Portfolio Choice with Borrowing Constraints

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Model Predictions

SHARE OF RISKY ASSETS IN PORTFOLIO DECREASES OVER THE LIFE-CYCLE WHEN THERE IS LABOR INCOME UNCERTAINTY.

CORRELATION BETWEEN LABOR INCOME FLUCTUATIONS AND RISKY ASSET RETURN FLUCTUATIONS INDUCES AGENTS TO HOLD MORE SAFE ASSETS.

Basic Idea

- Life-cycle model
- Partial equilibrium
- Retirement with no bequest motive
- Life begins at age 20 and ends at age 80

Timing

t ———————————— t+1

↑ ↑

AGENTS OBSERVE:

Wealth: W_t

Age: t

Income: Y_t

Timing

AGENTS CHOOSE:

Consumption: C_t

Risky Asset Holding: A_t

Risk Free Asset Holding: S_t

↓

t —————————————————— t+1

↑ ↑

AGENTS OBSERVE:

Wealth: W_t

Age: t

Income: Y_t

Timing

AGENTS CHOOSE:

Consumption: C_t

Risky Asset Holding: A_t

Risk Free Asset Holding: S_t

↓

t ——————————————— t+1

↑ ↑

AGENTS OBSERVE:

Wealth: W_t

Age: t

Income: Y_t

STATES EVOLVE:

$W_{t+1} = (1 + r_f)S_t + (1 + r_t^a)A_t$

$r_{t+1}^a = f(r_t^a, \epsilon^a)$

$Y_{t+1} = f(Y_t, \epsilon^y)$

Households

Period Utility	$u(C_t, K_t) = \frac{(C_t)^{1-\gamma}}{1-\gamma}$
Budget Constraint	$C_t + A_t + S_t = W_t + Y_t$
Nonnegativity Constraint	$C_t \geq 0$
Borrowing Constraint	$S_t \geq \underline{S}$
Short-selling Constraint	$A_t \geq 0$

State Transitions

$$
\begin{aligned}
W_{t+1} &= (1 + r_f)S_t + (1 + r_t^a)A_t \\
r_{t+1}^a &= f(r_t^a, \epsilon^a) \\
Y_{t+1} &= f(Y_t, \epsilon^y)
\end{aligned}
$$

Dynamic Decision Problem

- In period t the agent chooses a vector $\mathbf{x} = [S_t \quad A_t]'$ to maximize expected life-time utility given a state vector $\mathbf{s}$:

$$V_t(\mathbf{s}) = \max_x u_t(\mathbf{x}, \mathbf{s}) + \beta \int \hat{V}_t(\mathbf{s'}; \mathbf{a}) dF(\mathbf{s'} | \mathbf{s}, \mathbf{x})$$

- One continuous state: Wealth (W)
- Two discrete states: Risky asset return (r^a) and Labor Income (Y)

Value Function Approximation

- Approximate using n Chebyshev nodes z and n Chebyshev basis functions T:

$$\hat{V}_t = \sum_{i=0}^{n} a_i T_i(z)$$

Method

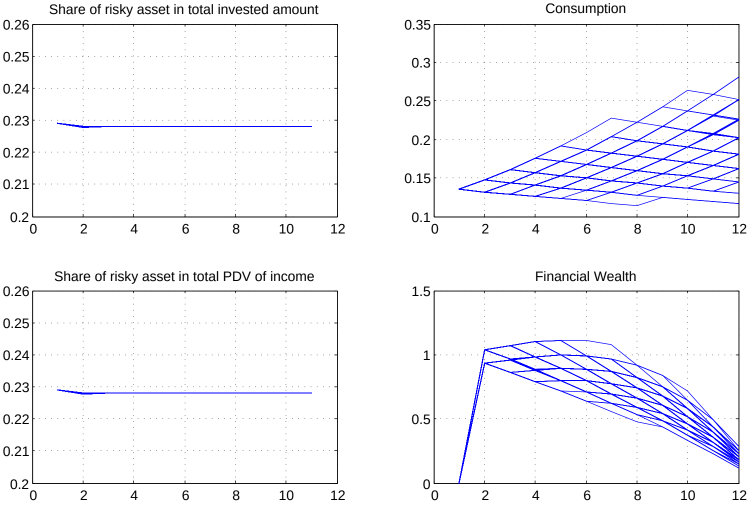
1. We approximate the value function and solve the problem via backward recursion using AMPL software.
2. Within AMPL we call the KNITRO nonlinear optimization solver to compute the optimal policy functions of the agents in each period.
3. We run Monte Carlo simulations and generate graphics in MATLAB.

Baseline Calibration

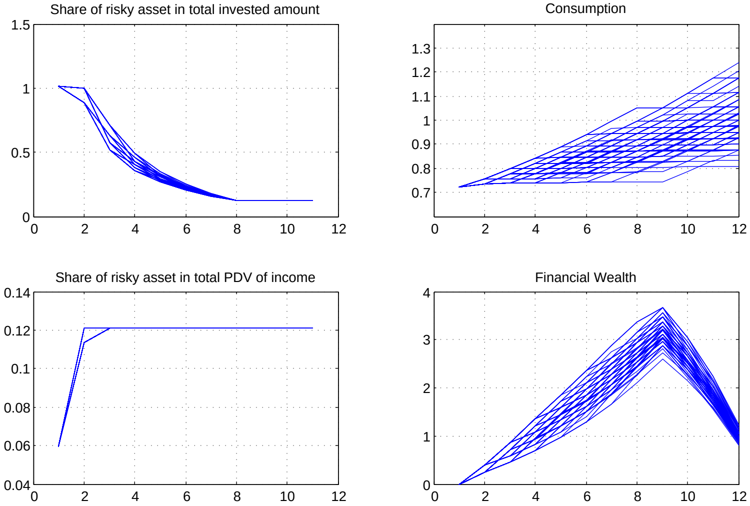
PARAMETER	VALUE	DESCRIPTION
γ	3.000	Coefficient of relative risk aversion
r_f	0.025	Risk free rate
β	0.990	Time discount factor
n	35	Order of approximation
$\underline{S}$	0.000	Borrowing constraint

- Discrete states r^a and Y take on two values each with i.i.d. shocks.

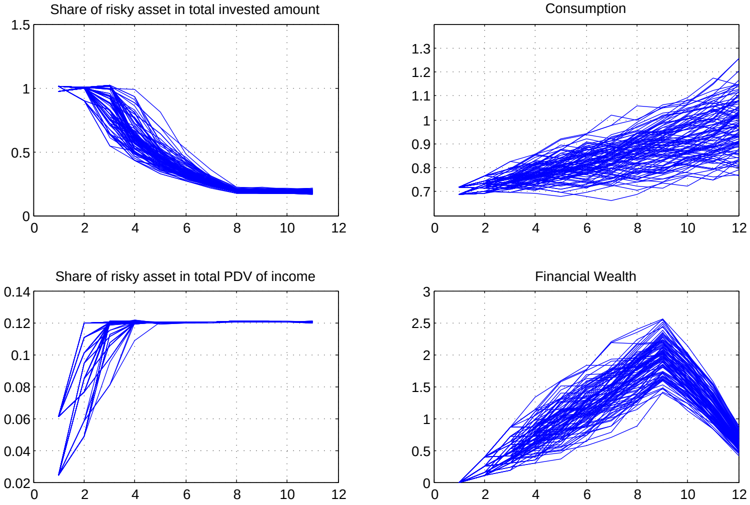
Income in First Period



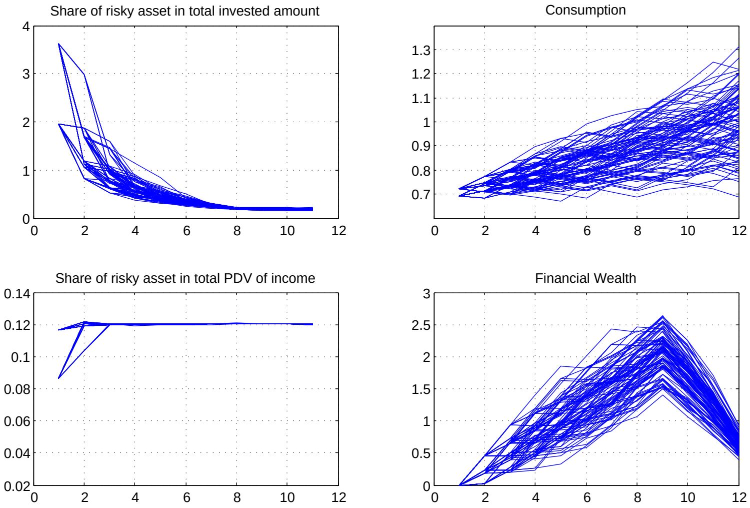
Deterministic Pre-retirement Income Stream



Income Risk and Retirement Payments



Borrowing Against Risky Asset Allowed



Correlation between Labor Income and Asset Risk