Optimization and Equations: Connections Between Economics and Numerical Analysis

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Optimization is Fundamental in Economics Modelling

- What is economics:
  - Definition: The study of the allocation of scarce resources
  - Assumption: actors make choices that maximize an objective function
  - Hence, economics problems are constrained optimization problems: maximize objective subject to scarcity constraints
- Examples
  - Consumer choice
  - Social planning problems
  - Principal-agent problems
  - Life-cycle problems
  - Profit maximization
  - Portfolio choice

Equations are Fundamental in Economics Modeling

- Equilibrium: a collection of choices by economic actors that are consistent with scarcity and individual rationality
- Demand equals supply
  - Competitive equilibrium
  - Asset market equilibrium
  - Dynamic market equilibrium
- Nash-Cournot
  - Oligopoly theory
  - Games of incomplete information
  - Games of asymmetric information
  - Political games

All Economic Analysis Uses Optimization and Equations

- Analysis of economic data is an optimization problem
- Unknown parameters are chosen so as to maximize the compatibility between statistical model and data
  - Least squares methods
  - Method of moments
  - Maximum likelihood
- Unknown parameters are chosen to fit data and satisfy equilibrium conditions
  - Structural estimation
  - A constrained optimization problem

Numerical Analysis is Applied Economics

- Numerical analysis is the development of computational tools that best use scarce computational resources to accomplish a computational task
- Scarce resources
  - Computer time
  - Programmer time
  - Progammer ability
- Objective
  - Accuracy
  - Speed
- Technologies
  - Memory
  - Processor
  - Communication links

Computation is About Using Computers

- You need to understand what computers do
  - Numbers - stored with infinite precision
  - Operations - executed with small errors
  - Storage methods and cache management
  - Interpreted versus compiled code
- As computer technologies change, the choice of algorithms changes
  - Single precision to double precision
  - Expensive memory to cheap memory
  - Serial to parallel processing (e.g., GPUs)

Progress in Hardware

- Moore's law for semiconductors (Moore gives Moore's law about another 10-15 years)
- Optical computing
- Quantum computing

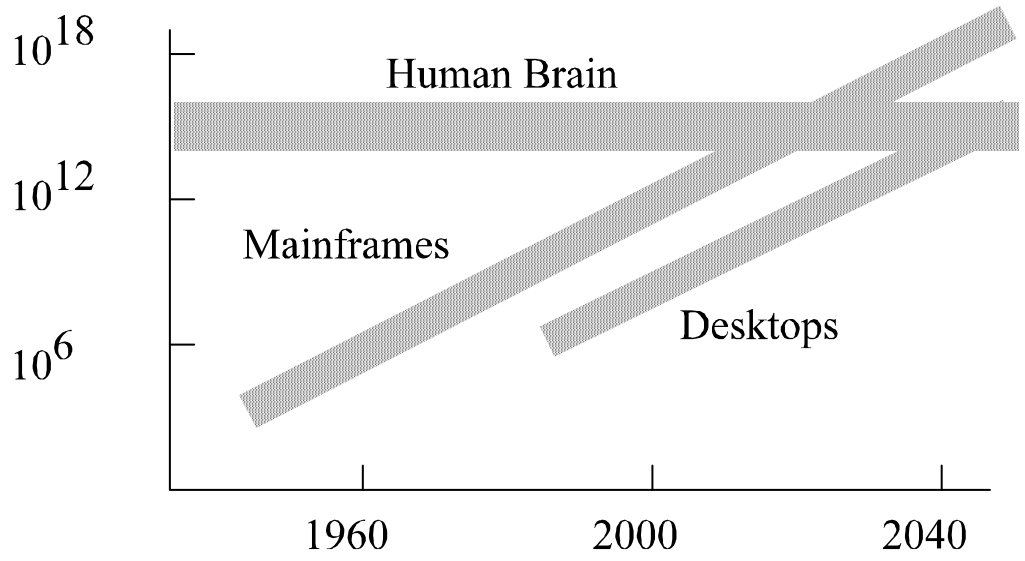

Figure 1: Trends in computation speed: flops vs. year

- Economists usually constrain themselves to using personal computers.

Hardware and Algorithms: Substitutes or Complements?

- Typical economist view: speed allows you to avoid learning about computation and methods
- Computational mathematicians' view
  - There are many possible methods, varying in fixed costs and marginal costs
  - Faster computers make it rational to invest in the fixed costs
    - Development costs
    - Use high fixed-cost methods that reduce marginal costs
  - Hardware speed and algorithm development are complements
- Historical pattern
  - Speed doubles every two years.
  - Algorithm efficiency grows at a similar, sometimes faster, rate

Objectives of ICE10

- Acquaint PhD students with the current state-of-the-art algorithms and software
- Teach the basic concepts behind algorithm development, now and in the future.