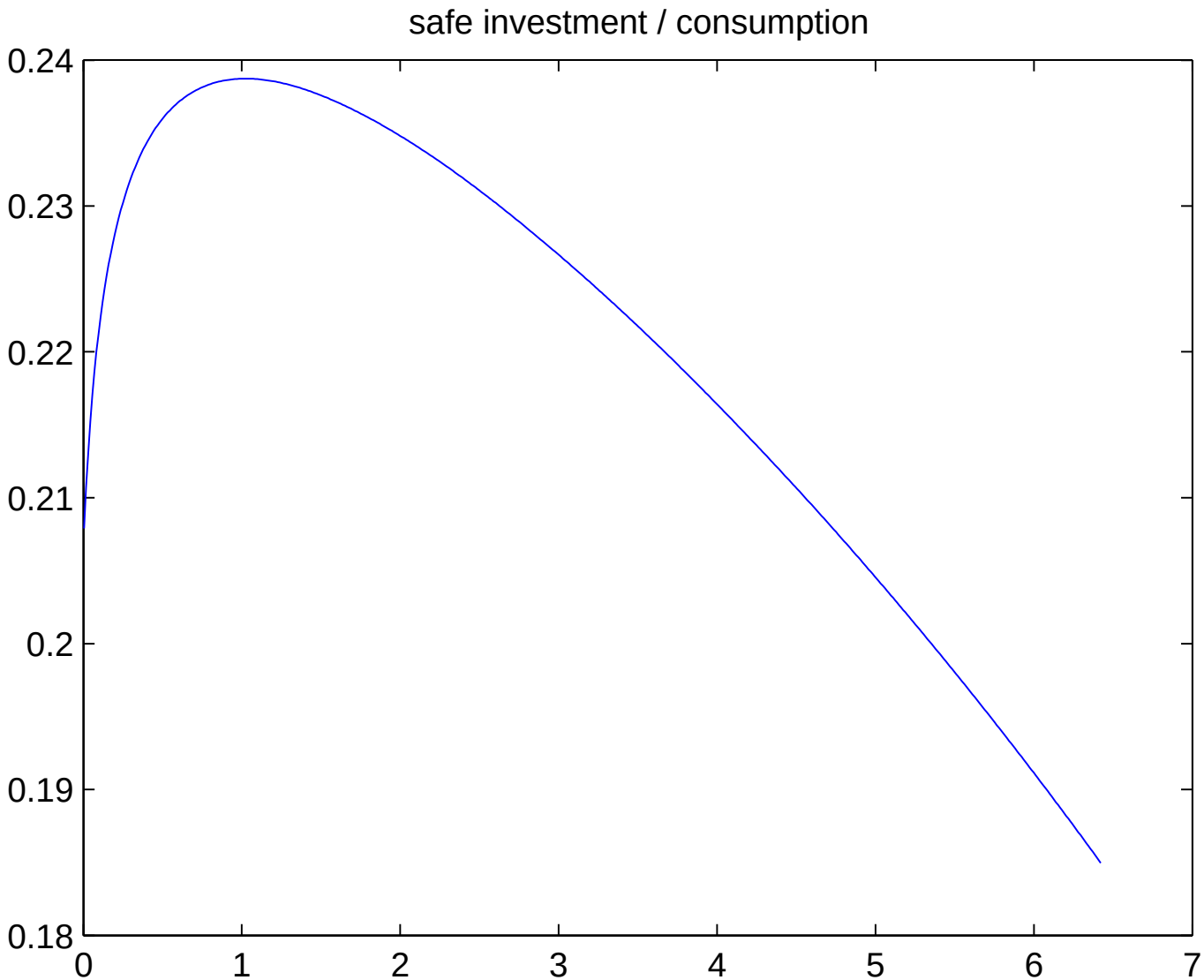
Results

Approximation Accuracy



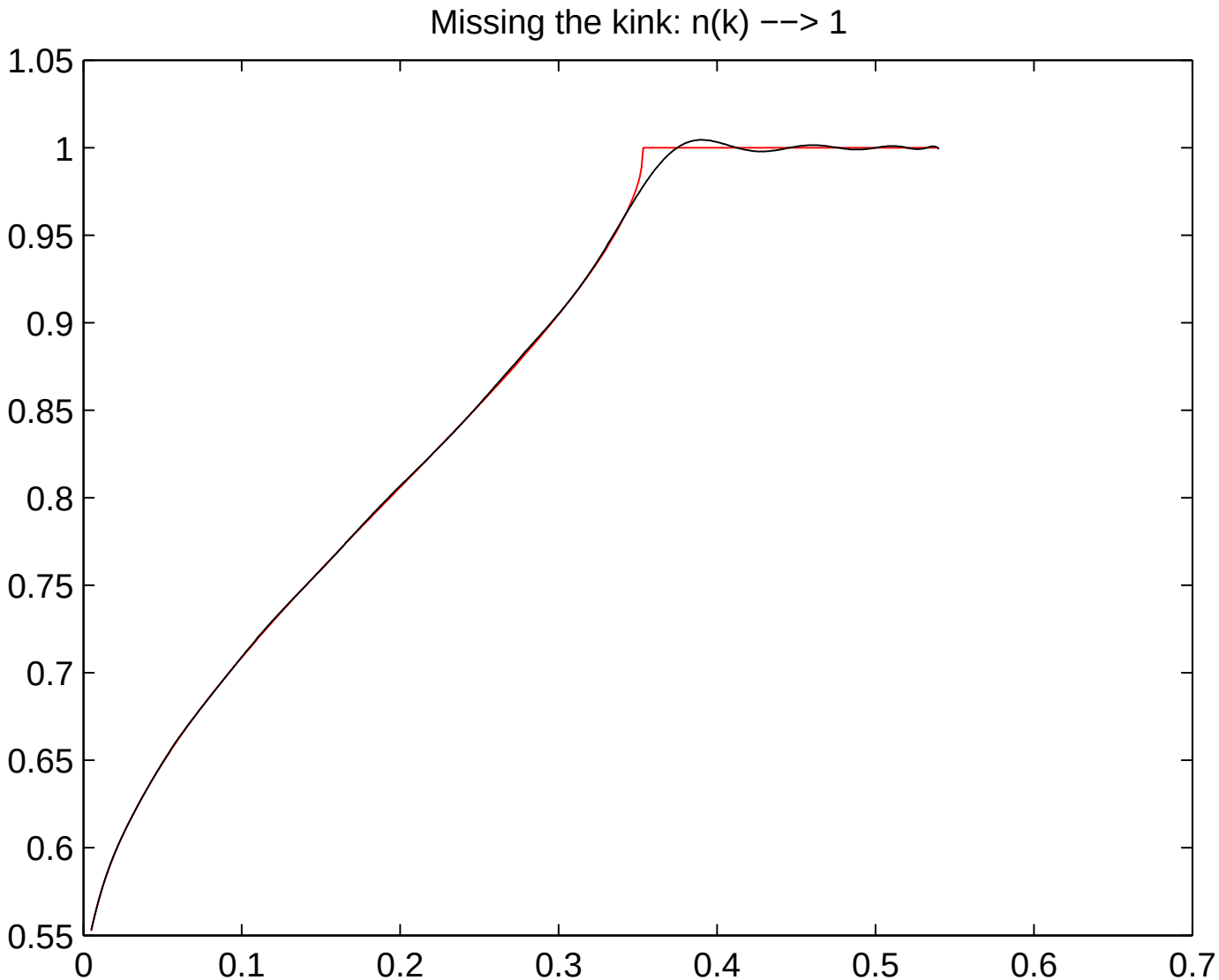
Results

Safe Investment



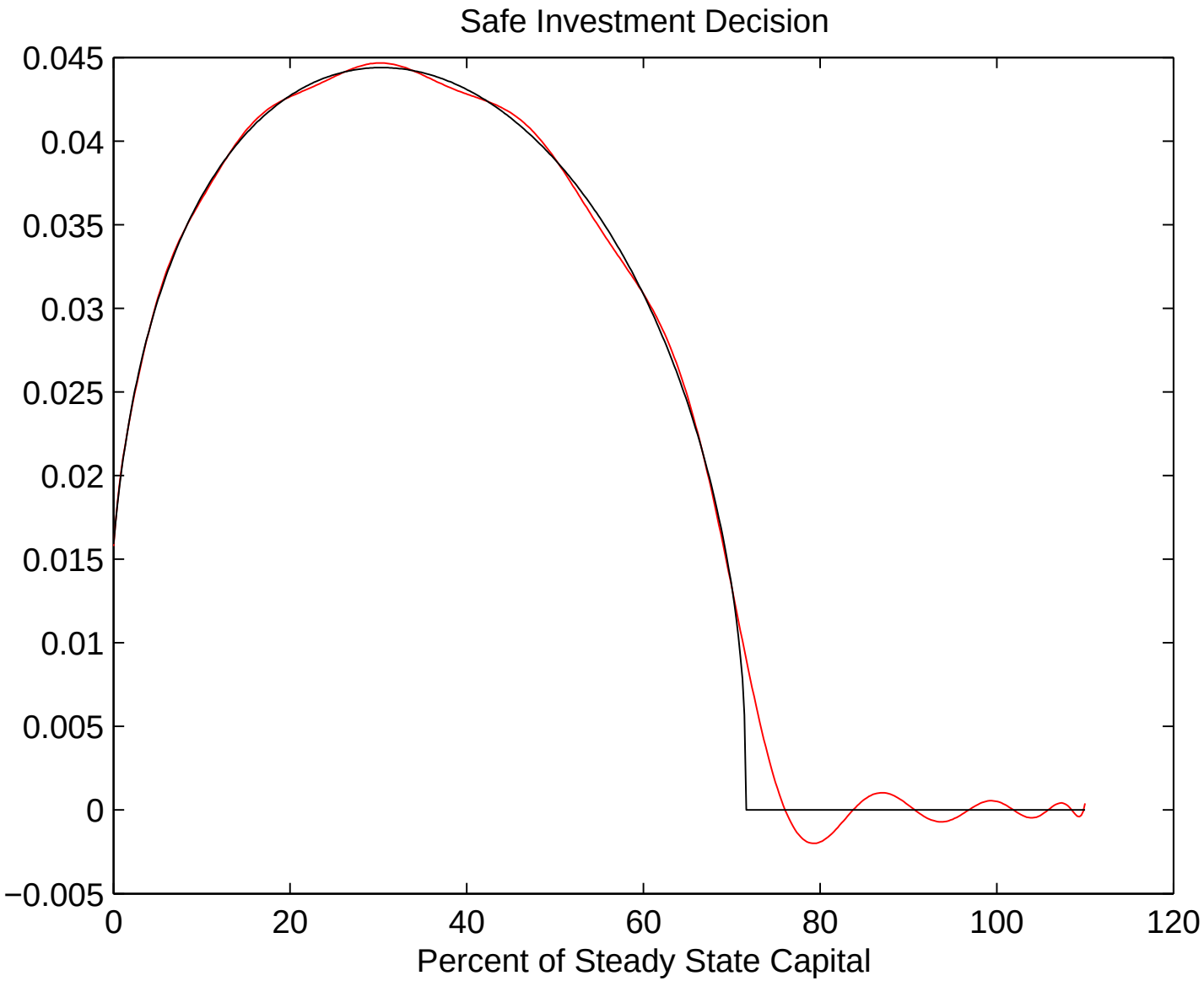
Results

Chebyshev Approximation at the Kink



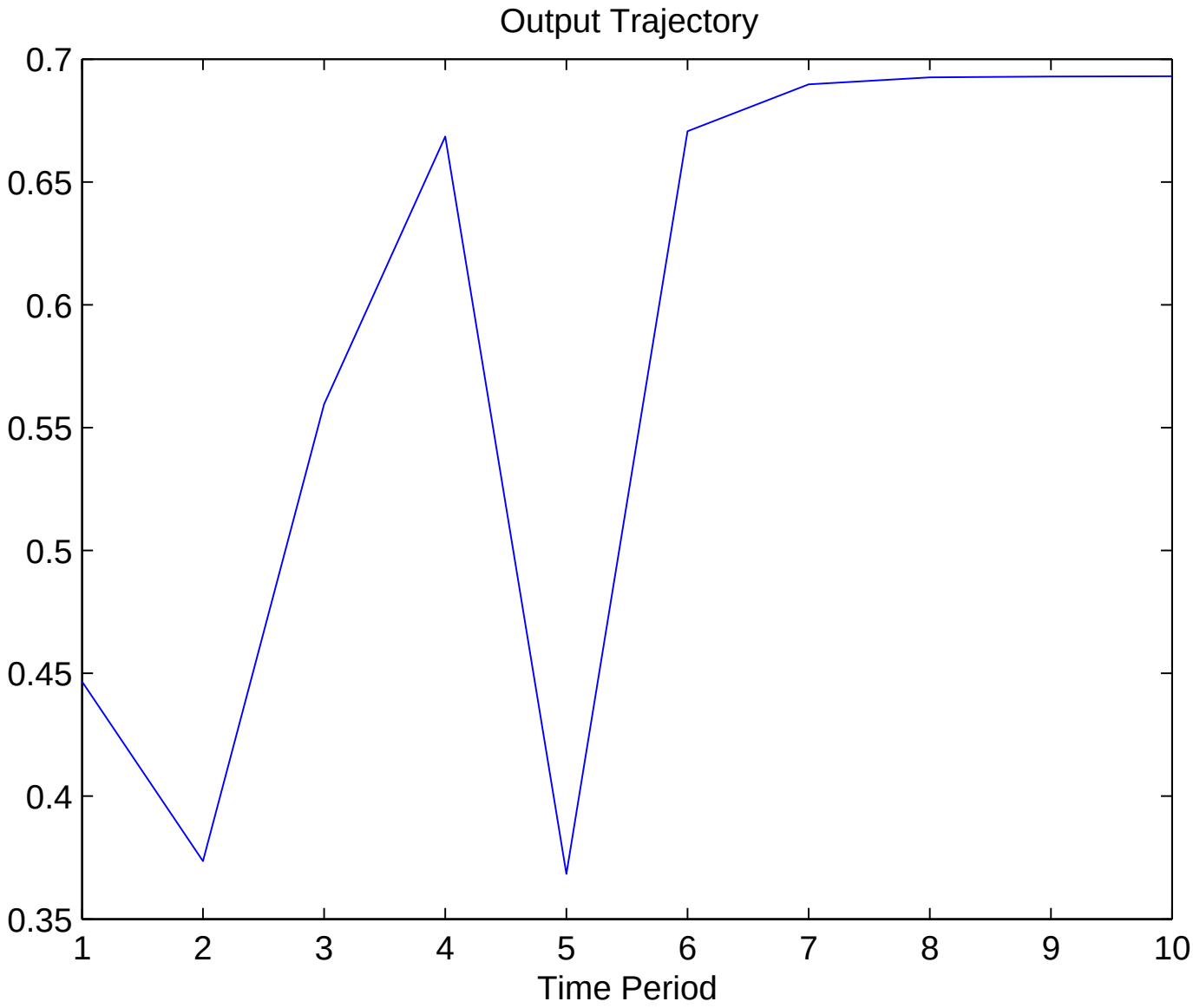
Results

Chebyshev Approximation at the Kink



Results

Simulated Trajectory



Data

Sample: 1970-2006

Year 1970	(*)
Per capita GDP mean for rich countries (A)	13 863
Per capita GDP mean for emerging countries (B)	2 075
Ratio (A/B)	6.68
Ratio of capital	225
Sample 1970-2006	
Mean GDP growth rate in rich countries	2.24%
Mean GDP growth rate in emerging countries	2.22%
Average variance in rich countries	0.05%
Average variance in emerging countries	0.17%

(*) GDP per capita (constant 2000 USD)

Data

Simulating Moments

- Back of the envelope calculation using a Cobb-Douglas production $y = k^{\alpha}1^{1-\alpha}$ function with parameter $\alpha = 0.35$, implies that an average developing country has only a .5% stock of capital of the richest.
- Fixing the coefficient or risk aversion to 4 and the share of risky activities without no fix cost in the economy to 10%. Fixing $q = 1$, given that what matters is the relative payment Q/q, we have have simulated the moments that our model predicts for different levels of D and Q.
- We have tried to match two moments: the average growth and variance of emerging countries relative to the developed. We have 2 free variables to match 2 moments.The solution seems to be around $D = .85, Q = 5.5$.