An Optimal Rule of Thumb for Pollution Permits Allocation

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Goal

- Solve a dynamic strategic game with discrete states and continuous choices (similar to the one that Karl Schmedders presented)
- Introduce complementarity conditions
- Consider this dynamic strategic game as the second stage of a leader-follower game (planner in 1st state, duopoly in the 2nd)

Environment

- 2 big polluters $\Rightarrow$ compete in a dynamic Cournot game
- There is another sector with small polluters
- Firms produce goods and need to back up emissions with pollution permits
- The 2 big polluters have market power in both the sectoral good market and the permits market
- Emissions depend on output and the emissions rate represented by θ_i $\Rightarrow$ this is our state variable
- Firms can invest to be cleaner in the future
- Transition to next state depends on investment

Game

- Firm's production q_i
- Demand function $P(Q) = A - b(q_1 + q_2)$
- Profit function of firm i in period t is:

$$\Pi_{it} = p_t q_{it} - c_t[\theta_{it} q_{it} - z_{it}] - dx_{it}^2$$

where
c_t: cost of pollution permit

$$c_t = \gamma(q^f + \sum e_i - z^f - \sum z_i)$$

q^f: emissions of small polluters before introducing permits
z^f: permits given to small polluters
z_{it}: permits given to big polluters
x_{it}: investment
dx_{it}^2: cost of investment
θ_{it}: efficiency

Dynamics

- States are $j = 1..S$
- $\theta_i = \Theta^{(j-1)}/(S-1)$ for some $\Theta \; \epsilon \; [0,1]$
- Transition probabilities depend on investment and current state
- We constrain the transition to contiguous states
  - Prob of going to lower state (more efficient - lower emission rate) $\Rightarrow \frac{\theta x}{1+\theta x+(1-\theta)}$
  - Prob of staying $\Rightarrow \frac{1}{1+\theta x+(1-\theta)}$
  - Prob of going to a higher state $\Rightarrow \frac{(1-\theta)}{1+\theta x+(1-\theta)}$
- Note that investment affects mainly the probability of reducing a firm's emissions rate

Equations

- Bellman equation
- FOC with respect to quantities q_i
- $x_i \geq 0 \perp \frac{dV}{dx_i} \leq 0$
- Three equations and one complementarity per firm per state

LOG FILE WITH PATH SOLVER

```
Major Iteration Log
  major minor  func  grad  residual    step  type prox    inorm  (label)
    0     0     3     3 7.1624e-01           I 0.0e+00 1.0e-01 (_scon[1200])
    1     1     4     4 5.9071e-03  1.0e+00 SO 0.0e+00 1.1e-03 (_scon[798])
    2     1     5     5 2.3457e-06  1.0e+00 SO 0.0e+00 5.3e-07 (_scon[1199])
    3     1     6     6 4.0926e-10  1.0e+00 SO 0.0e+00 2.5e-10 (_scon[285])
Major Iterations. . . . 3
Minor Iterations. . . . 3
Restarts. . . . . . . . 0
Crash Iterations. . . . 2
Gradient Steps. . . . . 0
Function Evaluations. . 6
Gradient Evaluations. . 6
Basis Time. . . . . . . 0.438000
Total Time. . . . . . . 0.657000
Residual. . . . . . . . 4.092554e-10
Path 4.7.01: Solution found.
5 iterations (2 for crash); 3 pivots.
6 function, 6 gradient evaluations.
```

Social Planner Problem

- Since there is market power in the permits market, initial allocation matters
- The planner uses the initial allocation as a policy instrument to maximize welfare
- A completely "non-parametric" rule is too highly dimensional (at least for us and by now)
- We solve for an optimal rule of thumb that depends on a single parameter ρ
- The planner sets for the optimal rule of thumb at time 0 and forever (taking into account strategic behavior of the firms)

$$z_i(\theta_i, \theta_{-i}) = \frac{\theta_{-i} + \rho}{\theta_i + \theta_{-i} + 2\rho} Z$$

where Z is the total number of permits

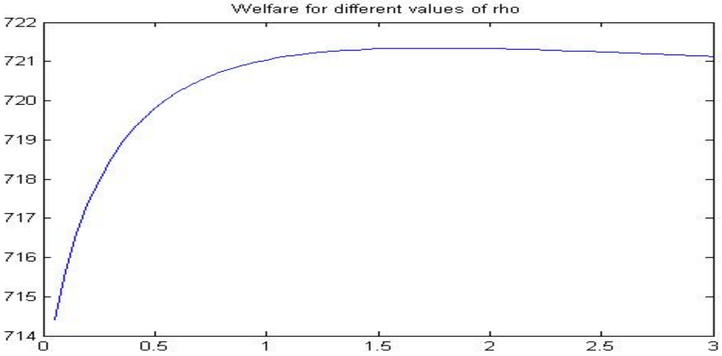
Social Planner Problem cont

$$\max_{\rho} \quad W(\theta_0;\rho)$$

$$\begin{aligned} s.t. \quad & W(\theta;\rho) = CS_{big}(\theta;\rho) + CS_{small}(\theta;\rho)\dots \\ & \qquad \dots - d\sum_i x_i(\theta;\rho)^2 + \beta\sum_s Pr(\theta'|\theta,x;\rho)W(\theta';\rho) \quad \forall i,\theta \\ & V(\theta;\rho) = \Pi(\theta;\rho) + \delta\sum_s Pr(\theta'|\theta,x;\rho)V(\theta';\rho) \quad \forall i,\theta \\ & \frac{dV}{dq} = 0 \quad \forall i,\theta \\ & x \geq 0 \perp \frac{dV}{dx} \leq 0 \quad \forall i,\theta \end{aligned}$$

Some computational issues

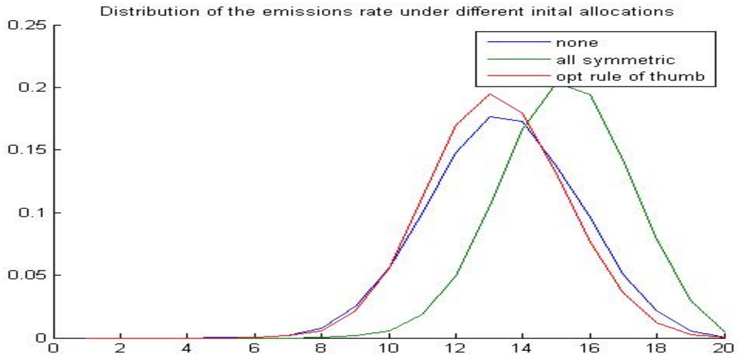
- Given that PATH was very efficient in the second stage, we thought of using the solver in this context as well
- However, the PATH solver does not allow for an objective function, so we need to move to KNITRO
- Luckily, KNITRO solves the optimization very rapidly and converges to our optimal rule of thumb
- We check SOC conditions
- We do a grid to evaluate our function and it seems well behaved. The maximum coincides with the solver solution.
- A starting value helps. We solve for the model for a fixed ρ using PATH in less than one second and then use it as an initial guess to solve the MPEC

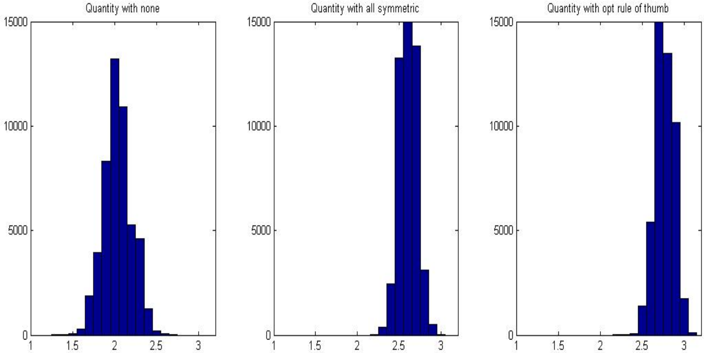

Welfare for different values of rho
722
721
720
719
718
717
716
715
714
0
0.5
1
1.5
2
2.5
3

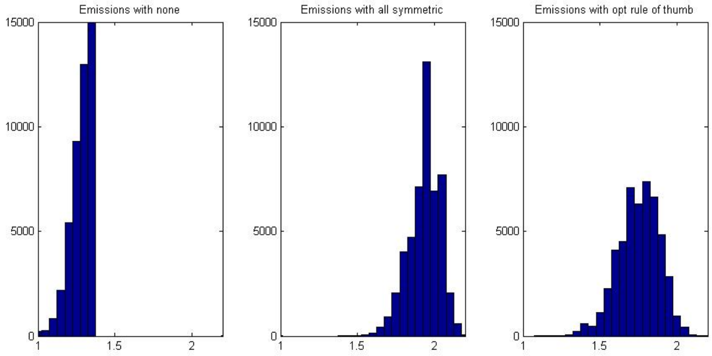
Our Experiment

We consider three different hypothetical cases:

1. The social planner gives no permits for free
2. The social planner distributes all permits to the oligopolists symmetrically
3. The social planner follows the optimal rule of thumb


Distribution of the emissions rate under different inital allocations
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10
12
14
16
18
20
none
all symmetric
opt rule of thumb


Quantity with none
Quantity with all symmetric
Quantity with opt rule of thumb
15000
10000
5000
0
1
1.5
2
2.5
3


Emissions with none
Emissions with all symmetric
Emissions with opt rule of thumb
15000
10000
5000
0
1
1.5
2

A more flexible rule of thumb

- We consider two possibilities to make our rule of thumb more flexible:
    1. Make the formula of the rule of thumb more non-parametric adding higher order terms (normalize c_1 to 1) - simulation with linear and squared terms solves well with KNITRO.

$$z_i(\theta) = \frac{c_0 + c_1\theta_{-i} + c_2\theta_i + c_3\theta_{-i}^2 + c_4\theta_i^2}{2c_0 + (c_1 + c_2)(\theta_i + \theta_{-i}) + (c_3 + c_4)(\theta_i^2 + \theta_{-i}^2)} Z$$

    2. Express the complete game as a complementarity problem by deriving the FOC conditions of the planner - taking policy functions into account. Solve using PATH.
- The polynomial approach works, and results suggest that other rules of thumb might perform better. To add more terms we should move to more adequate approximation functions
- We have tried to push the second approach, but we did not quite get there

Conclusions

- We have presented a dynamic game with complementarities within a leader-follower framework
- The dynamic game with complementarities worked efficiently with the PATH solver
- The leader-follower game did not work with the PATH solver but worked with KNITRO
- The fact that the solvers have different purposes highlights the importance of understanding the details (or looking for optimization people)
- In our application, we solved for an optimal rule of thumb to allocate pollution permits that performed well
- It could be interesting to see if we can allow for a more flexible rule that accommodates more parameters