An Empirical, Dynamic, Two-Sided Matching Game Applied to Market Thickness and Switching

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ICE August 1, 2008

Perfect information, matching, structural empirical work

- Structural empirical work tries to determine relative importance of agent characteristics in sorting pattern
  - Data on realized matches, exogenous agent characteristics
    - $\langle$Johnson Controls, General Motors$\rangle$, $\langle$Bosch, Volkswagen$\rangle$, ...
  - Estimate utility / match surplus functions
- Recent but growing literature
  - Ahlin (2006); Akkus and Hortacsu (2006); Angelov (2006); Bajari & Fox (2007); Boyd, Lankford, Loeb and Wyckoff (2003); Choo and Siow (2006); Dagsvik (2000); Ferrall, Salavanes and E. Sørensen (2004); Fox (2007); Gordon and Knight (2005); Levine (2008); Park (2007); M. Sørensen (2007); Weiss (2007); Yang (2006)

Perfect information matching / assignment games

- Allow endogenous prices $w_{\langle a,i \rangle}$, match $\langle a, i \rangle$
  - $u_{\langle a,i \rangle} = \tilde{u}_{\langle a,i \rangle} + \gamma_w w_{\langle a,i \rangle}$
  - Agents price takers
- Theory
  - Tinbergen (1947), Koopmans and Beckmann (1957), Shapley and Shubik (1972), Becker (1973), etc.
- One-to-one, two-sided matching theorems
  - Equilibrium assignment maximizes sum of match utilities for economy
  - Complementarities / substitutabilities of partners' characteristics drive sorting pattern

Dynamic (repeated) matching game

- Many examples where data on switching matches of interest
  - Marriage and divorce
  - Workers switching employers
  - Retailers switching suppliers
  - Previously independent firms merge
- Dependent variable switching combines information on
  - Characteristics of origin, destination firms
  - Incidence of switching by presence of rivals
- Switching can be more informative, sometimes
  - How many people work at Saab informative of demand for Saab's cars
    - Product market more than labor market issues
  - More informative from who leaves Saab, who switches to Saab
    - Workers from Volvo?

Dynamic (repeated) matching game cntd.

- Need forward looking agents
  - Both workers and firms
  - Benefit of a switch is in part how long you plan to stay
  - How long you stay is endogenous in a dynamic model
- This paper introduces repeated matching game
  - Each period matching market clears, most agents matched
  - Agents forward looking and can switch next period
    - No unemployment / search time
  - No knowledge of a direct theoretical antecedent

Application: Market thickness and switching

- Markets vary in thickness
- Do thicker markets lead to more switching?
- Here: elite engineers in Sweden, 1970–1990
  - Almost all private sector employers in Sweden, can track workers across firms
  - Men only
- Plants and jobs exist in characteristic space
  - Geographic location, industry, corporate parent, occupation, plant size
  - Workers distinguished by age, previous job
- Estimate switching model as repeated matching game
  - Switching costs in firm / job characteristics
- Big picture: Silicon Valley, etc.

Not testing competing models for switching

- How do market conditions affect switching?
  - Closeness of rival workers / jobs in characteristic space
  - Switching driven by i.i.d. logit shock for all choices
- Not: Distinguish switching costs from similar models that can fit match persistence data
  - Unlike Dube, Hitsch and Rossi (2007)
- Not: Observably identical worker *a* and *b* at same plant and job, why does *a* and not *b* switch?
  - Learning
  - Match quality / persistence of heterogeneous tastes for matches

Four computational curses of dimensionality

- Dynamic programming
  - d state variables, x values each: x^d states
- Matching game
  - e workers, e firms: e^2 matches and $e!$ assignments
  - $e = 100 \Longrightarrow e! \approx 9.33 \times 10^{157}$
- Repeated matching game
  - Combines dynamic programming & matching
  - Uncertainty over next period's matches
  - e matches in state, $e!$ values for next period's state
  - Address using steady state beliefs assumption...
- Econometric: integrate out error terms
  - Integrand repeated matching game?
  - Address using timing assumption on errors...

Estimation details

- Microdata, potentially small matching markets
  - Maximum likelihood
- Dependent variable data on matches only
  - Not prices despite their presence in the model
  - Privacy: prices not disclosed, Becker (1973) marriage prices abstraction
- Solving model once time consuming
  - Advocate estimation using Su Judd (2007) suggestion

Outline today's talk

1. Repeated matching game
2. Computation & estimation
3. Parameter estimates & data fit
4. Counterfactuals

Model: Agents: workers and jobs

- Outline model using generic notation like Rust (1987)
  - No market thickness until end
- One-to-one matching between workers and jobs
  - Similar to many-to-one matching between workers and employers
  - Sotomayor (1992) assumption of additive separability of profit from multiple workers at same firm
- $a = 1, \ldots, N$ workers
- $i = 1, \ldots, J$ jobs

Calendar time

- t is a year (say)
- Data on years $t = 0, \ldots, T$
- Each period matching market is repeated
- Worker a has age $d_{a,t}$ in year t
- Workers retire at age 60
  - Different calendar year D_a for each worker

Timing within a period

1. All agents have state variables known to them and all other agents.
2. Equilibrium wages set to equate *expected* supply and demand for all pairs of one worker and one job state
3. Each worker receives a taste shock for each job; each job receives a taste shock for each worker
4. Workers unilaterally choose a job. If 0 or ≥ 2 pick a job, they are accommodated with fewer or more slots.
    - Firms cannot screen workers, even if firms have negative taste shock

- $\delta = 0$ (no forward looking version) similar to Dagsvik (2000), Choo & Siow (2006), Weiss (2007)

Matches and states

- $\langle a, i, t \rangle$ is match of worker a to job i in period t
- $s_{\langle a,i,t-1 \rangle} = \left\{ x_{a,t}, x^f_{i,t} \right\}$, characteristics of both partners
    - superscript f refers to firms
- Say matches $\langle a, j, t-1 \rangle$ and $\langle b, i, t-1 \rangle$ occurred in $t-1$ & $\langle a, i, t \rangle$ in t
- State transition rule is

$$h_{\theta_1} \left(s_{\langle a,i,t \rangle} \mid s_{\langle a,j,t-1 \rangle}, s_{\langle b,i,t-1 \rangle}, \langle a, i, t \rangle \right)$$

    - Allows firm, occupation specific human capital accumulation
- θ_1 estimable parameters
- $s_{\langle a,0,t-1 \rangle} = \{ x_{a,t}, 0 \}$ worker unmatched
    - Job eliminated, new entrant
- $s_{\langle 0,i,t-1 \rangle} = \left\{ 0, x^f_{i,t} \right\}$ job unmatched
    - New position, old worker retired

Game states in individual agents' states

- In IO, aggregate state $\left\{s_{\langle a,i,t-1\rangle}\right\}_{a=1}^{N}$ in all players' states
- Compete for consumers, need to know how match options will change next period
- Computational approaches rely on modifying game
  - Doraszelski & Judd (2007)
  - Weintraub, Benkard & Van Roy (2007)
- Could keep track of states of rivals
  - Computationally demanding

Steady state beliefs

Assumption

If a worker with state $s_{\langle a,i,t-1\rangle}$ in period t has a continuation value $V\left(s_{\langle a,i,t-1\rangle}\right)$, workers in period t who might end up at state $s_{\langle a,i,\tau\rangle} = s_{\langle a,i,t\rangle}$ in some period $\tau > t$ expect to have continuation value $V\left(s_{\langle a,i,\tau-1\rangle}\right) = V\left(s_{\langle a,i,t-1\rangle}\right)$ when they reach that state.

- Model not in steady state (require very strong assumptions) but agents believe it is
- Age 40 workers imagine that the experiences of age 50 workers today reflect their options in 10 years
- Dynamic programming reflects only the trajectories of individual agents

Worker current-period utility functions

- Worker a of state $s_{\langle a,j,t-1\rangle}$, flow utility from match $\langle a,i,t\rangle$ with job i

$$u_\beta\left(\langle a,i,t\rangle, s_{\langle a,j,t-1\rangle}\right)+\varepsilon_{\langle a,i,t\rangle}=\tilde{u}_{\tilde{\beta}}\left(\langle a,i,t\rangle, s_{\langle a,j,t-1\rangle}\right)+\beta_w w_{\langle a,i,t\rangle}+\varepsilon_{\langle a,i,t\rangle}$$

- β utility parameters to estimate
  - β_w value of wage in utility
- $w_{\langle a,i,t\rangle}$ wage (endogenous in equilibrium)
  - Tildes: vector or sum excluding the component or term referring to wages
- $\varepsilon_{\langle a,i,t\rangle}$ match $\langle a,i,t\rangle$ specific taste shock

Worker decisions

- Choose match $\langle a, i, t \rangle$ to maximize

$$E\left[\sum_{\tau=t}^{D_a} \delta^{\tau-t}\left(u_\beta\left(\langle a, i, \tau\rangle, s_{\langle a,j,\tau-1\rangle}\right) + \varepsilon_{\langle a,i,\tau\rangle}\right) \mid s_{\langle a,j,t-1\rangle}, \varepsilon_{a,t}\right]$$

- $\delta \in [0, 1]$ is the discount factor
- $\varepsilon_{a,t}$ vector of all $\varepsilon_{\langle a,i,t\rangle}$'s

Dynamic programming problem

- Finite-horizon, discrete-time
    - Infinite horizon no issue
- Integrated continuation value $V\left(s_{\langle a,j,t-1\rangle}\right)$

$$= \int \max_{\langle a,i,t\rangle} E\left[\sum_{\tau=t}^{D_a} \delta^{\tau-t}\left(u_\beta\left(\langle a,i,\tau\rangle, s_{\langle a,j,\tau-1\rangle}\right) + \varepsilon_{\langle a,i,\tau\rangle}\right) \mid s_{\langle a,j,t-1\rangle}, \varepsilon_{a,t}\right]$$
$$\cdot g_{\theta_2}\left(\varepsilon_{a,t}\right) d\varepsilon_{a,t}$$
$$= \int \max_{\langle a,i,t\rangle}\left[u_\beta\left(\langle a,i,t\rangle, s_{\langle a,j,t-1\rangle}\right) + \varepsilon_{\langle a,i,t\rangle}\right.$$
$$\left. + \delta \int_{s_{\langle a,i,t\rangle}\in S} V\left(s_{\langle a,i,t\rangle}\right) h_{\theta_1}\left(s_{\langle a,i,t\rangle} \mid s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}, \langle a,i,t\rangle\right) ds_{\langle a,i,t\rangle}\right]$$
$$\cdot g_{\theta_2}\left(\varepsilon_{a,t}\right) d\varepsilon_{a,t}$$
$$= \int \max_{\langle a,i,t\rangle}\left[v\left(\langle a,i,t\rangle, s_{\langle a,j,t-1\rangle}\right) + \varepsilon_{\langle a,i,t\rangle}\right] \cdot g_{\theta_2}\left(\varepsilon_{a,t}\right) d\varepsilon_{a,t}$$

Worker match probabilities

- Wages are endogenous, set before realization of $\varepsilon_{\langle a,i,t\rangle}$
- Integrate $\varepsilon_{\langle a,i,t\rangle}$'s to form match probabilities

$$\Pr_t\left(\langle a,i,t\rangle \mid s_{\langle a,j,t-1\rangle}\right) = \int 1\left[v\left(\langle a,i,t\rangle, s_{\langle a,j,t-1\rangle}\right) + \varepsilon_{\langle a,i,t\rangle} > v\left(\langle a,k,t\rangle, s_{\langle a,j,t-1\rangle}\right) + \varepsilon_{\langle a,k,t\rangle} \forall k \neq i\right] \cdot g_{\theta_2}\left(\varepsilon_{a,t}\right) d\varepsilon_{a,t}$$

- McFadden / Rust: GEV class for $g_{\theta_2}\left(\varepsilon_{a,t}\right)$
  - Closed forms for integrals $V\left(s_{\langle a,j,t-1\rangle}\right)$ & $\Pr_t\left(\langle a,i,t\rangle \mid s_{\langle a,j,t-1\rangle}\right)$
- If wages exogenous
  - Like Kennan & Walker (2002)
  - Switching between US states

Job current-period profit functions

- Job i in period t with state $s_{\langle b,i,t\rangle}$ profit function for match $\langle a,i,t\rangle$

$$\pi_\gamma\left(\langle a,i,t\rangle, s_{\langle b,i,t-1\rangle}\right) + \varepsilon^f_{\langle a,i,t\rangle} = \tilde{\pi}_\gamma\left(\langle a,i,t\rangle, s_{\langle b,i,t-1\rangle}\right) - \gamma_w w_{\langle a,i,t\rangle} + \varepsilon^f_{\langle a,i,t\rangle}$$

- γ parameters to estimate
- $\varepsilon^f_{\langle a,i,t\rangle}$ match-specific unobserved profit
- Jobs forward looking with $\delta \in [0,1]$
- Continuation values $V^f\left(s_{\langle b,i,t-1\rangle}\right)$ and $v^f\left(\langle a,i,t\rangle, s_{\langle b,i,t-1\rangle}\right)$

$$\Pr^f_t\left(\langle a,i,t\rangle \mid s_{\langle b,i,t-1\rangle}\right) = \int 1\Big[v^f\left(\langle a,i,t\rangle, s_{\langle b,i,t-1\rangle}\right) + \varepsilon^f_{\langle a,i,t\rangle} > v^f\left(\langle c,i,t\rangle, s_{\langle b,i,t-1\rangle}\right) + \varepsilon^f_{\langle c,i,t\rangle} \,\forall c \neq a\Big] \cdot g^f_{\theta_3}\left(\varepsilon^f_{i,t}\right) d\varepsilon^f_{i,t}$$

State counts for period t

- N total workers, J total jobs
- S set and # of states
  - Finite number of states $s_{\langle a,i,t-1\rangle} = \left(x_{a,t}, x^{f}_{i,t}\right)$
- $N\left(s_{\langle a,i,t-1\rangle}\right)$ workers at state $s_{\langle a,i,t-1\rangle}$
- $N^{f}\left(s_{\langle a,i,t-1\rangle}\right)$ jobs at state $s_{\langle a,i,t-1\rangle}$
- Usually $N\left(s_{\langle a,i,t-1\rangle}\right) = N^{f}\left(s_{\langle a,i,t-1\rangle}\right)$
  - Except for partner 0, unmatched

Equilibrium in period t

- A wage function:

$$w_{\langle a,i,t\rangle} = w\left(s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right),$$

  for all pairs $\left(s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right)$
- Expected labor supply equals expected labor demand for all matches and pairs of states:

$$N\left(s_{\langle a,j,t-1\rangle}\right) \Pr_t\left(\langle a,i,t\rangle \mid s_{\langle a,j,t-1\rangle}\right) = N^f\left(s_{\langle b,i,t-1\rangle}\right) \Pr_t^f\left(\langle a,i,t\rangle \mid s_{\langle b,i,t-1\rangle}\right)$$

  - for all pairs $\left(s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right)$
  - for all period t matches $\langle a,i,t\rangle$
  - where $a \in N\left(s_{\langle a,j,t-1\rangle}\right)$ and $i \in N^f\left(s_{\langle b,i,t-1\rangle}\right)$

Model as a system of equations

S is number of match states. Model is a set of nonlinear equations:

1. $S \times S$ supply equals demand conditions
2. S workers' Bellman equations
3. S jobs' Bellman equations

Model unknowns:

1. $S \times S$ wages $w\left(s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right)$
2. S workers' continuation values $V\left(s_{\langle a,j,t-1\rangle}\right)$
3. S jobs' continuation values $V^f\left(s_{\langle b,i,t-1\rangle}\right)$

Equilibrium match probabilities when $J = N = 2$

Logit taste shocks

$$\Pr_t^\star\left(\langle a,i,t\rangle \mid s_{\langle a,j,t-1\rangle}\right) = \Pr_t^\star\left(\langle b,j,t\rangle \mid s_{\langle b,i,t-1\rangle}\right) = \frac{A}{A+B}$$

where

$$A = \exp\left(\frac{1}{2\left(1+\alpha_w\right)}\left(\alpha_w\left(\tilde{v}\left(\langle a,i,t\rangle, s_{\langle a,j,t-1\rangle}\right) + \tilde{v}\left(\langle b,j,t\rangle, s_{\langle b,i,t-1\rangle}\right)\right)\right.\right.$$
$$\left.\left. + \tilde{v}^f\left(\langle a,i,t\rangle, s_{\langle b,i,t-1\rangle}\right) + \tilde{v}^f\left(\langle b,j,t\rangle, s_{\langle a,j,t-1\rangle}\right)\right)\right)$$

and

$$B = \exp\left(\frac{1}{2\left(1+\alpha_w\right)}\left(\alpha_w\left(\tilde{v}\left(\langle a,j,t\rangle, s_{\langle a,j,t-1\rangle}\right) + \tilde{v}\left(\langle b,i,t\rangle, s_{\langle b,i,t-1\rangle}\right)\right)\right.\right.$$
$$\left.\left. + \tilde{v}^f\left(\langle b,i,t\rangle, s_{\langle b,i,t-1\rangle}\right) + \tilde{v}^f\left(\langle b,j,t\rangle, s_{\langle b,j,t-1\rangle}\right)\right)\right)$$

$\alpha_w = \gamma_w / \beta_w$ ratio wage parameters jobs & workers

Solving the model on the computer

- Schmedders (2008): use Newton's method to solve nonlinear equations
  - Quadratic convergence near solution
- Use a nonlinear solver to solve constrained max problem

$$\max_{\{V(s), V^f(s), w_{s,s'}\}} 0 \text{ s.t. model equations}$$

- Use restart feature to try many starting values
  - Counterfactuals easier as starting closer to solution
  - Got KNITRO 5.2 to add features to stop at first solution, record solutions
  - AMPL automatic differentiation exploits sparsity of model equations

Reducing model unknowns & equations

- Two curses of dimensionality addressed already
- $S + S + S \times S$ equations and unknowns
  - For each of T periods
- Dimension problem is in wages or supply equals demand, $S \times S$
- Alternative: group states into larger equivalent classes
  - Wages attached to job assignments and plants
- Group $\left(s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right)$ into equivalence classes $\mathscr{X}_t$
  - Equilibrium wage function $w\left(\mathscr{X}_t\right)$
  - Supply equals demand

$$\sum_{\left\{s_{\langle a,j,t-1\rangle} \mid \left(s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right) \in \mathscr{X}_t\right\}} N\left(s_{\langle a,j,t-1\rangle}\right) \Pr_t\left(\langle a,i,t\rangle \mid s_{\langle a,j,t-1\rangle}\right)$$

$$= \sum_{\left\{s_{\langle b,i,t-1\rangle} \mid \left(s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right) \in \mathscr{X}_t\right\}} N^f\left(s_{\langle b,i,t-1\rangle}\right) \Pr_t^f\left(\langle a,i,t\rangle \mid s_{\langle b,i,t-1\rangle}\right)$$

Existence and uniqueness

- **Existence** Never found model where $J = N$ without solution
  - Caveat: logit error terms
- **Uniqueness** Have not found any example with multiple matching probabilities
  - Wages determined up to a constant (no outside good)
  - May be sensitive to logit (Anderson, de Palma and Thisse 1992)

Empirical: matches more robust than prices

- Many authors show any stable match is efficient
  - First welfare theorem
  - Maximizes sum of agents payoffs
  - Not identical model but similar property here
- Competition limit of many imperfect competition models
- One-to-one matching, equilibrium physical matches are unique
  - Prices may not be, lie in intervals

Alternative pricing institutions can enforce efficient assignment

- Say iid shocks put positive mass on all matches
- Match-specific prices
  - Worker of state s quoted price by all firms
  - Worker chooses firm based on the price
  - Firm quotes a low price if the firm doesn't want the worker
  - Data: might only observe transacted price
- Job-specific prices plus screening
  - Firm administratively sets one wage for all workers
  - Firm knows equilibrium match, screens workers (resumes, interviews) to weed out less desirable workers
  - Data: wages only available to qualified workers, not all switchers
  - Wage regression will not distinguish worker, firm characteristics

Same matches in two pricing institutions

- Both match-specific wage and one wage + screening support same assignment
- Parallels in other literatures
  - Revelation principle gives actions in contract
    - Real-life contracts implement actions from revelation principle
  - Preferences
    - Many utility functions represent same preference ordering
- Matches more model-invariant than prices

Estimation: Data

- Year t
- A statistical observation for match $\langle a, i, t \rangle$

$$\left(\langle a, i, t \rangle, s_{\langle a,j,t-1 \rangle}, s_{\langle b,i,t-1 \rangle}, s_{\langle a,i,t \rangle} \right)$$

  - Match, current non-wage states, next period's state
- Need data on all firms, workers in matching market
  - Full solution approach
  - Unlike maximum score approach in Fox (2007)

Maximum likelihood

- Can factor likelihood into match, state components
  - Logit shocks occur after wages
- Match $\langle a, i, t \rangle$'s contribution to the likelihood

$$L\left(\langle a,i,t\rangle, s_{\langle a,i,t\rangle} \mid s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}\right) =$$

$$h_{\theta_1}\left(s_{\langle a,i,t\rangle} \mid s_{\langle a,j,t-1\rangle}, s_{\langle b,i,t-1\rangle}, \langle a,i,t\rangle\right) \Pr^{\star}_{t,\tilde{\beta},\tilde{\gamma},\theta_1,\theta_2,\theta_3}\left(\langle a,i,t\rangle \mid s_{\langle a,j,t-1\rangle}\right)$$

- Rust: estimate $\hat{\theta}_1$ in first stage, plug into second stage

Need wages, continuation values

- To compute $\Pr^{\star}_{t,\tilde{\beta},\tilde{\gamma},\theta_1,\theta_2,\theta_3}\left(\langle a,i,t\rangle \mid s_{\langle a,j,t-1\rangle}\right)$
  - *Equilibrium* matching probabilities
- Wages and continuation values determined by
  - Parameters
  - Distribution over states
- Two step estimator
  - Hotz and Miller (1993), many others
  - Large number of firm states relative to workers per state
  - Noisy nonparametric estimates of match probabilities
- Nested fixed point approach
  - Solving model requires many starting values
  - Computationally infeasible to nest inside likelihood evaluation

Estimate as constrained optimization (MPEC)

- Su & Judd (2007) suggest

$$\max_{\lambda} \sum_{t=1}^{T} \sum_{a=1}^{N_t} \log \Pr^{\star}_{t,\tilde{\beta},\tilde{\gamma},\theta_1,\theta_2,\theta_3} \left(\langle a, i, t \rangle \mid s_{\langle a,j,t-1 \rangle} \right)$$

  subject to supply equals demand, firms' Bellmans, workers' Bellmans, and where

$$\lambda = \tilde{\beta}, \tilde{\gamma}, \theta_3, \theta_4, (V(s))_{s \in S}, \left\{ V^f(s) \right\}_{s \in S}, \left\{ w_{s,s'} \right\}_{s \in S, s' \in S}$$

- Statistically almost same estimator as Rust, only computer program different
  - Have verified on small problems
  - Su & Judd handle multiple equilibria better

Su & Judd computational advantages

- Speed same order of magnitude as solving economic model once
  - Evaluating likelihood using nested fixed point once!
  - Matching, often
    - # structural parameters << # economic model unknowns
- Similar code to estimate as to solve model
- Standard errors
  - Silvey (1959)
  - Easier said than done!

Identification

- Identify workers' utility flows if know workers' continuation values?
  - Rust (1994), Magnac and Thesmar (2002), Aguirregabiria (2003), Bajari and Hong (2006), Heckman and Navarro (2007), Pesendorfer & Schmidt-Dengler (2006)
- Identify workers' and jobs' profit continuation values separately?
  - Not without unmatched people or exclusion restrictions
  - Fox (2007), Choo & Siow (2006)
- Normalizations: if no wage data, can estimation proceed with $\beta_w \neq \gamma_w$ unknown?
  - No, need $\beta_w = \gamma_w$

Engineers' in Sweden flow utility

$$u_\beta\left(\langle a,i,t\rangle, s_{\langle a,j,t-1\rangle}\right)+\varepsilon_{\langle a,i,t\rangle}=\beta_w w_{\langle a,i,t\rangle}+\sum_{l=1}^{4}\beta_{\mathrm{sc}_l,\mathrm{age}}\mathrm{sc}_l\left(\langle a,j,t-1\rangle,\langle a,i,t\rangle\right)+$$
$$\beta_{\mathrm{age,geo}}\mathrm{geo}\left(\langle a,j,t-1\rangle,\langle a,i,t\rangle\right)+\beta_{\mathrm{age,size,1}}\left|\Delta\mathrm{size}\left(\langle a,j,t-1\rangle,\langle a,i,t\rangle\right)\right|+\varepsilon_{\langle a,i,t\rangle}$$

- Switching cost in binary characteristics
  - Switch parent corporation (no transfer between plants)
  - Switch any plant at all
    - Fox & Smeets (2007) use firm output data to show firm tenure compatible with a newcomer vs. all others effect
  - Switch occupation of job
  - Switch industry of the plant
  - Switch county
    - $\mathrm{geo}\left(\langle a,j,t-1\rangle,\langle a,i,t\rangle\right)=$ $\log\mathrm{distance}\left(\mathrm{geo}\left(\langle a,j,t-1\rangle,\langle a,i,t\rangle\right)\right)$
  - Plant size
    - $\Delta\mathrm{size}\left(\langle a,j,t-1\rangle,\langle a,i,t\rangle\right)=\log\mathrm{size}\left(j_{t-1}\right)-\log\mathrm{size}\left(i_t\right)$
- $\delta=0.96^5\approx 0.82$

Firm's flow profits

$$\pi_\gamma\left(\langle a,i,t\rangle, s_{\langle b,i,t-1\rangle}\right) + \varepsilon^f_{\langle a,i,t\rangle} = \gamma_{\text{sizeage}} \log \text{size}\,(i_t) \cdot \text{age}_{a,t} - \gamma_w w_{\langle a,i,t\rangle} + \varepsilon^f_{\langle a,i,t\rangle}$$

- Term $\log \text{size}\,(i_t) \cdot \text{age}_{a,t}$ captures sorting by worker age, firm size
  - $\gamma_{\text{sizeage}} > 0$, efficient for older workers to be at larger plants
  - $\gamma_{\text{sizeage}} < 0$, efficient for older workers to be at smaller plants
  - In data, $\text{size}\,(i_t)$ is total number of white collar workers at an establishment
- Because of computer memory constraints, today only including 35 largest plant*job assignment categories
  - Not much variation in $\text{size}\,(i_t)$ in the sample
  - Downside of AMPL's automatic differentiation

Swedish data

- Data on almost all private sector workers in national labor market, 1970–1990
- Focus on elite group
  - Five-year engineering degree
  - Most elite undergraduate degree in Sweden
  - Only available from small number of university equivalents

Number of states (small example)

- Use 7 five year gaps as ages, and 35 "firms"
  - $7 \cdot 35 = 245$ worker states
  - 35 more $(1 \cdot 20)$ firm states (total 280) for positions currently vacant
    - Ex: retirement
- $N^f(\langle a, j, t-1 \rangle)$: "Firm" j is an occupation at a particular plant
  - Production engineer at plant 7
  - How wages vary

Computational burden of example

- 245 worker states
  - 245 continuation values + 35 $V(\text{retired}) = 0$ terms for finite horizon
- 280 firm states
  - 280 firm continuation values
- 2 wages (switchers, stayers) for each firm
  - $35 \cdot 2 = 70$ wages
- $245 + 35 + 280 + 70 = 630$ unknowns
  - times 2 years of data (1988 and 1989) = 1260 unknowns
  - 1260 constraints
- 7 structural parameters, 1267 Su Judd optimization variables

Switching cost point estimates

- Relative to standard deviation (≈ 1.28) of logit error term

Switching cost	Point estimate
Plant	-6.81
Corporation	-1.40
Industry	-2.18
Geo distance (log km)	-0.62
Occupation	-7.00
Plant size ($\lvert\Delta \log \text{size}\rvert$)	0.16

- Plant size gives positive switching cost, small magnitude relative to others
- Gothenburg to Stockholm is ≈ 400 km, or $\log 400 = 5.98$, $-0.62 \cdot 5.98 = -3.71$

Firm parameter

Firm parameter	Point estimate
$\log \text{size} \cdot \lfloor (\text{age} - 20)/5 \rfloor$	0.43

- Should be negative with full sample of small, large plants
- Only using top 35 plants: sorting by age not in sample

In sample statistical fit, geography

- Counties, 1984–1989, switch job at all (occupation, plant, etc.)

County	Data	Model
Stockholm	0.20	0.23
Ostergotland	0.16	0.28
Malmohus	0.08	0.14
Goteborg	0.26	0.21
Alvsborg	0.16	0.22
Orebo	0.22	0.23
Kopparberg	0.14	0.18
Gavleborg	0	0.06
Vasternorrland	0.15	0.27
Norrbotten	0.16	0.21

Statistical fit, industry

- Again, switching rate 1984–1989

Job assignment	Data	Model
Chemicals	0.17	0.22
Groceries	0.24	0.27
Manufacturing	0.08	0.15
Electrical	0.09	0.07
Construction	0.26	0.27
Transportation	0	0.03

Statistical fit, job assignments

- Again, switching rate 1984–1989

Job assignment	Data	Model
Production management	0.39	0.41
Research & development	0.17	0.21
Construction & design	0.11	0.20
Technical, planning, control etc.	0.05	0.11
Business fields	0.18	0.16

Statistical fit, fraction of switchers that move between category X

Recall, data on only 35 top occupation*plant categories

- Statistical fit for destination
- Base is all plant (or job assignment) switchers 1984–1989

Category	Data	Model
Counties	0.92	0.92
Industries	0.98	0.97
Parent firms	0.50	0.52
Plant size quantiles	0.50	0.28
Job assignments	0.37	0.49

- Predicted switching rate in Sweden in 1984–1989 if
  - Perturb one dimension plant / job characteristics
- Fix 7 structural parameters, resolve for the 1260 endogenous variables

Counterfactual	Change	Switching rate
County	Same	0.28
	Different	0.20
Industry	Same	0.21
	Different	0.10
Occup	Same	0.63
	Different	0.15
Plant	Same	0.72
	Different	0.14
Corporation	Same	0.26
	Different	0.16

Conclusions

- Matching games important new research tool for empirical work
- Datasets on repeated matches
- Idea: look at destination match of switchers, not just incidence of switching
- Estimated repeated matching game
    - Both sides of the market are forward looking
- Can explore counterfactual industry structures
- Application to market thickness and switching