

**Exercise 1** Suppose the random variables  $x_t$  ( $t = 1, \dots, T$ ) are independent, and  $P(x_t = \theta - 1 | \theta) = P(x_t = \theta + 1 | \theta) = 1/2$ .

Experiment 1 consists of collecting a sample of size  $T = 2$ . Show that a 75% confidence interval of smallest size for  $\theta$  is

$$C(x_1, x_2) = \begin{cases} \text{the point } (x_1 + x_2) / 2 & \text{if } x_1 \neq x_2 \\ \text{the point } x_1 - 1 & \text{if } x_1 = x_2 \end{cases}$$

(Thus if used repeatedly  $C(x_1, x_2)$  would contain  $\theta$  with probability 0.75.) The evidence in Experiment 1 is that  $C(x_1, x_2)$  constitutes a 75% confidence interval for  $\theta$ .

**Question:** In the context of (a), suppose you observe  $x_1 = 2$  and  $x_2 = 4$ . Then the 75% confidence interval in (a) is the point 3. Is this consistent with common sense about the reliability of the conclusion that  $\theta = 3$ ?