DOES IT PAY TO GET A REVERSE MORTGAGE?

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Many of today's households are at risk of having inadequate resources to support their retirement. For most retirees their home is their major asset. The bulk of their savings at retirement is typically locked in home equity and cannot be extracted for old age needs except through selling and moving out. The reverse mortgage allows one to convert equity into an income stream without making periodic loan payments or moving out. But principal plus the accumulated interest has to be paid back if the retiree definitively moves. Therefore, assessing the potential of this financial instrument requires jointly analyzing consumption, housing and mobility decisions.

This paper presents a structural, dynamic model for these decisions. More specifically, using the Mathematical Programming with Equilibrium Constraints approach, we estimate elderly preferences for consumption and housing using a subsample of single retirees from the Health and Retirement Study (HRS). We first study the optimal choice of consumption and elderly mobility in the absence of a reverse mortgage. Then, we calculate the welfare gain for HRS respondents in taking out a reverse mortgage. This exercise shows that the reverse mortgage eases the liquidity problem and provides longevity insurance. However, it introduces a new risk: the moving risk. The risk of moving and having to repay the cumulated debt on reverse mortgages creates a major welfare loss for those with initial low financial wealth. Common belief is that a reverse mortgage benefits those with resources tied up in home equity. This paper shows otherwise.

KEYWORDS: Housing, Consumption, Elderly Mobility, Dynamic Discrete and Continuous Choices, Constrained optimization approach

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1 Introduction

Baby boomers started retiring in 2001, signing the beginning of an accelerated rate of growth in the number of retirees in the US. By 2030, one out of five people is projected to be 65 years or older. Despite this rapid augment in the retired population, the financial situation for future retirees remains uncertain. As a matter the fact, increases in the cost of living and in health care costs, curtailments in medical coverage and other employee benefits plans and cutbacks in Social Security benefits expose many of today's households at risk of having to adjust to a decreased standard on living in retirement. An analysis of their financial portfolio shows clearly that for most retirees their house is their major asset. More than 80 percent of older households own their homes (Munnel et al, 2007), which are worth approximately $4 trillion. Economists and policymakers look at these financial assets as a potential source of savings to finance consumption in the elderly. According to the "life cycle" hypothesis of saving and consumption, individuals build savings during their working years and divest those savings to support consumption in retirement. However, when it comes to home equity, this pattern is not followed. Typically, older households do not divest home equity. Instead, homeownership rates remain stable until late in life and median home equity increases with age as older homeowners pay off mortgages and home value appreciates.

Before the advent of the reverse mortgage, only two alternatives were available to older homeowners to divest their home equity. They could sell and move out or they could borrow against their house property taking a conventional loan, such as mortgage or home equity loan. Traditional loans have to be repaid, either through installments or on maturity and older homeowners are often neither eager nor able to incur in new monthly obligations. Therefore, selling and moving out represented the best way to en-cash the savings locked up in residential property.

In the 1990s, reverse mortgages became available and provided a new way to convert home equity into cash. A reverse mortgage is a financial instrument that allows borrowers to access the equity in their home that would otherwise not be liquid, by providing income while not requiring payment as long as the borrower lives in the same house. When the retiree moves out or dies, the reverse mortgage lender keeps the minimum between the house value and the outstanding debt. The amount of money that could be borrowed via a reverse mortgage generally depends on the borrower's age and the value of the home. The minimum age for almost all reverse mortgage programs is 62.

A 2005 study by Stucki estimated the potential market at 13.2 million older households. However, at the end of 2007, only 265,234 federally insured reverse mortgages were issued (Department of Housing and Urban Development, 2007b). This represents about 1% of the 30.8 million households with at least one member age 62 and older in 2006 (U.S. Census Bureau,2006). The small percentage of older households with reverse mortgages makes us ask the following question: does it pay to get a reverse mortgage? Will reverse mortgages remain in the future a small niche product in the future or will they become a commonly used tool to finance consumption in retirement? Is moving out still the best option available to older retirees?

Moving for financial reasons is only one type of elderly mobility. Other types of elderly mobility include moving for assistance reasons, changes in marital status, climate or weather, health problem or services, desire to change neighborhoods or the location, shopping or other consumption services, and public transportation.

This paper presents a structural dynamic model of retiree consumption, housing and moving decisions. More specifically, in each period the individual chooses whether to continue living in the same house or move to a new house. If she moves out, she can either buy or rent a new house and she chooses the new house value. The main sources of income during retirement are social security, pensions and investment income, which are assumed not to vary over time for the single retiree. Given our focus on retirement, income from labor is not considered.

Data on households with a reverse mortgage are not available, hence we select a sub-sample of single retirees from the Health and Retirement Study (HRS) that could represent a potential target segment for this financial instrument. We first estimate the preference parameter between housing and consumption and the risk aversion of the households in our sample ,then, simulate how better or worse off they would be if they choose a reverse mortgage contract. We compare this result with the case in which households, instead of closing a reverse mortgage contract, simply liquidate their assets by moving out and renting. Our analysis shows that a reverse mortgage provides liquidity and longevity insurance, however, moving becomes a risky proposition. As a matter the fact, if the homeowners move out, they have to repay the minimum between the house value and the outstanding debt. Both consumption and housing profiles are affected in the periods following the move. We also show that moving and renting a new house generate the household's highest welfare gain. This choice provides liquidity, does not increase the household's level of indebtedness and does not introduce any moving risks. This result might explain why, after almost twenty years from its first appearance, the reverse mortgage market is still at 1% of its potential.

The solution method is innovative in two main respects. This is the first application of the MPEC (Mathematical Programming with Equilibrium Constraints) to an empirical structural model with a life-cycle dynamic programming problem involving a continuous state variable. The current econometric literature seems to dismiss the MPEC approach as computationally infeasible. However, Judd and Su (2008) shows that this approach is feasible if one uses the standard methods in the mathematical programming literature. They apply the MPEC approach to the canonical Zurcher bus repair model (Rust,1987). Second, there is an extensive literature focusing on the solution of discrete choice models. The framework was first introduced by Rust (1987,1988). Both the theoretical literature and most of the subsequent applications focus on discrete decision processes. However, given that our study involves both discrete and continuos choices and these data are present in our sample, we extend the existing literature including also continuous choices.

The structure of the paper is as follow. First, we present the features of a reverse mortgage contract and evaluate the lender's expected gain. Second, we present the household's life-cycle model. Third, we describe the solution method. Fourth, we show the results and the welfare analysis.

2 Reverse Mortgage

Reverse mortgages are home loans that do not have to be repaid as long as the borrower lives in the house. The borrower can receive the proceeds in one of the following ways: a lump sum at the beginning, monthly payments until a fixed term or a life-long annuity, by establishing a credit-line with or without accrual of interest on the credit balance, or a combination of the aforementioned. To be eligible for a reverse mortgage, a borrower must be 62 or older, own the home outright (or have a low loan balance) and have no other liens against the home. The retiree does not have to satisfy any credit or income requirements. A reverse mortgage accrues interest charges, beginning when the first payment is made to the borrower. When she dies or relocates, the minimum between the house value and the loan plus the cumulated interest has to be repaid. Even if the accumulated loan and interest exceed the realizable value of the house at disposal, the repayment is capped at that value only.

The amount of loan is a function of the age of the borrower and any co-applicant, the current value of the property and expected property appreciation rate, the current interest rate and interest rate volatility, closure and servicing costs and other specific features chosen.

A reverse mortgage is just one of several financial instruments that allow a homeowner to secure liquid funds against the equity in a house. In general, Home Equity Conversion Products could be useful to all those who are "house-rich but cash-poor" and, together, could enhance social welfare. [1]

Most of the empirical estimates fOR the potential of reverse mortgages to meet the financial needs of the elderly are from the public policy perspective . They are based on various federally sponsored surveys (American Housing

[1] The Home Equity Conversion Products include the following products, in addition to reverse mortgage. Home reversion / sale and lease back allows the homeowner to sell his house outright now, but she keeps the right to live in it for life for a nominal/reduced rent. The sale profits could be paid in a lump sum or as an annuity. The interest-only mortgage allows the borrower to get an immediate lump sum. She is required to make only interest payments during the tenure of the loan and the principal is due only on maturity or death or a permanent move or sale. The mortgage annuity/ home income enables the individual to use the loan amount buy a life annuity. The interest on the mortgage is deducted from the annuity and the balance is paid as periodic income. The principal is repaid on death or sale of the house. The shared appreciation mortgage provides loans at a below market interest rate. The lender obtains a pre-agreed share in any appreciation in the property value over the accumulated value of the loan The loan is due at death or moving or sale.

Survey, Survey of Income and Program Participation), rather than on surveys specifically to assess the potential for RM. Meyer and Simons (2004) estimated that over six-million homeowners in the United States could increase their effective monthly income by at least 20% by using a reverse mortgage. Of these, more than 1.3 million have no children. In addition, over 1.4 million poor elderly persons could raise their income above the poverty line. They also show that almost five million households could receive a lump sum twice as large as their current holdings of liquid assets, giving them access to resources in case of financial emergencies without losing their home.

Merrill, Finkel and Kutty (1998) use a different data set and more restrictive criteria to identify the prime target group for RM: the relatively older among the elderly (age>70 yrs); low income (<30,000 $/yr); high home equity (between 100,000 to 200,000 $); and a strong desire to remain in their current home (length of stay >10 yrs). They estimated such households to be 800,000 in late 1980s.

Kutty (1994) focused specifically on the potential of RM to lift the elderly above the poverty line. Based on a 1991 survey, the author estimated that 621820 such households, constituting 18% of all elderly poor households could be brought above the poverty line. The author advocated strong public policy support for RM. However, when we consider the actual reverse mortgage market, it is very small. At the end of 2007, only 265,234 federally insured reverse mortgages were issued (Department of Housing and Urban Development, 2007b). This represents about 1% of the 30.8 million households with at least one member age 62 and older in 2006 (U.S. Census Bureau, 2006). Stucki(2005) assumed that about half of older persons own homes with sufficient equity to be considered candidates for a reverse mortgage, which yielded an estimate of only about 2% of the potential market.

3 Is the Reverse Mortgage a fair contract? Lender's Perspective

In our model, we suppose that the reverse mortgage borrower i chooses to receive the proceeds as a lump sum at the closure of the contract in time j.

At the closure of the reverse mortgage contract, the lender's initial cost is the lump sum payment $\overline{B}$ and the revenue stream includes origination fees and service fees F.

If the borrower moves out of the house or dies at time t, that person would be required to repay the minimum between the house value and the outstanding debt:

$$\min(H_{it}, G_{it}^{RM})$$

where G_{it}^{RM} is the outstanding debt at time t. It is given by the initial lump sum payment $\overline{B}$ and cumulated interests:

$$G_{it}^{RM} = \overline{B} \sum_{j=1..t} (1+i_D)^{t-j} \tag{1}$$

where i_D is the nominal interest rate on reverse mortgage. In present value, the repayment in period t for household i is:

$$RM_{it} = \frac{\min(H_{it}, G_{it}^{RM})}{R^{t-j}} \tag{2}$$

Let $n_{i,t}$ household i's probability of being alive at time t and $m_{i,t}$ her probability of moving at time t. The expected gain for the lender is:

$$EGain_{j,i} = F + \sum_{t=j+1..T} n_{i,t-1}\{(1-n_{i,t})(1-m_{i,t}) + n_{i,t}m_{i,t}\}RM_{it} \tag{3}$$

A simple calculation, without taking into account the interest rate and the house price risk, shows that a homeowner with a house value equals to $100,000 could borrow about $47,000, $31,000 , or $10,000 respectively if she closes a Monthly Adjusting HECM, a Annually Adjusting HECM, and a Fannie Mae HomeKeeper contract at age 62. This represents the actual cost for the lender. Given women survival probabilities and US mobility rate, the expected gain for the lender is about $74,000, $64,000, $ 30,000. If we assume that F is a cost for the lender and we do not include it in equation (3), the expected gain is $54,000, $63,000, $22,000 respectively.

4 The Model

This section describes a model of post-retirement decision making. We consider the optimal consumption and housing decision for an individual from age 64 until age T. The individual dynamically chooses consumption, housing tenure and housing size. We allow the investor to acquire housing services through either renting or owning a house. In each period the individual chooses whether to continue living in the same house or move to a new house. When the individual decides to move, transaction costs are incurred. Consistent with the data, we assume that the household that moves cannot buy a larger house size and can rent any possible house size.

4.0.1 Preferences

Individual i's plan is to maximize her expected lifetime utility at age t, $t = 64, ..., T$. T is set exogenously and equals 95. In each period she receives utility U_{it}, from non-durable consumption C_{it} and housing services H_{it}, so that $U(C_{it}, H_{it})$.

The within-period retiree's preference over consumption and housing services are represented by the Cobb-Douglas utility function:

$$U_{it}(C_{it}, H_{it}) = \frac{(C_{it}^{1-\omega} H_{it}^{\omega})^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it}) \tag{4}$$

where C_{it} denotes consumption, H_{it} the housing value, ω measures the relative importance of housing services versus numeraire nondurable good consumption, γ is the coefficient of relative risk aversion.

We assume that $\varepsilon_{it}(d_{it})$ is independent across individuals and time. It is Extreme Value Type I distributed. It represents housing preference shock. Individuals move out of their home for several reasons, which are explained in detail in our survey. Some households move out for financial reasons, looking for a smaller or less expensive house Others because they desire to live near or with their children or other relatives, for health problem, for climate or weather reasons, for reasons related to leisure activities or public transportation and for change in marital status. We model this unobserved utility from moving as housing preference shock.

When the individual dies, her terminal wealth, TW_{it}, is bequeathed according to a bequest function $b(TW_{it})$:

$$b(TW_{it}) = \theta_B \frac{TW_{it}^{1-\gamma}}{1-\gamma} \tag{5}$$

4.1 Choice set

In each discrete period t, the household makes both discrete and continuous choices. Without loss of generality, we assume that the household makes two main decisions, housing and consumption. In particular, the household makes the discrete housing decision and, conditional on the housing decision, she makes the continuous consumption decision.

4.1.1 Discrete Choice: Housing

The housing choice is a multi-stage decision. First, the household decides whether to move or stay. The household that moves out makes the choice of owning or renting a new house and the house size of the new house. Consistent with our data, the retiree that moves could not afford a larger house.

Let d_{it} define household i's choice of housing.

First of all, the household makes the discrete choice of staying or moving out in period t:

$$d_{it}^1 = \begin{cases} D_{it}^M = 1 & \text{if household } i \text{ moves out of her house in period } t \\ D_{it}^M = 0 & \text{otherwise} \end{cases}$$

Second, if she moves out of the house, she makes the binary choice of owning or renting a new house.

$$d_{it}^2|d_{it}^1 = \begin{cases} D_{it}^O = 1 & \text{if household } i \text{ owns her house in period } t \\ D_{it}^O = 0 & \text{if household } i \text{ rents her house in period } t \end{cases}$$

The third stage decision over housing is the house value.

$$d_{it}^3|d_{it}^1, d_{it}^2 = H_{it+1}$$

Therefore, the discrete choice set is:

$$d_{it} = \{d_{it}^1, d_{it}^2, d_{it}^3\}$$

4.1.2 Continuous Choices: Consumption

Let C_{it} be the choice of consumption, conditional on the first choice of housing.

4.2 Housing Expenses

Per-period housing expenses ψ are a fraction of the market value of the house. We assume that ψ is constant across individuals with the same housing level and deterministic. They depends on D_i^O, the housing tenure indicator variable which is equal to one for homeowners and zero for renters, and on D_{it}^M. For homeowners, housing expenses represent a maintenance cost, sustained to keep the house at a constant quality. For renters, housing expenses represent the rental cost. The market value of the house is a function of house price per unit and house size. For both homeowners and renters, the housing expenses are assumed to be a constant value over time, denoted by ψ^{own} and ψ^{rent} respectively.

$$\psi_{it} = [D_i^O \psi^{own} + (1 - D_i^O)\psi^{rent}][D_{it}^M H_{it+1} + (1 - D_{it}^M) H_{it}] \tag{6}$$

If the retiree decides to sell her house at time t and move to another house, she pays (receives) the difference in owner-occupied housing wealth. In addition, she incurs a one-time transaction cost $\phi(D_{it+1}^O)$. We assume that households could not buy a larger house but could rent a house of any size. The renters that move are allowed only to rent a new house. The cost of moving is:

$$M_{it} = D_{it}^M D_{it}^O [D_{it+1}^O H_{it+1} - H_{it} + H_{it+1}\phi(D_{it+1}^O)] + D_{it}^M (1 - D_{it}^O) H_{it+1} \phi^{rent} \tag{7}$$

The transaction cost equals a fraction $\phi^{own}(\phi^{rent})$ of the market value of the new house when the investor moves to an owner-occupied (a rental) house, i.e.

$$\phi(D_{it+1}^O) = [D_{it+1}^O \phi^{own} + (1 - D_{it+1}^O)\phi^{rent}] \tag{8}$$

Typically we have larger moving costs for the case of a retiree that buys a new house, that is $\phi^{own} > \phi^{rent}$.

5 Solution Method

The solution method is innovative in two main respects. First, this is the first application of the MPEC approach to a dynamic life-cycle model with structural estimation of the preference parameters. Second, we estimate the structural model including both discrete and continuous choices.

The traditional approach in solving dynamic structural models consists of repeatedly taking a guess of the structural parameters and solving for the corresponding endogenous variables. This is computationally very demanding. We use the MPEC approach (Mathematical Programming with Equilibrium Constraints) to solve our empirical model. This approach consists in formulating the life-cycle dynamic programming problem and the maximum likelihood estimation of the preferences as a constrained optimization problem. The idea behind the MPEC approach is to choose structural parameters and endogenous economic variables simultaneously and symmetrically, to solve the dynamic programming and the maximum likelihood problems in a one-step procedure. After having formulated the expressions defining the objective and the equilibrium equations (the constraints), we submit them to one of the state-of-art optimization solvers. The direct use of the state-of-art algorithms implies that we do not need to make any decision about the algorithmic details (such as, for example, choosing between value function iteration or policy iteration).

In several respects, the MPEC approach could not be considered completely new, since it is based on ideas and methods developed in statistics and econometrics literature. Nevertheless, the current econometric literature tends to dismiss this approach as computationally infeasible. Judd and Su (2008) argue that the constrained optimization approach is feasible if one uses standard methods in the mathematical programming literature. They apply the MPEC approach to the canonical Zucher bus repair model. We extend their approach, presenting the MPEC with Dynamic Programming.

There is an extensive literature focusing on the solution of discrete choice models. The framework was introduced by Rust (1987,1988) and then extended in Hotz and Miller (1993) and Aguirragabiria and Mira (2002). The focus, both in the theorical literature and in most of the subsequent applications, is on discrete decision processes. However, given that our study involves both discrete and continuous choices and these data are present in our sample, we extend the existing literature including also continuous choices in the model. The continuous state variable is the financial assets and the continuous choice variables is consumption. The discrete and discretized variables are the moving decision, the owning decision, and the housing level.

The solution method is described in detail in the Technical Appendix.

5.1 State Space

The state space in period t consists of variables that are observed by the agent and the econometrician X_{it} and by variables observed only by the agent ε_{it}.

$$X_{it} = \{W_{it}, H_{it}, D^O_{it}, Age_t\}$$

where W_{it} is household i's non-housing financial wealth at time t, H_{it} the beginning of period house value, and D^O_{it} the beginning of period housing tenure.

The term ε_{it} references a vector of unobserved utility components determined by the discrete alternative:

$$\varepsilon_{it} = \{\varepsilon_{it}(d_{it})\}$$

5.2 Heterogeneity

Given the observed realization of household choices and states $\{d_{it}, C_{it}, X_{it}\}$, the objective is to estimate the preferences denoted as $\theta = \{\gamma, \omega, \sigma\}$

Optimal decisions depend on the state variables, X_{it} and ε_{it}.

The constants are denoted as $K = \{\psi^{own}, \psi^{rent}, \phi^{own}, \phi^{rent}, C_{MIN}\}$

We allow for heterogeneity in the state variables, X_{it} and ε_{it}, but not in preferences θ.

5.3 The Household's Problem

The household maximizes the expected lifetime utility over housing tenure $\{D^o_{it}, D^M_{it}\}$, consumption C_{it} and housing value H_{it}.

We assume that the starting period is age 64. The value function of the problem is defined as:

$$V_t(X_{it}, \varepsilon_{it}) = \max_{d_{it}, C_{it}} E_t \left[\sum_{t=62}^{T} \beta^{t-64} (N(t-1,t) n_t U(C_{it}, H_{it}) | X_{it}, \varepsilon_{it}) + b(TW_{it}) \right] \tag{9}$$

s.t

$$W_{it+1} = RW_{it} + ss - C_{it} - G_{it} - \psi_{it} - M_{it} + \varepsilon_{it}(d_{it}) \tag{10}$$

$$C_{it} \geq C_{MIN}, \qquad H_{it} \geq H_{MIN}, \tag{11}$$

where q_t denote the probability of being alive at age t conditional on being alive at age $(t-1)$, and let $N(t,j) = (1/n_j) \prod_{k=1}^{t} n_k$ denote the probability of living to age t, conditional on being alive at age j. W_{it} denotes non-housing financial wealth after the house loan payment. $\varepsilon_{it}(d_{it})$ is i.i.d. error distributed as a normal with mean 0 and variance σ^2_y which represents out-of-pocket expenses.

The choice variables are included in d_{it}.and C_{it}.

Eq. (10) denotes period t retiree i's budget constraint. Let ss denote the retiree's income, which includes Social Security benefits and pension .

Eq. (11) defines the retiree i's constraints on consumption and house size at age t.

5.4 The Bellman Equation

The retiree faces two initial alternatives: *stay* or *move*. If the individual stays in the previous house, the choice variable is consumption. If the individual moves, the choice variables are consumption, housing tenure and new house value.

The value function for period t is given by the following expression:

$$\begin{aligned} V_{it}(X_{it},\varepsilon_{it}) &= \max_{d_{it},C_{it}} \tfrac{(C_{it}^{1-\omega}(D_{it}^M H_{it+1}^{\omega}+(1-D_{it}^M)H_{it}^{\omega}))^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it}) + \\ & \beta(n_{t+1}E[V_{it}(W_{it+1},(D_{it}^M H_{it+1}+(1-D_{it}^M)H_{it}^s),\varepsilon_{it+1})]+b(TW_{it+1})) \\ & st \\ W_{it+1} &= RW_{it}+ss-G_{it}-C_{it}-\psi_{it}-M_{it} \\ C_{it} &\geq C_{MIN}, \qquad H_{it}\geq H_{MIN}, \end{aligned}$$

The optimal policy function for period t is given by:

$$\begin{aligned} \pi_{it}(X_{it},\varepsilon_{it},\theta) = & \arg\max_{d_{it},C_{it}} \tfrac{(C_{it}^{1-\omega}(D_{it}^M H_{it+1}^{\omega}+(1-D_{it}^M)H_{it}^{\omega}))^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it}) + \\ & \beta(n_{t+1}E[V_{it}(W_{it+1},(D_{it}^M H_{it+1}+(1-D_{it}^M)H_{it}^s),\varepsilon_{it+1})]+b(TW_{it+1})) \end{aligned}$$

The computation of the optimal policy functions is complicated due to the presence of the vector $\varepsilon_{it}(d_{it})$.It enters nonlinearly in the unknown value function EV_{it+1} Therefore, following Rust (1988), we make the following assumptions:

ASSUMPTION 1 (Additivity): The within period utility function has the additively separable representation:

$$U(d_{it},C_{it},X_{it},\theta) = \frac{(C_{it}^{1-\omega}H_{it}^{\omega})^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it})$$

ASSUMPTION 2 (Conditional Independence): The conditional probability function of the state variables is given by:

$$f(X_{it+1},\varepsilon_{it+1}|d_{it},C_{it},X_{it},\varepsilon_{it},\theta) = q(\varepsilon_{it+1}|X_{it+1})g(X_{it+1}|d_{it},C_{it},X_{it},\theta)$$

Therefore EV_{t+1} does not depend on ε_{it} and the Bellman equation can be rewritten as:

$$
\begin{aligned}
V_t(X_{it}, \varepsilon_{it}) &= \max_{d_{it}, C_{it}} [U(d_{it}, C_{it}, X_{it}, \theta) + \varepsilon_{it}(d_{it}) + \beta q_{it} EV(X_{it+1})] \\
&= \max_{d_{it}} \left\{ \left[\max_{C_{it}} \{[U(d_{it}, C_{it}, X_{it}, \theta) + \beta q_{it} V(X_{it+1})] | d_{it}\} \right] + \varepsilon_{it}(d_{it}) \right\}
\end{aligned}
$$

Therefore, the solution of period t's problem could be divided in two parts without loss of generality. There is an inner maximization with respect to the second stage continuous choices conditional on the first stage discrete choices and then an outer maximization with respect to the first stage discrete choices.

5.5 Outer Maximization

Let $r(X_{it}, d_{it})$ represent the indirect utility function associated to the first stage choice d_{it} :

$$
r(X_{it}, d_{it}) = \max_{C_{it}} \{[U(d_{it}, C_{it}, X_{it}, \theta) + \beta n_{it+1} EV_{t+1}(X_{it+1})] | d_{it}]
$$

Under the assumption that $\varepsilon(d_{it})$ is distributed as a Type I Extreme Value error, the conditional choice probabilities are given by the following formula:

$$
P(j | X_{it}, \theta) = \frac{\exp\{r(X_{it}, j, \theta)\}}{\sum_{k \in d_{it}(X_{it})} \exp\{r(X_{it}, k, \theta)\}}
$$

and $EV_{t+1}(X_{it+1})$ is given by:

$$
EV_{t+1}(X_{it+1}) = \ln \left[\sum_{k \in d_t(X_t)} \exp\{r(X_{it}, k, \theta)\} \right]
$$

5.6 Inner Maximization

The first step in the solution of period t's involves solving the following *choice-specific value function:*

$$
r(X_{it}, d_{it}) = \max_{C_{it}} \{[U(d_{it}, C_{it}, X_{it}, \theta) + \beta n_{it+1} EV_{t+1}(X_{it+1})] | d_{it}]
$$

This function has to be computed for each possible d_{it}, subject to the contemporaneous budget constraint and the constraints on consumption and house size.

5.7 Estimation of Preferences: Constrained Optimization Approach to Maximum Likelihood

Given the observed panel data $(C_{it}, H_{it}, D_{it}^O, D_{it}^M)_{\substack{t=1\dots T\\ i=1\dots N}}$ we infer the unknown parameter vector θ by maximizing the likelihood function:

$$L(\theta) = \prod_{i=1}^{N}\prod_{t=1}^{T} P(d \mid X_{it}, \theta) \tag{12}$$

We estimate the parameter vector θ by solving the constrained optimization problem:

$$\max_{\theta, \{V_{it}(X_{it},\theta)\}_{\substack{t=1\dots T\\ i=1\dots N}}} \log L(\theta) - \Lambda \tag{13}$$
$$s.t. \qquad Equilibrium\ Constraints$$

where Λ is the sum of Bellman, Euler, Envelope and Policy function errors. See the Technical Appendix for details.

6 The Data

The Health ad Retirement Study (HRS) is a US national panel study which covers a wide range of topics. In particular, questions on family structure, employment status, demographic characteristics, housing, stocks, bonds, IRA, other financial assets, income, pension, social security, and benefits are relevant for our analysis. Questionnaires assessing individual activities and household patterns of consumption were mailed to a sub-sample of the Health and Retirement Study (HRS).The Consumption and Activities Mail Survey (CAMS), the survey including this information on consumption, was first conducted in 2001.

We select a group of households that is the potential target segment for RM, according to estimates from the public policy perspective.

Our sample includes single, retired homeowners, 62 years old or older. They are retiree. Social security is the homeowners' main source of income. Pensions and earned interests on financial assets contribute much less as a source of per-period income. We eliminate all households with incomplete records or missing information about their consumption and financial situation for the years 2000, 2002, 2004. After these cuts were made, a sample of 175 single households remain. The range of their non-housing financial wealth is between \$0 and \$300,000.

In our study, we use the net value of non-housing financial wealth, which includes stocks, bonds, saving accounts, mutual funds, individual retirement accounts (IRAs), other assets less liabilities, e.g. house loan, credit card debt and other liabilities. It does not include the value of any real estate, vehicles, or business. Consumption includes vehicles, washing machine, drier, dishwasher, television, computer, telephone, cable, internet, vehicle finance charges, vehicle insurance, health insurance, food and beverages, dining/drinking out, clothing and apparel, gasoline, vehicles, prescription and nonprescription medications, health care services, medical supplies, trip and vacations, tickets to movies, sorting events and performing arts, hobbies, contribution to religious, educational, charitable or political organizations, cash or gift to family, friends outside your household.

Housing expenses for homeowners represent the maintenance cost incurred to keep the house at a constant quality, and for renters, represent the rental cost. Table 1 shows the descriptive statistics for house value, financial wealth, consumption, social security income and age for the first year in the sample. The number of households observed are 175.

Table 1. Descriptive Statistics

		Percentiles		Min	Max	Mean
	25%	50%	75%			
H	\$40,000	\$70,000	\$92,000	\$ 2,500	\$170,000	\$71,000
W	\$5,000	\$17,500	\$63,000	\$0	\$276,548	\$45,950
H/W	0.86	2.5	7.5	0.11	1500	23.4
C	\$6,270	\$9,774	\$15,090	\$110	\$84,380	\$13,873
ss	\$6,972	\$9,468	\$11,340	\$0	\$ 24.701	\$9,087
Age	69	74	79	64	84	74

Our empirical data show that housing represents an increasing proportion of the total wealth as people age, that is their resources are greatly tied up in their house.

In each period, about 10% of the households in our sample moves out of her house. Among those who moved, about 35% decide to rent a new house, while about 65% buy a new house. At the end of the three years of the panel, about 25% of the population moved and about 10% rented a new house. The moving decision does not appear to be strictly related with age.

7 Calibration and Results

The subjective discount rate is set at $\beta = 0.96$. The real interest rate is $r = 0.04$. Following Yao and Zhang (2005a), the rental rate is $\psi^{rent} = 6\%$ and maintenance cost is $\psi^{own} = 1.5\%$. Transaction costs are $\phi^{own} = 6\%$ and $\phi^{rent} = 1\%$, respectively, when moving to an owner-occupied house and when moving to a rental house.

We estimate the parameter γ using a grid search approach. Given the parameter γ, we use the MPEC approach to estimate ω and σ. Table 2 shows the estimation results

Table 2

Parameter	Estimate	Asymptotic s.e.	Bootstrap s.e.
γ	3.87	(1.07e-009)	(0.04)
ω	0.85	(1.82e-009)	(0.0002)
σ	0.87	(6.82e-004)	(0.05)

The asymptotic standard errors, calculated using a finite difference approach, are very small. Several other papers have noted a significant downward bias of asymptotic standard errors of maximum likelihood estimates of non-linear systems. Therefore, in order to reduce the bias, we compute the standard errors using a bootstrap procedure. Resampling was conducted by sampling with replacement across households as is standard practice in panel models. To obtain the global optimum we use different initial starting points. However, the optimal solution was not influenced by the initial starting points. In total the standard errors are calculated with 30 bootstraps. The results for the standard errors are preliminary and incomplete.

Do Reverse Mortgages Pay?

A reverse mortgage is a loan against the retiree's home that does not have to be paid back for as long as the retiree lives there. We assume that the retiree chooses to receive the proceeds as a single lump sum of cash at the closure of the contract.

Differently from forward mortgages,the retiree has to pay some start up costs, which we denote as F. They are assumed to be a fraction of the house value plus some additional cash for closing costs f. The up-front costs include an origination fee (2% value of the house), an up-front mortgage insurance premium (2% value of the house), an appraisal fee and certain other standard closing costs (about $2000-4000).

$$F = \lambda H_{it} + f \tag{14}$$

The maximum amount that the household can initially borrow is assumed to be a fraction of the house value and of the borrower's age:

$$V_{it} = \kappa_i H_{it} \tag{15}$$

We assume that the date t real interest rate on reverse mortgage is equal to the real return on a one-period bond plus a premium θ^D:

$$r_D = r + \theta^D \tag{16}$$

Let $\overline{B}$ denote the fixed reverse mortgage payment at time t for household i. This amount depends on several factors including the age of the borrower and the appraised value of the home. In general, the higher the age of the borrower, the larger is the amount that can borrowed.

A reverse mortgage accrues interest charges, beginning when the first payment is made to the borrower and then the interest is compounded annually. Let G_{it}^{RM} be the outstanding debt at time t:

$$G_{it}^{RM} = \overline{B} \sum_{j=1..t} R_D^{t-j} \tag{17}$$

If the retiree decides to move out of the house, she has to repay the minimum between the value of the house and the accumulated debt plus a one-time transaction cost $\phi(D_{it}^{O})$. The cost of moving is:

$$M_{it} = D_{it}^{O} D_{it}^{M} [D_{it+1}^{O} H_{it+1} - \max(0, H_{it} - G_{it}^{RM}) + H_{it+1}\phi(D_{it+1}^{O})] \tag{18}$$

The welfare gain from reverse mortgage is calculated as the percentage increase in the initial financial wealth that makes the household without the reverse mortgage as well off in expected utility terms as with the reverse mortgage .

For each household in our sample, we calculate the expected lifetime utility from closing the reverse mortgage contract in the first year of our panel. Then, we calculate the percentage increase in their initial financial wealth that makes them as well off as with the reverse mortgage.

Table 3 shows the median financial wealth and welfare gain for each house value.

Table 3.

House Value	Financial Wealth	Welfare Gain
\$40,000	\$10,300	-37.95%
\$80,000	\$26,000	3.16%
\$120,000	\$48,800	25.56%

Table 4 shows median financial wealth, welfare gain and house value as a function of non-housing financial wealth. Let LW denote initial financial wealth less than \$10,000, MW between \$10,000 and \$80,000 and HW greater than \$80000.

Table 4.

	Financial Wealth	Welfare Gain	House Value
LW	\$1,540	-63.7%	\$40,000
MW	\$26,500	13.44%	\$80,000
HW	\$138,000	34.61%	\$80,000

On average, our simulations show that two groups of households have a welfare loss from a reverse mortgage. First, those with house values less than or equal to \$40,000. Second, those with low financial wealth.

The benefits from closing a reverse mortgage contract are liquidity and longevity insurance. The consumer can en-cash some of the saving locked in the house and would be able to experience higher levels of consumption than otherwise possible. In addition, she can live in the same house while alive, regardless of the amount of the outstanding debt. However, households with a smaller house experience a significant welfare loss due to the initial high transaction costs, which represent a significant fraction of the consumer's reverse mortgage payment. In addition, the consumer is facing a new risk, the moving risk. If the household has to move, for any exogenous reason, her future well-being, her future consumption profile and housing choices are significantly affected. Home equity is an important component of precautionary savings. If a homeowner has drawn down on his equity through an RM, his ability to meet unforeseen costs or move into alternative housing may be limited. This is specifically true for households with initial low financial wealth, as a matter the fact that some of the choices over consumption and housing, available before closing the reverse mortgage contract, are not affordable anymore after the closure of the contract. Therefore the common belief is that RM benefits those with resources tied up in home equity, those defined. "house rich, cash poor". This simulation shows otherwise.

Furthermore, the result reveals that some households would be better off in closing a reverse mortgage contract than without. However, even though it is about twenty years that the reverse mortgage is available, its market is still at 1% of its potential. For about 20% of the households that move in our sample, the main motivation was the financial one but nobody chooses to have a reverse mortgage. This seems to imply that the traditional way of moving out to encash the savings locked in the house is preferred to a reverse mortgage. To assess whether the households make the right choice of not taking a reverse mortgage we compute the welfare gain from moving out and renting a new house of the same value.

Table 5 shows the median financial wealth, welfare gain as a function house value. Table 6 reports the median financial wealth, welfare gain and house value as a function of the initial wealth.

Table 5. Welfare gain from Moving and Renting

House Value	Financial Wealth	Welfare Gain
\$40,000	\$10,300	498%
\$80,000	\$26,000	366%
\$120,000	\$48,800	205%

Table 6. Welfare gain from Moving and Renting

	Financial Wealth	Welfare Gain	House Value
LW	\$1,540	5200%	\$40,000
MW	\$26,500	215%	\$80,000
HW	\$138,000	23%	\$80,000

Households with very low financial wealth have a significant welfare loss from closing a reverse mortgage contract, due to the moving risk, the high transaction costs and the fact that they could borrow only a percentage of the house value. On the other side, if equity in their house is released by moving out and renting, they would be able to en-cash the full amount of the saving locked in their house without increasing their level of indebtedness and without suffering the moving risk.

8 Conclusion

Increases in the cost of living and in health care costs, curtailments in medical coverage and other employee benefits plans, and cutbacks in Social Security benefits expose many of today's households to the risk of having inadequate resources to support their retirement. Empirical evidence shows that for most retirees the majority of resources is tied up in their house. However, tipically retirees do not divest their home equity to support consumption in retirement and, instead, they adjust to a decreased living standard. Before the advent of the reverse mortgage, the best way to divest those savings locked in the residential property was selling and moving out. In the 1990s, the reverse mortgage , a financial instrument designed for "house rich but cash poor" households, was introduced as a potential tool able to enhance social welfare. A reverse mortgage is a home loan that allows the retiree to borrow money against the house without having to make periodic loan repayment or moving out. After about twenty years from its first appearance, the reverse mortgage market is at 1% of its potentiality. We ask the following question: "Does it pay to get a reverse mortgage?".

In order to answer this question, we build a dynamic structural model of housing, consumption and moving decision. We present the first application of the MPEC approach to a dynamic life-cycle model with structural estimation of the preference parameters. We allow households to make both a multistage discrete housing choice and a continuous consumption choice, extending the literature on discrete choice processes. We use a sample of single retiree in the HRS that could be a potential target for the reverse mortgage. We first calculate their expected life-time utility without reverse mortgage and then we simulate how better off they would be with a reverse mortgage. Our analysis shows that reverse mortgages provide liquidity and longevity insurance, but introduce a new risk, the moving risk. We also show that households are significantly better off by moving out and renting a new house than staying in the same house with a reverse mortgage. This might explains why reverse mortgages are and could remain in the future niche products.

Further policy analysis needs to be conducted to design other financial instruments, appealing to the older households, that would allow them to easily access the equity in their home.

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10 Technical Appendix

I am currently working on this appendix.

MPEC with DP: Discrete and Continuous Choices

The panel data used in this study involves 3 years and about 170 individuals. The available data are both continuous and discrete.

The continuous data include data on consumption and financial wealth. The discrete (or discretized) data are the individual's housing tenure (own-rent), her moving decision and her house size. We have additional data on the individuals' demographics, including age.

The MPEC with DP approach consists in solving simultaneously the dynamic programming problem and the maximum likelihood estimation of the preference parameters.

11 Dynamic Programming with Approximation of the Value Function

Life Cycle Model:

One continuous state variable: wealth

Two discrete state variables: previous period housing tenure and previous period house size

One continuous choice variable: consumption

Many discrete choices: Not Move(N), Move to House size h with housing tenure q (Mhq), where q ={Own,Rent}

11.1 Backward Solution from Time T for True Value Functions

In each period, the household chooses whether to stay in her house or to move out. If she moves out, she can either buy or rent a new house and she can choose her new house size. Let the subscripts d^N, d^{Mhq} denote respectively the decision not to move, the decision to move to house size h and housing tenure q. The housing tenure is a binary variable that takes value 1 if the household own the house

The last period value function is known and equal to $V_T(W, H, Q)$ where W is the household's financial wealth, H her initial period house size and Q her initial period housing tenure..

In periods $t = 1..(T-1)$ we define:

$$
\begin{aligned}
V_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + ss; H, Q) + \varepsilon_t^N \\
V_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + ss; h, q) + \varepsilon_t^{Mqh}
\end{aligned}
$$

where M is the transaction cost:

$$
M = Q(H - qh + \phi^{own}qh + \phi^{rent}(1-q)h) + (1-Q)(1-q)\phi^{rent}h
$$

and ψ is the per-period housing expense:

$$
\psi = [Q\psi^{own} + (1-Q)\psi^{rent}]H + [q\psi^{own} + (1-q)\psi^{rent}]h
$$

c_{d^N} and $c_{d^{Mqh}}$ are the consumption levels respectively if the individual does not move and if she moves to house size h choosing the housing tenure q. ss is the household's per-period income. η_{t+1} is her survival probability. ε_t^N and ε_t^{Mqh} are type I extreme value errors.

Following Rust, we assume that the additivity and the conditional indipendence assumptions hold.

To simplify the notation, we introduce the following expressions, which are evaluated at the optimal consumption level:

$$
\begin{aligned}
\widehat{V}_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + ss; H, Q) \\
\widehat{V}_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + ss; h, q)
\end{aligned}
$$

The extreme value assumption on ε_t implies that we can reduce the dimensionality of the dynamic programming problem. The Bellman equation is given by the following closed form solution:

$$
\begin{aligned}
V_t(W, H, Q) &= \Pr(N|W, H, Q) \cdot \widehat{V}_{d^N,t} + E(\varepsilon_t^N|N) \\
&\quad + \sum_h \sum_q \{\Pr(Mhq|W, H, Q) \cdot \widehat{V}_{d^{Mhq},t} + E(\varepsilon_t^{Mhq}|M)\} \\
&= \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}
\end{aligned}
$$

Given V_{t+1}, the Bellman equation implies, for each wealth level W, three set of equations that determine the optimal consumption, $c^*_{d^N}$,$c^*_{d^{Mhq}}$, $V_t(W, H, Q)$, and $V_t'(W, H, Q)$

Euler Equations:

$$
\begin{aligned}
u'(c^*_{d^N}, H) - \beta\eta_{t+1}RV'_{t+1}(RW - c^*_{d^N} - \psi + ss; H, Q) &= 0 \\
u'(c^*_{d^{Mhq}}, h) - \beta\eta_{t+1}RV'_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + ss; h, q) &= 0
\end{aligned}
$$

Envelope Condition:

$$V_t'(W,H,Q) = \Pr(N|W,H,Q) \cdot \widehat{V}'_{d^N,t} + \sum_h \sum_q \Pr(Mhq|W,H,Q) \cdot \widehat{V}'_{d^{Mhq},t}$$

Bellman equation:

$$V_t(W,H,Q) = \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}$$

The time $t = 1..(T-1)$ probabilities of not moving and moving to house size h with housing tenure q are:

$$\Pr(N|W,H,Q) = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(V_t(W,H,Q))}$$

$$\Pr(Mhq|W,H,Q) = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(V_t(W,H,Q))}$$

11.2 Backward Solution from Time T for Approximate Value Functions

Let $\Phi(W,H,Q;a)$ and $\Phi_d(W,H,Q;b)$ be the functions that we use to approximate respectively the value functions, $V(W,H,Q)$. and the policy functions $c_d^*(W,H,Q)$, with $d=\{d^N,d^{Mhq}\}$. If we assume that they are a seventh-order polynomials centered at $\overline{W}$, then

$$\Phi(W,H,Q;a,\overline{W})=\sum_{k=0}^{7}a_{k,H,Q}(W-\overline{W})^k$$

The time t value function is approximated by

$$V_t\left(W,H,Q\right)=\Phi(W,H,Q;a_t,\overline{W}_t)=\sum_{k=0}^{7}a_{k+1,H,Q,t}(W-\overline{W}_t)^k$$

The time t policy functions are approximated by

$$c_{d,t}^*\left(W,H,Q\right)=\Phi(W,H,Q;b_{d,t},\overline{W}_t)=\sum_{k=0}^{7}b_{k+1,H,Q,d,t}(W-\overline{W}_t)^k$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{W}$ on time and the.dependence of the policy functions on time is represented by the dependence of the b coefficients and the center $\overline{W}$.

We will choose $\overline{W}_t=(W_t^{\max}+W_t^{\min})/2$, the period t average wealth. Note that $\overline{W}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, we will drop it as an explicit argument of Φ. So, $\Phi(W,H,Q;a_t)$ will mean $\Phi(W,H,Q;a_t,\overline{W}_t)$.

We would like to find coefficients a_t and $b_{d,t}$ such that each time t Bellman equation, along with the Euler and envelope conditions, holds with the Φ approximation; that is, for each time $t<T-2$, we want to find coefficients a_t such that for all W

$$\Phi(W,H,Q;a_t)=\ln\left\{\exp(\widehat{V}_{d^N,t})+\sum_h\sum_q\exp(\widehat{V}_{d^{Mhq},t})\right\}$$

where

$$\begin{aligned}
\widehat{V}_{d^N,t} &= u(c_{d^N}^*,H)+\beta\eta_{t+1}\Phi_{t+1}(RW-c_{d^N}^*-\psi+ss;H,Q;a_{t+1})\\
\widehat{V}_{d^{Mhq},t} &= u(c_{d^{Mhq}}^*,h)+\beta\eta_{t+1}\Phi_{t+1}(RW-c_{d^{Mhq}}^*-\psi-M+ss;h,q;a_{t+1})
\end{aligned}$$

and for time $t=T-1$, we want to find coefficients a_t given that

$$\begin{aligned}
\widehat{V}_{d^N,T-1} &= u(c_{d^N}^*,H)+\beta\eta_T V_T(RW-c_{d^N}^*-\psi+ss;H,Q)\\
\widehat{V}_{d^{Mhq},T-1} &= u(c_{d^{Mhq}}^*,h)+\beta\eta_T V_T(RW-c_{d^{Mhq}}^*-\psi-M+ss;h,q)
\end{aligned}$$

This is not possible unless the solution is a degree 7 polynomial. We need to approximately solve the Bellman equation. To this end, we define various errors.

First, we create a finite grid of wealth levels we will use for approximating the value functions. Let $W_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. Then we create a grid of house sizes. Let $H_{j,t}$ be grid point j in the time t grid.

Next we need to specify the various errors that may arise in our approximation. We will consider three errors and one side condition.

First, at each time t and each $W_{i,t}$ and each initial period house size $H_{j,t}$ and housing tenure Q_t, the absolute value of the Euler equations if consumption is respectively $c^*_{i,j,d^N,t}$ and $c^*_{i,j,Q,d^{Mhq},t}$, which we denote as $\lambda^e_{i,j,Q,t} \geq 0$, satisfies the inequality

$$-\lambda^e_{i,j,Q,t} \leq u'(c^*_{i,j,d^N,t}, H_{j,t}) - \beta\eta_{t+1} R\Phi'(RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss; H_{j,t}, Q_t; a_{t+1}) \leq \lambda^e_{i,j,Q,t}$$

$$-\lambda^e_{i,j,Q,t} \leq u'(c^*_{i,j,d^{Mhq},t}, H_{t+1})$$

$$-\beta\eta_{t+1} R\Phi'(RW_{i,t} - c^*_{i,j,d^{Mhq},t} - \psi - M + ss; H_{t+1}, Q_{t+1}; a_{t+1}) \leq \lambda^e_{i,j,Q,t}$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $W_{i,t}$ with consumption $c_{i,j,d^N,t}$ and $c_{i,j,d^{Mhq},t}$is denoted by $\lambda^b_{j,Q,t}$ and satisfies

$$-\lambda^b_{j,Q,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_t; a_t) - \ln\left\{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^b_{j,Q,t}$$

where

$$\begin{aligned}
\widehat{V}_{i,j,d^N,t} &= u(c^*_{i,j,d^N,t}, H_{j,t}) + \beta\eta_{t+1}\Phi(RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss; H_{j,t}, Q_t; a_{t+1}) \\
\widehat{V}_{i,j,d^{Mhq},t} &= u(c^*_{i,j,d^{Mhq},t}, H_{t+1}) + \beta\eta_{t+1}\Phi(RW - c^*_{i,j,d^{Mhq},t} - \psi - M + ss; H_{t+1}, Q_{t+1}; a_{t+1})
\end{aligned}$$

Third, the envelope condition errors, λ^{env}_t, satisfies

$$-\lambda^{env}_{j,Q,t} \leq \Phi'(W_{i,t}, H_{j,t}, Q_t; a_t) - \{f_{i,j,d^N,t} \cdot \Phi'(RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss; H_{j,t}, Q_t; a_{t+1})$$

$$+ \sum_h \sum_q [f_{i,j,d^{Mhq},t} \cdot \Phi'(RW_{i,t} - c_{i,j,d^{Mhq},t} - \psi^{Mhq} - M + ss; H_{t+1}, Q_{t+1}; a_{t+1})]\} \leq \lambda^{env}_{j,Q,t}$$

where $\Phi'(x;a_t)$ is the derivative of $\Phi'(x;a_t)$ with respect to x and

$$f_{i,j,d,t} = \Pr(d|W_{i,t}, H_{j,t}, Q_t) = \frac{\exp(\widehat{V}_{i,j,d,t})}{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})}$$

Fourth, we introduce the policy function errors:

$$-\lambda^{cons}_{i,j,Q,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_t; b_t) - c^*_{i,j,d,t}(W_{i,t}, H_{j,t}, Q_t) \leq \lambda^{cons}_{i,j,Q,d,t}$$

11.3 Empirical Part

In the theorical DP part we obtain the coefficients used in the approximation of the value function.

In this part, for any individual data of financial wealth, initial period house size and age, we calculate the predicted consumption and probabilities of moving. First, the individual makes the housing decision $d^H_{n,tp}$, with $d^H = \{d^N, d^{Mhq}\}$, then she makes her consumption decision.

Let $c^{pred}_{n,tp}$ and $c^{data}_{n,tp}$ denote respectively the predicted and the true value of consumption for household n at time tp.

For any given discrete choice on housing $d^H_{n,tp}$, using the real data on consumption, we calculate the measurement error:

$$\Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c^{data}_{n,tp} - c^{pred}_{n,tp})^2}{2\sigma^2}}$$

The probability for the discrete choice on housing is given by:

$$\Pr(d^H_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{e^{V_{d,n,tp}}}{\sum_m e^{Vm,n,tp}}$$

Therefore the joint probability of making the discrete housing choice $d^H_{n,t}$ and the continuous consumption choice $c_{n,t}$ is given by:

$$\Pr(d^H_{n,tp}, c_{n,tp}|W^{data,}_{n,tp} H^{data}_{n,tp}, Q^{data}_{n,tp}) = \Pr(d^H_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) * \Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp})$$

The Log-Likelihood is given by:

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d^H_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}, \theta)$$

where N denotes the number of individuals in the sample and TP the number of time periods in the panel data.

11.4 MPEC

With these definitions,let

$$\Lambda = \sum_t \sum_i \sum_j \sum_Q \lambda^e_{i,j,Q,t} + \sum_t \sum_j \sum_Q \lambda^b_{j,Q,t} + \sum_t \sum_j \sum_Q \lambda^{env}_{j,Q,t} + \sum_t \sum_i \sum_j \sum_Q \sum_d \lambda^{cons}_{i,j,Q,d,t}$$

and let P be a penalty parameter.

The MPEC approach to the estimation of the preference paramenters is:

$$\underset{\theta,a,c}{Max} \mathcal{L}(\theta) - P\Lambda$$

subject to:

Bellman error:

$$-\lambda^b_{j,Q,t} \leq \Phi(W_{i,t}; a_t) - \ln \left\{ \exp(\widehat{V}_{i,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t}) \right\} \leq \lambda^b_{j,Q,t}$$

Euler error

$$\begin{aligned} -\lambda^e_{i,j,Q,t} &\leq u_{c;i,j,d^N,t} + \beta\Phi^+_{W;i,j,d^N,t} \leq \lambda^e_{i,j,Q,t} \\ -\lambda^e_{i,j,Q,t} &\leq u_{c;i,j,d^{Mhq},t} + \beta\Phi^+_{W;i,j,d^{Mhq},t} \leq \lambda^e_{i,j,Q,t} \end{aligned}$$

Envelope error

$$-\lambda^{env}_{j,Q,t} \leq \Phi_{W;i,d^N,t} - \{f_{i,d^N,t} \cdot \Phi^+_{W;i,j,d^N,t} + \sum_h \sum_q [f_{i,j,d^{Mhq},t} \cdot \Phi^+_{W;i,j,,d^{Mhq},t}]\} \leq \lambda^{env}_{j,Q,t}$$

Policy function error

$$-\lambda^{cons}_{i,j,Q,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_t; b_{d,t}) - c^*_{i,j,d,t}(W_{i,t}, H_{j,t}, Q_t) \leq \lambda^{cons}_{i,j,Q,d,t}$$

The probability of decision d :

$$f_{i,j,d,t} = \frac{\exp(\widehat{V}_{i,j,d,t})}{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})}$$

11.5 AMPL

11.5.1 Backward Solution from Time T for Approximate Value Functions in AMPL

In order to formulate this problem in AMPL, we need to list every quantity that is computed.

The time-specific wealth grids $W_{i,t}$ are fixed. We discretize the house size.

The parameters are

$$W_{i,t}, \beta, \eta_t, R, \psi^{own}, \psi^{rent}, \phi^{own}, \phi^{rent}, \alpha^{bequest}$$

The basic variables of interest are

$$\begin{array}{l} c_{i,j,d^N,t},\ c_{i,j,d^{Mhq},t} \\ a_{k,j,Q,t}, b_{k,j,Q,d,t} \\ \lambda^e_{i,j,Q,t}, \lambda^b_{j,Q,t}, \lambda^{env}_{j,Q,t}, \lambda^{cons}_{i,j,Q,d,t} \end{array}$$

AMPL does not allow procedure programming; therefore, we need to define other variables to represent quantities defined in terms of other variables. We first need

$$\begin{array}{rcl} u_{i,j,d^N,t} & \equiv & u\left(c^*_{i,j,d^N,t}, H_{j,t}\right) \\ u_{c;i,j,d^N,t} & \equiv & u'\left(c^*_{i,j,d^N,t}, H_{j,t}\right) \\ W^+_{i,j,d^N,t} & \equiv & RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss \\ f_{i,j,d^N,t} & = & \Pr(N|W_{i,t}, H_{j,t}, Q_t) \end{array}$$

$$\begin{array}{rcl} u_{i,j,d^{Mhq},t} & \equiv & u\left(c^*_{i,j,d^{Mhq},t}, H_{t+1}\right) \\ u_{c;i,j,d^{Mhq},t} & \equiv & u'\left(c^*_{i,j,d^{Mhq},t}, H_{t+1}\right) \\ W^+_{i,j,d^{Mhq},t} & \equiv & RW_{i,t} - c^*_{i,j,d^{Mh},t} - \psi - M + ss \\ f_{i,j,d^{Mhq},t} & = & \Pr(Mhq|W_{i,t}, H_{j,t}, Q_t) \end{array}$$

We next use those variables to build more variables

$$\begin{array}{rcl} \Phi_{i,j,Q,t} & \equiv & \Phi(W_{i,t}, H_{j,t}, Q_t; a_t) \\ \Phi_{W;i,j,Q,t} & \equiv & \Phi'(W_{i,t}, H_{j,t}, Q_t; a_t) \\ \Phi^+_{i,j,d^{MQ},t} & \equiv & \Phi(W^+_{i,j,d^{NM},t}, H_{j,t}, Q_t; a_{t+1}) \\ \Phi^+_{W;i,d^N,t} & \equiv & \Phi'(W^+_{i,j,d^{NM},t}, H_{j,t}, Q_t; a_{t+1}) \\ \Phi^+_{i,j,d^{Mhq},t} & \equiv & \Phi(W^+_{i,j,d^{Mh},t}, H_{j,t+1}, Q_{t+1}; a_{t+1}) \\ \Phi^+_{W;i,j,d^{Mhq},t} & \equiv & \Phi'(W^+_{i,j,d^{Mhq},t}, H_{j,t+1}, Q_{t+1}; a_{t+1}) \\ \Psi_{i,j,d,t} & \equiv & \Phi(W_{i,t}, H_{j,t}, Q_t; b_{d,t}) \end{array}$$

With these variables defined, the Bellman equation error inequality becomes

$$-\lambda^{b}_{j,Q,t} \leq \Phi_{i,j,Q,d,t} - \ln\left\{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^{b}_{j,Q,t}$$

where

$$\begin{aligned}
\widehat{V}_{i,j,d^N,t} &= u_{i,j,d^N,t} + \beta\eta_{t+1}\Phi^{+}_{i,j,d^N,t} \\
\widehat{V}_{i,j,d^{Mhq},t} &= u_{i,j,d^{Mhq},t} + \beta\eta_{t+1}\Phi^{+}_{i,j,d^{Mhq},t}
\end{aligned}$$

the Euler equation error inequalities become

$$\begin{aligned}
-\lambda^{e}_{i,j,Q,t} &\leq u_{c;i,j,d^N,t} + \beta\Phi^{+}_{W;i,j,d^N,t} \leq \lambda^{e}_{i,j,Q,t} \\
-\lambda^{e}_{i,j,Q,t} &\leq u_{c;i,j,d^{Mhq},t} + \beta\Phi^{+}_{W;i,j,d^{Mhq},t} \leq \lambda^{e}_{i,j,Q,t}
\end{aligned}$$

and the envelope error inequality becomes

$$-\lambda^{env}_{j,Q,t} \leq \Phi_{W;i,d^N,t} - \{f_{i,j,d^N,t}\cdot\Phi^{+}_{W;i,j,d^N,t} + \sum_h \sum_h [f_{i,j,d^{Mhq},t}\cdot\Phi^{+}_{W;i,j,,d^{Mhq},t}]\} \leq \lambda^{env}_{j,Q,t}$$

The probability of decision d :

$$f_{i,j,d,t} = \frac{\exp(\widehat{V}_{i,j,d,t})}{\exp(\widehat{V}_{i,j,d^{NM},t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})}$$

The policy function errors are

$$-\lambda^{cons}_{i,j,Q,d,t} \leq \Psi_{i,jQ,,d,t} - c^{*}_{i,j,Q,d,t} \leq \lambda^{cons}_{i,j,Q,d,t}$$

11.5.2 Empirical Part in AMPL

We consider individuals in our sample such that $Age^{data}_{n,tp} = 1..(T-2)$.

Let $W^{data}_{n,tp}$, $Age^{data}_{n,tp}$, $H^{data}_{n,tp}$ and $Q^{data}_{n,tp}$ denote respectively the data on financial wealth, age, house size and housing tenure for household n in year tp in the panel data. Given these data, the variables of interest are:

$$\begin{aligned}
c^{pred}_{d^N,n,tp} &= \Psi_{d^{NM}}(W^{data}_{n,tp}, Age^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) \\
c^{pred}_{d^{Mhq},n,tp} &= \Psi_{d^{Mh}}(W^{data}_{n,tp}, Age^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp})
\end{aligned}$$

$$\begin{aligned}
u^{pred}_{d^N,n,tp} &\equiv u\left(c^{pred}_{d^{NM},n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}\right) \\
u_{c;d^N,n,tp} &\equiv u'\left(c_{d^{NM},n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}\right) \\
W^{+}_{d^N,n,tp} &\equiv RW^{data}_{n,tp} - c^{pred}_{d^N,n,tp} - \psi(H^{data}_{n,tp}, Q^{data}_{n,tp}) + ss \\
f^{pred}_{d^{NM},n,tp} &= \Pr(NM|W^{data}_{n,t}, H^{data}_{n,tp}, Age^{data}_{n,tp}, Q^{data}_{n,tp})
\end{aligned}$$

$$\begin{aligned}
u^{pred}_{d^{Mhq},n,tp} &\equiv u\left(c^{pred}_{d^{Mh},n,tp}, H^{data}_{n,tp}, H^{choice}_{n,tp}, Q^{data}_{n,tp}, Q^{choice}_{n,tp}\right) \\
u_{c;d^{Mhq},n,tp} &\equiv u'\left(c_{d^{Mhq},n,tp}, H^{data}_{n,tp}, H^{choice}_{n,tp}, Q^{data}_{n,tp}, Q^{choice}_{n,tp}\right) \\
W^{+}_{d^{Mhq},n,tp} &\equiv RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) - M(H^{data}_{n,tp}, Q^{data}_{n,tp}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) + ss \\
f^{pred}_{d^{Mhq},n,tp} &= \Pr(Mhq|W^{data}_{n,tp}, H^{data}_{n,tp}, Age^{data}_{n,tp})
\end{aligned}$$

We next use those variables to build more variables

$$\begin{aligned}
\Phi^{data}_{n,tp} &\equiv \Phi(W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}; a_{Age^{data}_{n,tp}}) \\
\Phi^{data}_{W;n,t} &\equiv (\Phi^{data})'(W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}; a_{Age^{data}_{n,tp}}) \\
\Phi^{+}_{d^{N},n,tp} &\equiv \Phi^{data}(W^{+}_{d^{NM},n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}; a_{Age^{data}_{n,tp}+1}) \\
\Phi^{+}_{W,d^{N},n,tp} &\equiv \Phi'(W^{+}_{d^{NM},n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}; a_{Age^{data}_{n,tp}+1}) \\
\Phi^{+}_{d^{Mhq},n,tp} &\equiv \Phi(W^{+}_{d^{M},n,tp}, H^{data}_{n,tp}, H^{choice}_{n,tp}, Q^{data}_{n,tp}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1}) \\
\Phi^{+}_{W;d^{Mhq},n,tp} &\equiv \Phi'(W^{+}_{d^{M},n,tp}, H^{data}_{n,tp}, H^{choice}_{n,tp}, H^{data}_{n,tp}, H^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})
\end{aligned}$$

$$\widehat{V}^{pred}_{d^{N},n,tp} = u(c^{pred}_{d^{NM},n,tp}, H^{data}_{n,tp}) + \beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{NM},n,tp} - \psi(H^{data}_{n,tp}, Q^{data}_{n,tp}) + ss; H^{data}_{n,tp}, Q^{data}_{n,tp}; a_{Age^{data}_{n,tp}+1})$$

$$\widehat{V}^{pred}_{d^{Mhq},n,tp} = u(c^{pred}_{d^{Mhq},n,tp}, H^{choice}_{n,tp})$$

$$\begin{aligned}
&+\beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) - M(H^{data}_{n,tp}, Q^{data}_{n,tp}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) \\
&+ss; H^{data}_{n,tp}, H^{choice}_{n,tp}, Q^{data}_{n,tp}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})
\end{aligned}$$

The probabilities of not moving and moving are:

$$f^{pred}_{d^{NM},n,tp} = \Pr(H_{d^{N},n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{NM},n,tp})}{\exp(\widehat{V}^{pred}_{d^{N},n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}$$

$$f^{pred}_{d^{Mhq},n,tp} = \Pr(H_{d^{Mhq},n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{Mh},n,tp})}{\exp(\widehat{V}^{pred}_{d^{N},n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}$$

We introduce the following constraints concerning the measurement error in consumption:

$$\Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(\frac{(c^{data}_{n,tp} - c^{pred}_{d,n,tp})^2}{2\sigma^2}$$

KNITRO Problem Characteristics
Objective goal: Maximize
Number of variables: 72746
bounded below: 23688
bounded above: 0
bounded below and above: 23521
fixed: 0
free: 25537
Number of constraints: 103488
linear equalities: 0
nonlinear equalities: 35280
linear inequalities: 35280
nonlinear inequalities: 32928
range: 0
Number of nonzeros in Jacobian: 960344
Number of nonzeros in Hessian: 287529