Economics Problems

1. Solve the dynamic life-cycle consumption problem (14.1) where $w = 1$ for all t, $T = 40$, $\beta = 0.96$, and $r = 0.06$. Next assume utility is $u(c) - v(\ell)$ where c is consumption and ℓ is labor supply. If c_t and ℓ_t are the consumer's consumption and labor supply choices in period t, then the dynamic budget constraint implies that S_t follows

$$\begin{aligned} S_0 &= 0 = S_T \\ S_{t+1} &= (1+r)S_t + w\ell_{t+1} - c_{t+1} \end{aligned}$$

The consumer chooses c_t and ℓ_t to maximize

$$\sum_{t=0}^{t=T} \beta^t \left(u(c_t) - v(\ell_t)\right)$$

subject to his dynamic budget constraint. Let $u\left(c\right) = c^{1-\gamma}/\left(1-\gamma\right)$, $\gamma = .5, 2, 3$, and let $v(\ell) = \ell^{1+\eta}/(1+\eta)$ where $\eta = 2, 10$. Compute the solutions of these life-cycle problems. Use any optimization method or software available to you.

2. Consider the endowment economy with m goods and n agents. Assume that agent i's utility function over the m goods is

$$u^i(x) = \sum_{j=1}^{m} a_j^i \log\left(\nu_j^i + x_j\right)$$

Suppose that agent i's endowment of good j is e_j^i. Assume that $a_j^i,\ e_j^i,\ \nu_j^i > 0$, $i = 1, \cdots, n$, $j = 1, \cdots, m$. Compute the competitive equilibrium using a nonlinear equation solver. Let $m = 4$ and $n = 3$, and solve for the case

$$\begin{aligned} a^1 &= (1,1,1,1),\ \nu^1 = (2.5, 1.6, 1.7, 1.9) \\ a^2 &= (1,2,3,4),\ \nu^2 = (2,3,2,3) \\ a^3 &= (2,1,1,5),\ \nu^3 = (0.6, 0.7, 2.5, 2.1) \end{aligned}$$

3. Consider the endowment economy with m goods and n agents. Assume that agent i's utility function over the m goods is

$$u^i(x) = \sum_{j=1}^{m} a_j^i \; (1 + x_j)^{1-\nu_j^i} \; (1 - \nu_j^i)^{-1} \; .$$

Suppose that agent i's endowment of good j is e_j^i. Assume that a_j^i, e_j^i, $\nu_j^i > 0$, $i = 1, \cdots, n$, $j = 1, \cdots, m$. The social planner's problem with n consumers is

$$\begin{aligned} \max_{x_j^i,\ i=1,..,n,\ j=1,...,m} \quad & \sum_{i=1}^{n} \lambda_i u^i(x^i) \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_j^i \leq \sum_{i=1}^{n} e_j^i, \; j = 1, .., m \end{aligned}$$

Write a program that will read in the ν_j^i, a_j^i, and e_j^i values and the social weights $\lambda_i > 0$, and output the solution to the social planner's problem. Let $m = 4$ and $n = 3$, and solve for the case

$$\begin{aligned} a^1 &= (1,1,1,1), \; \nu^1 = (1.5, 1.6, 1.7, 1.9) \\ a^2 &= (2,3,5,7), \; \nu^2 = (2,3,4,2) \\ a^3 &= (2,1,3,5), \; \nu^3 = (0.4, 0.6, 1.5, 2.5) \end{aligned}$$

where $e_j^i = 1$ for all i and j, and the social weights are $\lambda = (1,1,1)$, $(2,4,7)$, $(1,5,4)$. Use the endowment vectors as the initial guesses.

4. Consider

$$\max_{c_t} \sum_{t=1}^{\infty} \beta^t u(c_t)$$
$$k_{t+1} = f(k_t) - c_t$$

The solution is expressed in terms of an unknown function

$$c_t = C(k_t) : \text{consumption function}$$

which satisfies the functional equation:

$$
\begin{aligned}
0 &= u'(C(k)) - \beta u'(C(f(k) - C(k)))f'(f(k) - C(k)) \\
&\equiv (\mathcal{N}(C))(k)
\end{aligned}
$$

Create an approximation:

$$\widehat{C} \equiv \sum_{i=0}^{n} a_i k^i$$

which "nearly" solves

$$\mathcal{N}\ (\widehat{C}) = 0$$

for $k \in [0.5, 1.5]$ for the example presented in the Mathematica notebook on perturbation. Compare the perturbation and projection solutions.