Solving the Bewley Model with AMPL

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The Income Fluctuation Problem

$$
\begin{aligned}
V(a,\varepsilon) &= \max_{c,a'} \left\{ u(c) + \beta E_{\varepsilon} V(a',\varepsilon') \right\} \\
\text{s.t.} \quad c &= (1+r)a + w\varepsilon - a' \\
a' &\geq \underline{a}
\end{aligned}
$$

Stationary RCE

$\{V, a', c, H, K, r, w, \lambda^*\}$ such that:

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2. Markets clear:

$$H = \int \varepsilon d\lambda^*$$

$$K = \int a'(a, \varepsilon) d\lambda^*$$

$$F(K, H) = \int c(a, \varepsilon) d\lambda^* + \delta K$$

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3. Stationary distribution:

$$\lambda^*(\mathcal{B}) = \int Q\left((a, \varepsilon), \mathcal{B}\right) d\lambda^*$$

$$Q((a, \varepsilon), \mathcal{B}) = \mathbb{I}(a'(a, \varepsilon) \in \mathcal{B}) \sum_{\varepsilon' \in \mathcal{B}} \pi(\varepsilon, \varepsilon')$$

The Dark Ages vs. Enlightenment

1. Traditionally: NFXP

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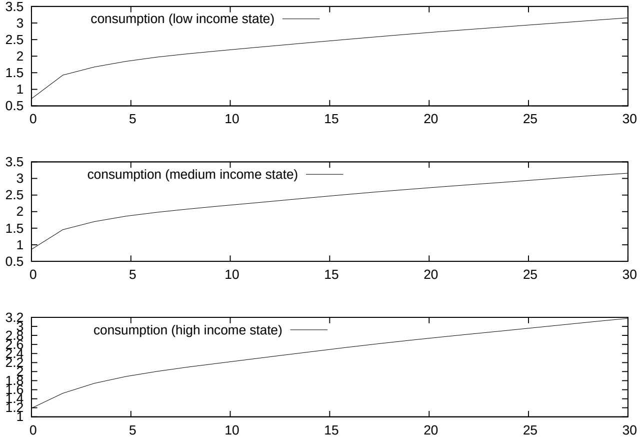
1. Traditionally: NFXP
2. ICE: Solve it at once

Our Approach - PE

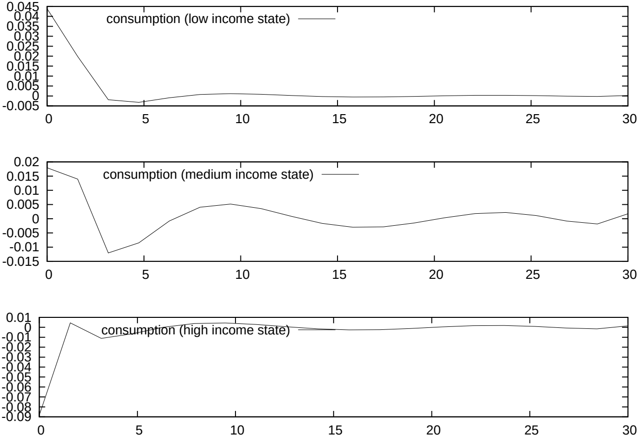
Problem: Binding borrowing constraint!

$$
\begin{aligned}
c_{con} &= (1+r)a + w\varepsilon - \underline{a} \\
u'(c_{unc}) &= \beta(1+r)E_{\varepsilon}(u'(c')) \\
c &= \min\{c_{con}, c_{unc}\} \\
a' &= (1+r)a + w\varepsilon - c \\
a' &\geq \underline{a}
\end{aligned}
$$

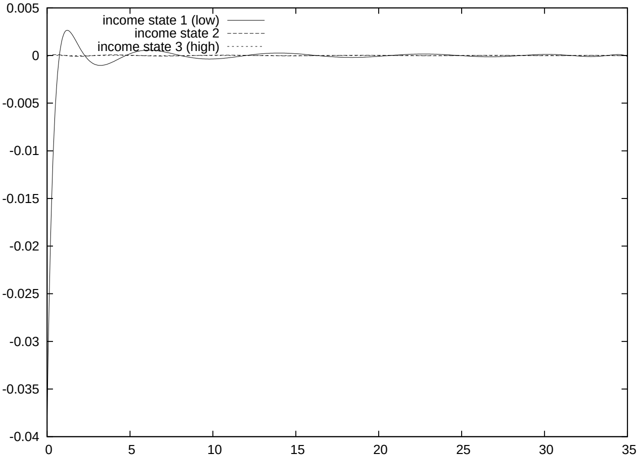
Policy Functions



Euler Equation Errors



Euler Equation Errors II



Our Approach - GE

- policy function iteration

$$c_{unc} = u'^{-1}\left[\beta(1+r)E_{\varepsilon}u'\left(c(a', \varepsilon')\right)\right]$$

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- Discretization of $a'(a,\varepsilon)$.

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- policy function iteration

$$c_{unc} = u'^{-1}\left[\beta(1+r)E_{\varepsilon}u'\left(c(a',\varepsilon^{'})\right)\right]$$

- Discretization of $a'(a,\varepsilon)$.
- $$P'\lambda = \lambda$$

The Equilibrium Solution

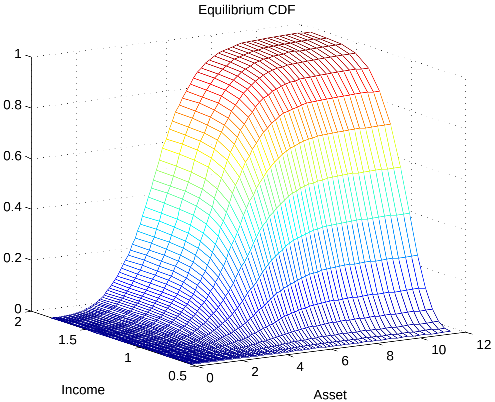

Figure: IID Income Shock

The Renaissance

- Starting values

The Renaissance

- Starting values
- Kinks (i)

The Renaissance

- Starting values
- Kinks (i)
- Kinks (ii)