Appendix for "The Option Value of Educational Choices And the Rate of Return to Educational Choices"

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Appendix

The Definition of an American Option in Finance[1]

- Let $S_t = (S_t^1, \dots, S_t^n)$ denote the vector of securities underlying an option, modeled as a Markov process on R^n with discrete time-parameter $t = 0, 1, \dots, T$.
- The argument $t = 0, 1, \dots, T$ indexes the set of time points (in increasing order) when the option is exercisable, also called *exercise opportunities* or simply *stages*.
- Let $h(t, s)$ be the payoff from exercise at time t in state s, discounted to time 0 with the (possibly stochastic) discount factor recorded in S_t.
- Assume S_t is Markovian.

[1] Source: Avramidis, Zinchenko, Coleman, and Verma (2000).

- The option value at (time, state) pair (t, s) is obtained by dynamic programming:

$$q(t,s) = \begin{cases} h(T,s) & \text{for } t = T \text{ and all } s \\ \max\{h(t,s), c(t,s)\} & \text{for } t < T \text{ and all } s \end{cases}.$$

- $c(t, s)$ is the discounted value of the option associated with the decision to "continue", i.e., not exercise the option at (t, s), thereby holding it until at least stage $t + 1$:

$$c(t,s) = E[q(t+1, S_{t+1} \mid S_t = s].$$

- The quantity $c(t, s)$ is called the *continuation value* at (t, s).

- Arbitrage-pricing theory suggests that the arbitrage-free price of the option is obtained when the conditional expectation in (4) is with respect to the Equivalent Martingale Measure (EMM).
- Under the EMM, the value of any tradeable security, discounted to time 0, is a martingale.
- The *option value* at time 0 is $q(0, s_0)$, where s_0 is the known state of underlying assets at time 0.
- The American option considers a *single* node in our setting (e.g. one year of school).

- The problem analyzed in the standard financial option literature is to consider when to exercise the option (when do you return to school and go to the next node in our notation).
- The S payoff R.

- In our notation, for a person at schooling level s, their payoff $h(t, s)$ corresponds to switching to the next node $V(s(t+1), t+1)$.
- $s(t+1) \neq s(t)$

$$c(t, s) \Longleftrightarrow E[V(s(t), t+1 \mid \mathcal{I}(s(t), t)].$$

- $E[V(s(t), t+1) \mid \mathcal{I}(s(t), t)]$ is the value of staying on at the schooling level at $s(t) = s$ one more period.
- In finance, the option value is the value of the program the *first time* you get the chance to take the option.
- Thus at time t^*, the first time at level s, the value of the program is

$$E[V(s \mid t^*), t^* \mid \mathcal{I}(s(t^*), t^*)].$$

- The continuation value is the value after t^* $(t > t^*)$ when you stay on at level $s(t^*)$ but retain the right to go on.
- Cashout reward is the reward that comes from cashing out and not continuing:

$$CO(\mathcal{I}(s(t^*), t) = E\left[\sum_{t=t^*}^{T} \frac{R(s(t^*), t)}{(1+r)^{t-t^*}} \mid \mathcal{I}(s(t^*), t)\right]. \quad (1)$$

- This term is analogous to $h(t, s)$.
- Thus we have that the *true* option value is return above stopping (a cashing out) i.e.

$$\max[\text{option value as } \textit{they} \text{ define it}; CO(\mathcal{I}(s(t^*), t)].$$

- For them, $h(t, s)$ is the cash-out value. For us, it is the value of going to the next schooling level.
- Thus if I am in schooling level $\bar{s}$ at time t $(s(t) = \bar{s})$, I have the option of going on to the "next" schooling level possible.
- In an ordered set of choices (e.g. grades), this is $\bar{s} + 1$.
- But in general it can be $s' \neq \bar{s}$ provided $s' \in \kappa(\bar{s}, t)$.
- Payoff at t is $E\left[V(s', t \mid \mathcal{I}(\bar{s}, t))\right]$ (this is like their h).
- Staying on has value $E[V(\bar{s}, t) \mid \mathcal{I}(\bar{s}, t)]$.
- This is the *continuation value of staying on* in the schooling level $\bar{s}$.

- At the time t^*, when I transit to s' from $\bar{s}$, I "strike on the option" and get a new option $s'' \in \kappa(s', t^*)$ the option value is

$$\max_{s'' \in \kappa(s', t^*)} E \max \left[V(s'', t^* \mid \mathcal{I}(s', t^*)\right], \left[V(s', t^*) \mid \mathcal{I}(s', t^*)\right]$$
$$- E \left[V(s', t^*) \mid \mathcal{I}(s', t^*)\right]$$

- The final term arises in our problem (but not theirs) because the fall back for us is staying in state s'.
- That could mean either stopping or delaying one period.

- As noted by Altonji (1993), one may learn something new that affects future choices (this produces a learning value s' and subsequent choices).
- Altonji (1993) calculates *ex ante* and *ex post* internal rates of return reflecting the learning value of schooling (two choices, two-period model, two sets of internal rates of return).
    - He focuses on learning about serially correlated unobserved components while the student is in school and resulting option values.
    - Considers agent switching among majors.
    - Focuses solely on learning in college and college dropout. He does not analyze dropping out of high school and associated learning.
    - Does not analyze uncertainty in post-schooling earnings.
    - Considers learning in school only.
    - Analyzes three period models.
    - Analyzes and estimates *ex ante* internal rates of return.

Relationship to Previous Work

- Like Weisbrod (1962) and Altonji (1993), we recognize the option value that comes from educational choices.
- Like Levhari and Weiss (1974) and Keane and Wolpin (1997), we recognize uncertainty in post-educational earnings.
- Like Altonji (1993), Arcidiacono (2004) and Santos (2008), we rocognize the learning value of schooling.
- Unlike Keane and Wolpin (1997, 2001), we consider serially persistent shocks which agents learn about (as in Miller 1984 and Pakes 1986)
- This produces much greater estimated option values than an independent shock model.
- We define and estimate the appropriate rate of return for a dynamic model with serially persistent shocks, nonlinearity and learning.