Mergers and Dynamic Oligopoly

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Abstract. Static oligopoly theories disagree on whether mergers are profitable. The Cournot model says that many potential mergers would be unprofitable whereas the Bertrand model says that all mergers are profitable. We show that, for economically sensible parameter values, mergers are profitable for merging firms when firms choose both price and output, using inventories to absorb differences between output and sales. Furthermore, substantial cost advantages are necessary for a merger to benefit consumers. The merger predictions of our dynamic model are most similar to predictions of static Bertrand analyses of differentiated products even though our model often behaves like the Cournot model in the long run.

JEL CLASSIFICATION:

KEYWORDS: Oligopoly, dynamic games, mergers

1. Introduction

A major task of antitrust policy is the regulation of mergers. A merger allows participating firms to coordinate their actions and, presumably, increase profits. This will harm consumers to the extent that this coordination allows firms to raise prices and reduce output, but it may benefit consumers if cost savings from the merger are so large that prices fall. Any discussion of antitrust policy requires an understanding of the impact of mergers on consumer welfare and producer profits. Despite the simple intuition, static economic theory is ambiguous in its predictions. Cournot analysis argues that if a merger does not reduce costs and does not nearly produce a monopoly, then the merging firms will lose profits. The Cournot view implies that regulators only need to prevent the formation of monopolies and that firms would pursue a nonmonopolistic merger only if it reduced

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their costs. Bertrand analysis argues that firms will always be able to enhance market power and profits through merging and argues for more activist merger regulation. Simple static models fail to give us a clear analysis of mergers. We re-examine basic merger questions using the dynamic model of oligopoly presented in Judd (1990), which allows firms to choose both prices and output, using inventories to absorb differences between output and sales. We find that mergers are generally profitable for participating firms in the Judd (1990) model for empirically reasonable specifications of taste and technology.

The results from static oligopoly analysis follow directly from their simple static assumptions. The intuitive view is supported by Bertrand-style analysis with differentiated goods. Deneckere and Davidson (1985) showed that merged firms will increase prices on their goods, and that the unmerged firms will follow by raising their prices, resulting in higher profits for all firms. Price increases in Bertrand competition are mutually beneficial, and mergers of any size are profitable. In contrast, mergers are not always profitable in Cournot games. If firms merge in a Cournot oligopoly of a homogeneous good, then the merged firm will want to reduce its output relative to the premerger total output. The unmerged firms will respond by increasing their output, a response which reduces profits to the merged firm. In fact, Salant, Switzer and Reynolds (1983) conclude that mergers involving fewer than 80% of the firms in the market are unprofitable. Thus, a potential merger which does not reduce costs will not happen unless it nearly results in monopoly.

The key fact is that Bertrand analyses assume that firms set prices and immediately adjust output to meet demand, whereas Cournot analyses assume that firms decide how much to produce and then accept whatever price is necessary to sell that output. Their simple structure make static models[1] unsatisfactory for studying any oligopoly question, particularly merger problems. They assume that firms may choose only price or quantity whereas real firms choose both. Unfortunately, the results depend critically on which is chosen by the firm and which is chosen by the market. Since real-world firms choose both price and output, it is desirable to examine merger issues without making arbitrary choices about strategic variables.

The pervasive ambiguities[2] in static oligopoly theory has lead some to argue that one

[1]Some have examined an alternative, Stackelberg-like, approach which assumes that after a merger the merged entity first chooses its output recognizing the later reactions of the other firms. Farrell and Shapiro(1990) and Gaudet and Salant(1991) use this approach. We stay with a dynamic Nash equilibrium concept wherein all actions within a period are simultaneous but each firm's actions today may affect any other firm's actions tomorrow. Our dynamic game approach captures some of the ideas in conjectural variations approaches but does so by explicitly modelling multiperiod interactions in a Nash equilibrium approach.

[2]The ambiguity is an example of how models with strategic complements behave very differently from models with strategic substitutes; see Bulow et al. (1985). This ambiguity is endemic in static merger analysis. For example, Lommerud and Sorgard (1997) examines the profitability of a merger if the unmerged firms react by introducing new varieties. They find that the Cournot and Bertrand models yield different predictions.The Bulow et al. approach will not be useful for us since our strategic variables

model of static oligopoly is better than the other. For example, Kreps and Scheinkman (1983) used a two-stage model where firms first choose capacity and then choose prices. The Kreps-Scheinkman model leads to the same outcomes as the Cournot model. However, even these results are fragile. The Kreps-Scheinkman game needs to specify rationing rules since they do not allow inventories. Deneckere and Davidson (1986) show that the equilibrium is sensitive to those rules.

We argue that it is ultimately futile to search for the best static model, and that instead we should investigate merger questions in more realistic dynamic models. In many industries, each firm decides the price at which it sells its products and how much to produce each period, and uses inventories to absorb differences between current output and current demand. Kirman and Sobel (1974) examined such a model, proving existence of equilibrium under certain conditions. Judd (1990) presented a linear-quadratic model with adjustment costs, investment, and learning. The addition of adjustment costs makes quantities more difficult to adjust than prices, a focus of the Kreps-Scheinkman (1983) model. Judd (1990) examines only cases with linear-quadratic specifications of tastes and technology, a choice we continue here. Since the inherently dynamic model in Judd (1990) allows the flexibility we see in many industries, we regard it as a more reasonable description of reality than either static model.

It first appears that this more complex model will be no more precise in its predictions. Judd (1990) shows that long-run profits, prices, and output replicate the static Cournot model if the costs of adjustment are high. In these cases, any merger would reduce the long-run profits of merger partners. Judd (1990) also shows that if the costs of output adjustment are low then the long- and short-run equilibria are essentially the Bertrand equilibrium, implying that any merger increases the merged firms' profits immediately and permanently. Therefore, the Judd (1990) model produces a mixture of the Bertrand and Cournot results for long-run profits and prices.

However, firms and consumers do not care only about the long-run. When firms consider a merger, they presumably care about the present value of profits, not the long-run profits. Also, the proper measure of the impact on consumers is the present value of consumer surplus. When we take a present value approach, we find surprisingly unambiguous results for merger questions. For a broad range of parameter values, we find that a merger increases the present value of profits for the merging firms, even in many cases where the long-run steady-state equilibrium nearly equals the static Cournot equilibrium. Since the static approach corresponds to a steady state analysis in our dynamic model, it is clearly invalid in a dynamic world where producers and consumers care about the discounted present value of profits and utility.

Salant et al. and Davidson-Deneckere focus on mergers which don't affect costs. A

will be a mixture of complements and substitutes, and which effects dominate depends on the parameters.

merger may be socially beneficial if it allows firms to reduce their costs. The lack of merger incentives in the Cournot model implies that if Cournot firms do merge short of monopoly it must be because of cost savings. However, that is not true in the case of Bertrand oligopolies. Froeb and Werden (1998) and Werden (1996) derive simple formulas for determining how much cost reduction is required for a profitable merger to not result in a price increase. These papers are limited to the static Bertrand and Cournot models. We reexamine this issue in our dynamic model.

This paper also uses a different methodological approach than typically taken in the industrial organization and antitrust policy literature. Because of the complexity of our dynamic model, we cannot express dynamic equilibrium in compact formulas. Instead, we use numerical methods to compare the pre- and post-merger dynamic equilibria. We perform these computations over a broad and realistic range of values for the critical demand and cost parameters. These calculations also produce evidence concerning the quantitative importance of various factors. We argue that dozens of numerical examples using realistic parameters in a realistic dynamic model and producing quantitative information are more useful in analyzing real-world merger problems than theorems producing qualitative results about unrealistic static models.

Our conclusions are limited by the special nature of the model but less so than is typical in oligopoly analysis[3] since its dynamic detail is far more realistic than static analyses. In general, this paper shows that adding realistic dynamic detail is both desirable and simple to accomplish, and has substantial impact on the answers to important merger questions.

2. Dynamic Oligopoly with Inventories and Adjustment Costs

We use a multi-firm extension of the linear-quadratic duopoly model of differentiated products with inventories developed in Judd (1990). Throughout this paper we assume that the representative consumer has the utility function

$$U = C\sum_{i=1}^{N} q_i - \left(\sum_{i=1}^{N} q_i\right)^2 - B\sum_{\substack{i,j=1\\ j>i}}^{N} (q_i - q_j)^2 - \sum_{i=1}^{N} p_i q_i + Y \tag{1}$$

where q_i is consumption of good i, p_i is the price of good i, and Y is money income. We fix the coefficient of $\left(\sum_{i=1}^{N} q_i\right)^2$ to be 1. This is not a limitation since a change in units used to measure q would allow us to change the coefficient of $\left(\sum_{i=1}^{N} q_i\right)^2$ to be 1 without

[3]Our analysis is limited to linear demand and cost structures, a limitation shared by much of the literature. Crooke et al. (1999) emphasizes how the price change after a merger is sensitive to the assumed functional forms in a Bertrand model. In particular, they find that linear specifications predict smaller price changes than many other specifications such as the constant elasticity form. A numerical approach for nonlinear structures could be executed using the methods described in Judd(1992); we leave those generalizations for future work.

loss of generality. The inverse demand functions are linear:

$$\begin{aligned} p_i &= -\psi q_i - \alpha \sum_{\substack{j=1 \\ j \neq i}}^{N} q_j + C \\ \psi &\equiv 2\left(1 + B(N-1)\right),\ \alpha \equiv 2(1-B) \end{aligned} \tag{2}$$

The demand for good i, d_i, is

$$\begin{aligned} d_i &= -bp_i + a \sum_{\substack{j=1 \\ j \neq i}}^{N} p_j + c \\ a &\equiv \frac{1-B}{2BN^2}, b \equiv \frac{B+N-1}{2BN^2}, c \equiv \frac{C}{2N} \end{aligned} \tag{3}$$

Note that C is the "choke price," that is, if all products sold at price C then demand for all goods is zero. The parameter B represents the degree of substitutability among products. Products are perfect substitutes if $B = 0$. As B increases, products are less substitutable, and their demands are independent if $B = 1$. If B exceeds 1 then the products are complements. Since the analysis of complementary products would be substantially different, and discussions of horizontal mergers typically focus on substitutable products, we assume B lies between 0 and 1.

Substitutes Assumption: $0 < B < 1$

The key feature in Kirman-Sobel (1974) and Judd (1990) is the presence of inventories which allow firms to choose price and output simultaneously. This specification adds substantial realism to the model. We follow Judd (1990) and assume adjustment costs in production. The inventory of product i follows the rule

$$I_{i,t+1} = \phi\ (I_{i,t} + q_{i,t} - d_{i,t}) \tag{4}$$

where I_t, q_t, d_t represent beginning-of-period inventory, output, and demand in period t respectively, and $1 - \phi = \delta \in [0, 1]$ is the depreciation rate. In our model, demand is determined by current prices only; hence, products are perishable for consumers, but are somewhat durable as long as stored by firms. Therefore, given positive inventory holding costs, each firm economizes on production and storage costs over time. We assume quadratic inventory holding costs; the holding costs for product i are $H_i(I_i) = h_i\, I_i + g_i I_i^2$.

We assume a cost function for product i that includes adjustment costs. The cost of producing $q_{i,t}$ at time t is

$$C_i(q_{i,t}) = m_i q_{i,t} + \frac{\gamma}{2}\ (q_{i,t} - q_{i,t-1})^2 \tag{5}$$

where m_i is specific to product i, and γ represents the common quantity adjustment cost[4]. Adjustment costs affect short-run marginal costs, but the long-run marginal cost for product i is m_i.

The quadratic adjustment cost in (5) models the realistic notion that it is costly to change output quickly. Kreps and Scheinkman (1983) appeal to adjustment costs to justify their sequential game where quantities are chosen first and prices second. However, their game has only these two stages followed by one period of sales and output. The game in Judd (1990) allows firms to simultaneously choose prices and quantities in each period, and allows inventories to absorb differences between output and sales, and avoids the rationing rules in Kreps-Scheinkman. The sequence of moves in this multi-period dynamic game is a more realistic description of dynamic interaction. The extra flexibility in a dynamic game allows us to model the relative cost of adjusting output and prices in a direct fashion instead of through careful construction of a simpler game. We assume that adjustment cost terms are quadratic since the dynamic equilibrium is then relatively easy to compute.

Let $s_{i,t}$ denote the lagged output of product i, $q_{i,t-1}$. The vector of state variables, y_t, is defined by[5]

$$y_t = [1,\ I_{1,t},\ s_{1,t},\ q_{1,t},\ p_{1,t}, \cdots I_{N,t},\ s_{N,t},\ q_{N,t},\ p_{N,t}]' \tag{6}$$

and the vector of control variables, x_t, is defined by

$$x_t = [q_{1,t},\ p_{1,t}, \cdots q_{N,t},\ p_{N,t}]'$$

The state variables follow the linear law of motion

$$y_t = \aleph\ y_{t-1} + \beth\ x_t \tag{7}$$

where $\aleph$ and $\beth$ are derived from (3,4) and the definitions for y and x. Current profits from product i are defined by the demand and inverse demand systems (2,3) and can be expressed in the quadratic form

$$\pi_i(y) = p_i d_i(p) - C_i(q_i, s_i) - H_i(I_i) = \frac{1}{2}\ y' R_i\, y \tag{8}$$

[4]We assume that γ is common in order to preserve the general symmetry of the problem. Heterogeneous γ would not affect the tractability of the analysis. In order to focus on merger issues and possible cost reductions, we assume symmetry in other aspects of problem, as is typical in this literature. The specification in (5) also assumes that long-run marginal cost is constant. We could add a $n_i q_{i,t}^2$ term to model nonconstant long-run marginal costs, but we focus on the constant LRMC case to keep the analysis simple and comparable to most papers in this literature. Furthermore, we are assuming that marginal costs are constant only in the long run, a far more reasonable assumption than assuming constant short-run marginal cost, the assumption implicit in static analyses which make no distinctions between the short and long run.

[5]We follow the trick of including the constant 1 in the list of state variables. This allows us to express profits as a simple quadratic form. The equations of motion are augmented by $y_{1,t+1} = y_{1,t}$ and the initial condition is $y_{1,0} = 1$.

These specifications define the structure of a linear-quadratic game.

2.1. Closed-Loop Nash Equilibrium and Merger Analysis. We now describe closed-loop Nash equilibrium and the exercise we perform to examine mergers. Each firm (which may produce many products) chooses outputs and prices in each period to maximize the present value of profits. Since the definition of a firm will change with a merger, we must include that possibility in our notation. We assume that there are M products produced by $N \leq M$ firms. Let ω denote the product configuration; specifically, firm $\omega(i)$ produces product i. Define $\Omega_j(\omega)$ to be the products produced by firm j in product configuration ω. Let $x_i = (q_i, p_i)$ denote output and price of product i, and let Π_j denote the profit flow for firm j. Firm j's profits is a function of the state y as well as the product configuration ω, and is expressed in

$$\Pi_j(y, \omega) = \sum_{i \in \Omega_j(\omega)} \pi_i(y) \tag{9}$$

The law of motion is (7) for any product configuration.

For each product configuration ω we have a linear-quadratic game. The value function for firm j is defined to be the present value of current and future profits. The Bellman equation (we follow here the indexing scheme used in Kydland (1975, 1977) where y_{t-1} is the state variable in the value function at time t) for firm j when ω is the product configuration is

$$\begin{aligned} V_j(y_{t-1}, \omega, t) \quad &= \quad \max_{x_i, i \in \Omega_j(\omega)} \{\Pi_j(y_t, \omega) + \beta V_j(y_t, \omega, t+1)\} \\ &\quad\ s.t. \quad y_t = \aleph\, y_{t-1} + \beth\, x_t \end{aligned} \tag{10}$$

Our Bellman equation makes explicit the dependence of equilibrium profits on the product configuration. The value function for each firm is specified differently depending upon the market structure, the control variables and current profits. When each firm produces a single product, the controls for firm i are $x_i = (q_i, p_i)$ and its profits are $\Pi_i = \pi_i$. We take this to be the pre-merger equilibrium. If all firms were to merge the result would be a single firm which would have a monopoly over the N goods.

Firms are no longer symmetric if some of them merge. If firm 1 and firm 2 merge, then the new merged entity controls the variables (q_1, p_1, q_2, p_2) and competes with firms which sell only one product. The profit function for the merged firm will be the sum of π_1 and π_2 but π_1 and/or π_2 may be different due to the merger because of cost savings from joint production. Formally, we allow costs to depend on the product configuration. We assume that technology transfers are free within a firm, and therefore free among merged firms. Thus, it is crucial how we specify the form of (joint) cost functions for a merger[6].

[6]See, for examples of different specifications, Perry and Porter[1985] or Farrell and Shapiro[1990].

If technologies are identical over firms and have no scale effects, a merger will produce no cost-savings.

We can compute the subgame perfect Nash equilibrium for any product configuration since we have assumed linear-quadratic utility and cost functions. We examine two simple cases, following standard merger analysis. We assume that we begin with N firms each producing a single product. After a merger of M firms, we have a non-cooperative game among $N - M + 1$ entities: one merged firm selling M products and $N - M$ unmerged independent firms selling a single product. By comparing the solution of this game with the solution of the initial non-cooperative game among N firms, we can provide possible answers to the questions about the profitability and social value of mergers.

We need to be clear about the dynamic details of the analysis. The critical features are: (i) the N single-product firms converge to the steady-state of the subgame-perfect Nash equilibrium before the merger occurs; (ii) there is an unanticipated merger of some of the firms; (iii) any cost saving among the merged firms is achieved immediately and permanently; and, (iv) firms proceed under the assumption that there will be no further mergers. To do this we compute the subgame perfect equilibrium strategies and value functions for two product configurations: the pre-merger case of single-product firms and the post-merger product configuration. We compute the changes in the present value of consumer and producer surplus, evaluated at the pre-merger steady state which occur as a result of the unanticipated merger and use these values to judge the impact of the merger on consumer and producer surplus.

2.2. Computational Details. We have specified a general linear-quadratic dynamic game. There are no closed-form solutions for the equilibrium of the infinite-horizon dynamic game. This is not surprising since there are no closed-form solutions for multivariate linear-quadratic control problems, which require the solution a Ricatti equation. There are some eigenvalue-eigenvector solution methods for LQ control problems which reduce the numerical problem to familiar matrix algebra operations, but our dynamic game is not the solution to a single optimal control problem, but the solution to interactions among the individual control problems. The absence of a closed-form solution makes it unlikely that we can prove theorems telling us when mergers are profitable, increase consumer surplus, or increase social surplus. There are closed-form solutions for finite horizon games given a terminal payoff. In fact, we will use them in our computations below. However, they are the result of hundreds of iterations of linear operations and would be impossible to even just display in this paper. They are also useless for proving theorems on the impact of mergers. We must resort to computation in order to address these issues in a quantitative fashion. In fact, our computations show that there are no simple theorems since we find examples covering a broad range of possibilities. The only way to proceed is to compute equilibria for empirically sensible cases, and find instructive patterns for those cases.

Fortunately, it is relatively easy to compute the equilibrium value functions for each configuration of firms and products. A linear-quadratic dynamic game reduces to a coupled system of Bellman's equations where each firm-specific Bellman equation is an optimal linear regulator problem. We obtain the subgame perfect solution (also known as the feedback solution and Markov Perfect equilibrium) numerically by recursive, backwards computation beginning with a concave terminal value functions for the firms. That is, we make a guess about the firms' value functions at some terminal time T, then compute the value functions at time $T-1$, $T-2$, $T-3$, etc. We iterate backwards for 500 periods; we find that the 500-period solution is indistinguishable from the 400-period solution and insensitive to the guess for the terminal value. The resulting collection of policy and value functions describe firm behavior during any transition period as well as at the steady state. This computation must be done for both the pre-merger dynamic game as well as any of the post-merger dynamic games we study.

The details of computing the dynamic oligopoly equilibrium are spelled out in the literature; see, for example, Basar and Olsder (1995) and Kydland (1975, 1977) for precise descriptions of the linear algebra used to compute the equilibrium value and strategy functions at time t given the value functions at time $t+1$. We will illustrate the ideas behind the computational technique by presenting the details for the simpler case of duopoly.

The solution to $V_j(y, \omega, t)$, the value function for firm j at time t in state y and product configuration ω,[7] takes the form of coupled Ricatti equations. The solution to V_j is a quadratic form

$$V_j(y_{t-1}, \omega, t) = (1/2)\, y_{t-1}' S_{j,t}\, y_{t-1}$$

Decompose $\beth$ in (7) into $\beth = [\beth_1,\ \beth_2]$. Define

$$H_t = \begin{bmatrix} \beth_1' \Sigma_{1,t} \\ \beth_2' \Sigma_{2,t} \end{bmatrix} \tag{11}$$

$$\Sigma_{j,t} = R_t + \beta S_{j,\,t+1},\ j = 1,\ 2 \tag{12}$$

Let $x_{j,t}$ denote the set of prices and outputs under the control of firm j in the product configuration ω. Given the value function at $t+1$, $(1/2)\, y_t' S_{j,t+1}\, y_t$, firm 1 chooses the variables under its control, $x_{1,t}$, to solve the Bellman equation (10), implying the first-order condition

$$0 = \beth_1' \Sigma_{1,t} \beth_1 x_{1,t} + \beth_1' \Sigma_{1,t} \beth_2 x_{2,t} + \beth_1' \Sigma_{1,t} \aleph y_{t-1} \tag{13}$$

[7] The product configuration for a duopoly is either one product per firm or a single firm controlling both products. We keep the ω notation since it is necessary in the cases we compute.

First-order conditions for firm 2 imply the symmetric condition

$$0 = \beth_2' \Sigma_{2,t} \beth_2 x_{2,t} + \beth_2' \Sigma_{2,t} \beth_1 x_{1,t} + \beth_2' \Sigma_{2,t} \aleph y_{t-1} \tag{14}$$

If we combine the first-order conditions in (13,14), we find that the equilibrium rule is

$$x_t = G_t y_{t-1}$$

where

$$G_t = (H_t B)^{-1} H_t \aleph \tag{15}$$

Furthermore, the quadratic form for the value function at t is

$$S_{j,t} = (\aleph + \beth G_t)' \Sigma_{j,t} (\aleph + \beth G_t)$$

The equations (11,12,15) form a recursive set of matrix equations which take us from the $t+1$ value functions represented in the $S_{j,t+1}$ matrices to the period t value functions represented in the $S_{i,t}$ matrices. If period T were the last period and $S_{j,T}$ were firm j's final payoff, then we can compute the payoff for periods $t < T$ by iterating the process in (11,12,15). For any time $t < T$ we can combine these expressions to produce a closed-form solution for $S_{j,t}$ but the result is of no value for our purposes. Therefore, we compute several examples.

Any computation is an approximation, so we need to examine their likely quality. Standard diagnostics indicate that the numerically computed equilibria for these cases are quite reliable. In every simulation, we found that the condition numbers of the matrices which arise in our computations are quite small. In particular, these log condition numbers always fall between 2 and 4, indicating that we lose at most 4 decimal digits (out of the 16 digits which our computer can carry) in critical matrix operations. Also, the first five digits of the results were insensitive to changes in the horizon and large changes in the terminal value function. These diagnostics indicate that our numerical results are good approximations to the infinite-horizon equilibrium to at least a few significant digits, an accuracy which is adequate for our purposes.

There is one detail about which we must be careful. This procedure uses just the first-order conditions for the players' optimization problems. It may produce value functions for a player which are not concave in that players' control variables, in which case the decision rules do not satisfy the second-order conditions for optimization. For example, this could arise if there were significant increasing returns to scale in a firm's production function. Our examples do not have increasing returns to scale in output. We sometimes allow mergers to have a spillover effect on costs, but that does not produce an increasing returns to scale since all firms have constant returns to scale with respect to output both before and after the merger. We have checked our examples and find that decision rules in our equilibria satisfy the second-order conditions for optimization and that solutions to the first-order conditions are unique at each stage. Therefore, all of our equilibria are unique.[8]

2.3. Limitations and Possible Extensions. There are many limitations of this analysis. First, we have made special functional form assumptions concerning demand and supply; however, this is not a weakness unique to this analysis. Crooke et al. (1997) show that the predictions for the price effects of mergers in the typical static model are sensitive to functional form specifications. We are sure that many of our quantitative results would also be sensitive to changes in the demand specification, However, it is unclear if the case of linear demand and supply substantially biases the qualitative results in any particular direction. In any case, extending our analysis to nonlinear cost and demand structures is well beyond the scope of this paper since it would require the solution of nonlinear dynamic games. The numerical methods presented in Judd (1992) could be used to analyze more general specifications, but we leave this for future work.

Second, we have ignored entry. The possibility of entry will affect merger decisions since entry may blunt the incentive of incumbents to merge and their ability to exploit any market power. This has been the focus of papers such as Kamien and Zang (1990), Werden and Froeb (1998), and Gowrisankaran (1999). Those studies have stayed with conventional Bertrand or Cournot modes of competition. Nonlinear extensions of our model could allow both price and output decisions by firms as well as entry, but they lie beyond the scope of this study.

2.4. Cases Examined. We compute the impact of mergers for several cases. We now list those cases and indicate why these represent a broad range of empirically reasonable cases.

We compute equilibria of markets with $N = 5$, 7 and 10 products. The long-run marginal cost for product i, m_i, may vary across products and may be affected by a merger. We assume firm 1 has the best technology with LRMC equal to m_1, and that it is involved in any merger. The pre-merger marginal cost for all products other than product 1 is m_{-1}.

We assume no depreciation of inventories, that is, $\delta = 0$; some experimentation shows that this does not affect our results. The inventory cost parameters are chosen to be $h_i = 0$, $g_i = 1, 10$, and 100, for all i. Our tables below will report on the case $g = 100$, a quadratic cost function with very steep curvature and an optimal steady state inventory level close to zero. Supplementary calculations indicate that the results do not depend on

[8]This uniqueness property would disappear if we allowed an infinite horizon since the folk theorem implies that there could be reputation-style equilibria as well as the ones we approximate. The objective of this paper is to examine the implications of long but finite horizon games with the implications of static analyses. Therefore, we do not examine reputation equilibria.

this. Also, no result is affected by h since h just affects the optimal inventory level[9].

The adjustment cost parameter, γ, is assumed to be $0, 5, 50$, or 500, so that the examples cover the cases ranging from the case of no adjustment cost to cases with expensive and slow output adjustment. There is little point examining cases where $\gamma > 500$ since the adjustment costs become unrealistically large. It is not immediately apparent what values γ should have. Fortunately, γ is closely related to the slope of the short-run marginal cost curve. We will use the implied values for short-run marginal costs to judge what γ should be.

We assume $C = 100$; this is arbitrary since long-run marginal costs are constant. The substitutability parameter, B, is critical; we allow it to range between 0.1 and 0.5. We will see that this range encompasses sensible values for own- and cross-price elasticities. The discount factor, β, is set to be 0.99; this corresponds to setting a unit of time equal to three months if the annual interest rate is 4%.

The complete list of critical parameter values used in our calculations are listed in Table 1. Our discussion below will present the results for some of these values to indicate the magnitudes of various effects. We will also provide statements summarizing the qualitative patterns found in the complete collection of computations.

[PUT TABLE 1 ABOUT HERE]

While this set of parameters is somewhat limiting, it focuses on economically sensible cases. We will see that the range for B covers sensible values for own- and cross-elasticities of demand, and the range for γ covers sensible values for the short-run elasticity of marginal cost. We make no effort to use estimated elasticity values for any particular industry; instead we choose values for the critical parameters which are plausible. Increasing the range of values will not affect the qualitative findings much since we find some ambiguities even with this range of values. The key fact is that our calculations can show how the results depend on demand and cost elasticities and the reader can ultimately judge what is plausible and what is not.

2.5. Qualitative Results. We next note some qualitative features of equilibrium that help us understand our results. First, when there are no adjustment costs ($\gamma = 0$) the dynamic equilibrium for the finite-horizon game is the same as the Bertrand equilibrium and the Bertrand equilibrium is an equilibrium of the infinite-horizon game. This happens because both short- and long-run marginal costs are constant when $\gamma = 0$. With no adjustment costs, current output affects neither current nor future marginal cost. Furthermore, unexpected movements in inventory are immediately neutralized in the following

[9]This is true also because we assume constant long-run marginal cost. If LRMC were not constant then the long-run level of output would be affected by a merger, affecting the LRMC and the marginal cost of changing inventories.

period without any effect on marginal cost. Price decisions affect only demand since any inventory implications of a price decision are absorbed by a change in output without affecting marginal cost. Therefore, there is no strategic value to inventories and output decisions focus on maintaining a target level of inventory. Therefore, pricing decisions are the same as in the static Bertrand equilibrium, and output is set to supply anticipated demand and maintain the efficient level of inventory.

Second, as the adjustment cost parameter γ increases, the slope of short-run marginal cost curve gets steeper and our dynamic model incorporates some of the capacity-commitment or quantity-setting features of static Cournot games. The steady state of the oligopoly when all firms have very large adjustment costs has nearly the same output and prices as a static Cournot equilibrium. These cases resemble the Kreps-Scheinkman model since Kreps-Scheinkman implicitly assume that the ex post cost of capacity adjustment is infinite.

Judd (1990) demonstrated these results for the case of two firms. Our analysis is more general since we analyze multifirm oligopolies but we find these same basic qualitative results. Since the parameter γ parameterizes the transition from Bertrand oligopoly to Cournot oligopoly, we might expect a comparable mixture of results for merger analysis. However, the steady-state results in our dynamic model are not relevant since convergence to the steady state is very slow when adjustment costs are large. In dynamic economics in general, we know that comparative steady state analysis is highly questionable, particularly as part of welfare analysis. In our dynamic model, the net result is much less ambiguous than static analyses.

3. Mergers with no Cost Saving

We first examine the case where all firms have the same costs, both before and after any merger. Our computations display several robust properties. We illustrate them in Figure 1 for the case of a two-product merger in a five-product industry when $m = 30$, $B = 0.1$, and $\gamma = 500$. This is a case where the products are good substitutes and adjustment costs are very high. The steady state of this case is indistinguishable from the static Cournot model. This case is a rather extreme one but gives us rich example of what may happen in equilibrium.

Figure 1a displays the per-product output for both the merged and unmerged firms. Equilibrium converges slowly to the steady state because of the high value of γ. The unmerged firms continuously expand their output and the merged firm continuously reduces its output of each product, as would be predicted by a Cournot model. Figure 1b displays product prices. All unmerged firms gradually increase their prices over pre-merger levels, as predicted by Bertrand analysis. The merged firm has a humped pattern for prices, first raising prices then retreating somewhat before settling in to a steady state with higher prices.

Figure 1c displays per-period profits. The evolution of profits is even more complex. The profits of the unmerged firms immediately increase above pre-merger levels and continue to rise forever. In this case, the merged firms' profits immediately falls below pre-merger levels but then rises above pre-merger levels, only to ultimately fall below pre-merger levels. This is one of the few cases where the merged firms lose in terms of present value, but the hump-shaped pattern in profits is seen often. In the short-run the merged firms absorb high adjustment costs to get output down. In the intermediate run, the unmerged firms have not yet attained their long-run market share and the merged firm extract extra profits by coordinating the output and pricing of its products. Firms may want to merge even though the resulting market power is temporary. In contrast, the traditional static approach would argue that a merger would not be desired since it implicitly only examines the steady state. Figure 1d tracks consumer, producer, and social surplus. This merger produces a growing transfer from consumers to producers. The net effect on social surplus was slightly negative.

[PUT FIGURE 1 ABOUT HERE]

The humped pattern for the merged firms' profits in Figure 1d was seen in several examples we plotted, even for some profitable mergers. Sometimes, the initial effect on the merged firms' profits is sometimes negative, even though there is a present value gain. The intuition for the overshooting pattern is clear. The intermediate gains in profits occur because it takes time for the unmerged firms to increase their output; in the meantime, the merged firm also cuts back on output but in a coordinated fashion across the products it sells, as in a Bertrand analysis.

3.1. Profits. We now focus on the impact of mergers on profits. Tables 2 and 3 display our results for the impact of a merger on the present value of profits. They report the percentage profit gain per product for the firms involved in a merger; e.g., the first row of Table 2 tells us that if four firms merge then the profits for the four-product firm will be 39.5% greater than the joint profits of the four pre-merger firms. We assume $m = 30$; the results are practically identical for $m = 20, 40$, and so are not reported here. In these tables, we see that larger mergers are more beneficial for the participating firms. As expected, γ is negatively related with the profitability measure. When the model collapses to a Bertrand game ($\gamma = 0$) firms instantly move to steady-state prices and adjust out mergers of any size are profitable, as predicted by the Bertrand approach to mergers. With $\gamma = 0$, firms can instantly set the new steady state levels of price and quantity, which makes our dynamic model equivalent to a static Bertrand game.

[PUT TABLES 2 AND 3 ABOUT HERE]

As γ increases, the importance of transition periods also increases. The steady states when $\gamma = 500$ almost equal the static Cournot equilibria. Many cases with high γ imply a reduction in steady state profits, as with the static Cournot model, but imply an increase in the present value of profits. This is the key reason why we find far more profitable mergers than static Cournot analysis does.

Whether or not the merged firms benefit from a merger depends on the parameter values. In general, the merged firms enjoy an increase in the present value of their combined profits. The only exceptions in Tables 2 and 3 are mergers of two firms with $B = 0.1$, $\gamma = 50$ in the five-product industry, and mergers of two and three firms with $B = 0.1$, $\gamma = 50$, merger of two firms with $B = 0.2$, and $\gamma = 50$ in the ten-product industry. Thus, partial market concentration is less likely to be profitable when products are close substitutes and firms are less able to change output. This is similar to the extreme case of homogeneous products in the traditional static Cournot game, such as in Salant et al.(1983).

Table 2 also reports the elasticities of short-run marginal cost in the pre-merger steady states. The elasticity of marginal cost is defined as the percentage increase of marginal cost caused by one percent increase of output quantity, and reflects the slope of marginal cost curve. This elasticity gives us a way to judge what are plausible values for γ. The case where a two-product merger was unprofitable also had a SRMC elasticity of nearly ten, implying that a 1% increase in output over one unit of time would increase marginal cost by nearly ten per cent. This is where our choice of $\beta = .99$ is important. Since safe assets return about 1% (after inflation) and risky assets like equity return about 7%, this implies that the period of time we use is at least two months and could be up to twelve months, depending on a firm's cost of capital. Given the fact that output is far more volatile than prices, elasticities of SRMC greater than ten appear to be, at best, marginally plausible. Cases with $\gamma = 500$ more often produce Cournot-like implications for profit gains but also imply implausible elasticities of SRMC on the order of 100. Therefore, these examples are not supportive of using static Cournot models for merger analysis since it implies extreme properties of the cost function.

While unprofitable mergers are possible, they seem to require unrealistically inflexible cost functions. Of course, in examining an actual merger, one could estimate the short-run elasticities to evaluate likely profitability. In any case, the reader can examine Tables 2 and 3 and judge for themselves whether the cases with Cournot-like results are plausible. We next summarize our results from Tables 2 and 3 and our complete set of computations.

Summary 1. *For the parameter values displayed in Table 1, mergers with no cost savings increase the present value of profits for the participants except when quantity adjustment costs are large, the products are good substitutes, and the number of merged firms is small. When adjustment costs are large, steady state profits for the merged firms may* *fall even when the present value of profits increases.*

3.2. Consumer and Social Surplus. Consumers surely lose from mergers when there are no cost savings, and social surplus will fall because of the rise in prices and fall in output. Tables 4 and 5 report the changes in consumer and social surplus from various mergers. They show that consumer losses from mergers are greater when adjustment costs are higher. This is because the initial pre-merger steady state has lower output and is less efficient as γ increases and any merger aggravates that inefficiency. Tables 4 and 5 show that social surplus losses are small unless the merger involves most of the products. Tables 4 and 5 assume that the constant marginal cost is $m = 30$; since the results are expressed in percentages, they are nearly the same for $m = 20,\ 40$; therefore, we display only the $m = 30$ case.

The dependence on B is ambiguous. Mergers affect output and prices if $B \neq 1$. When $B = 1$, product demands are independent and mergers have no impact on anything. Similarly, the case $B = 0$ and $\gamma = 0$ reduces to Bertrand competition among perfect substitutes, wherein price always equals marginal cost and any merger which does not lead to monopoly has no impact on price, profits, or total output. Therefore, mergers short of monopoly reduce consumer and social surplus for intermediate values of B but not for the extreme cases.

Tables 4 and 5 also show that surplus changes depend intuitively on M, the number of merged firms. The loss is convex in M, implying that larger mergers are disproportionately bad for consumers and society. This is expected since a couple of firms which merge have little additional market power but that the marginal ability to collude increases as more competitors are eliminated. This corresponds to the standard static result that the efficiency cost of oligopoly is (roughly) inversely proportional to the square of the number of competitors.

[PUT TABLES 4 AND 5 ABOUT HERE]

We next summarize the results from Tables 4 and 5 and our complete set of computations.

Summary 2. *For the parameter values displayed in Table 1, consumer and social surplus are reduced by mergers. The loss is convex in the number of merged firms. The loss is greatest for goods with intermediate substitutability.*

4. Mergers with Cost Savings

If a merger reduces, there will be a trade-off between efficiency gains and market concentration. To study this trade-off, we examine mergers where one firm with a superior technology merges with other firms, allowing the merged firm to use the low-cost technology to produce all of its products. Specifically, we assume Firm 1 has the lower-cost technology initially. Such a merger will have ambiguous effects. First, the merged entity raises the price of good 1 due to market concentration. Second, it may reduce the price of the other goods sold by the merged entity due to cost reductions. We will see that the critical determinants are various elasticities, the size of the efficiency gain, and the size of merger.

We examine these issues using the same model and basic parameters as before, except that m_1, firm (and product) 1's long-run average and marginal cost, differs from the other firms' cost, which we denote m_{-1}. We examine cost reductions of 10% and 30%, representing a small and large cost reduction. We computed equilibria for the parameter values in Table 1 and find that cases with different parameter values and cost savings can be inferred by interpolating the results we report in tables.

4.1. Profits. When mergers reduce costs, profits for the merger partners should rise by more than when there are no cost savings. The profits for product 1 do not always increase, since it loses its monopoly of the low-cost technology as Firm 1 merges with other firms. We found several small merger cases where the profits from product 1 falls. However, the costs of producing the products other than product 1 are reduced, and the merged firm gains from both increased efficiency and market concentration. Ignoring the distribution problem within a merger, we look at the total profit for the merged firm relative to their pre-merger firms.

Tables 6 and 7 report the change in profits for the merged firm relative to the total profits of its pre-merger firms. Tables 6 and 7 presents the case of a 10% cost reduction. Table 6 reports pre-merger demand and cost elasticities to indicate the reasonability of the examples. The elasticities are averages over the pre-merger firms in the $N = 5$ case; an average is necessary since the asymmetry in costs imply asymmetric pre-merger equilibria. Again we find that our choices for B and γ bracket a wide range of economically plausible demand and cost elasticities. We do not repeat this for the $N = 10$ case since there are few differences. We see that the profit gains from mergers are substantial in all cases, and that there are no cases in Tables 6 and 7 where profits fall. Even a 10% cost advantage of a merger wiped out the possibility of losses in the cases where mergers lost money in Tables 4 and 5.

[PUT TABLES 6 AND 7 ABOUT HERE]

We next summarize the results from our complete set of computations.

Summary 3. *For all the parameter values displayed in Table 1, mergers are profitable for merger participants when $m_1 < .9m_{-1}$. The profit gains are greater for larger mergers, but decrease with γ, the cost of adjusting output.*

4.2. Consumer and Social Surplus. The cost savings from a merger will reduce the total costs of production. The key welfare issue is whether this gain is enough to overcome the increase in market power and lead to lower consumer prices. To examine this we compute the change in present values of consumer surplus and social surplus due to the merger. Tables 8 and 9 report the percentage change in the present value of consumer and social surplus after a merger relative to the pre-merger steady state.

[PUT TABLES 8 AND 9 ABOUT HERE]

The results are robust across the five- and ten-product cases, and also represent the results for other choices of N. First, Tables 8 and 9 show that consumers suffer from mergers unless the cost savings are large or the market concentration of the merged firm is small. Even when the cost savings are 30%, consumer surplus falls when two out of five firms merge. Consumers benefit only when a few firms out of several merge, such as in Table 9 where there are initially ten firms. Consumers also suffer more as γ increases. This is because larger γ means that the firms are playing more of a Cournot game in the long run and that the merged firm takes less advantage of its cost advantage.

Second, the implications for social surplus are more benign. Social surplus includes the profits of the unmerged firms as well as the consumer surplus and the merged firms' profits. The unmerged firms gain when there are no cost savings, but they may lose when the merger reduces costs. The merged products with lower costs are now more competitive and may take sales from the unmerged firms. Mergers are often socially beneficial when $\gamma = 0$ since then our dynamic model reduces to a static Bertrand game where equilibrium prices are close to production costs. As γ increases, the social surplus losses from mergers are greater.

Third, the relation between social surplus and the size of mergers is not monotone. In particular, Table 9 shows that there is often a nontrivial merger that maximizes social surplus. For example, when $N = 10$, a five-product merger enhances social surplus more than the other mergers displayed when $B = 0.2$ and $\gamma = 0$, but a three-firm merger is best when $\gamma = 50$.

Fourth, we see most mergers still reduce social surplus unless cost savings are large. With large cost savings, even a monopoly may be desirable if that is the only way for the superior technology to diffuse. Comparing these tables with Tables 6 and 7, we found all the mergers that benefit consumers also enhance social surplus. Mergers always involve some redistribution since we found no Pareto-improving mergers. The reason is that unmerged firms suffer a loss in profits whenever consumers gain. In this model, consumers and the merger's competitors have opposite interests.

We now summarize the results from Tables 8 and 9, and our complete set of computations.

Summary 4. *For the parameter values displayed in Table 1, mergers can increase social surplus when cost savings are nontrivial, but consumers gain only when cost savings are large, the merged firms have small market share, and/or the game is nearly a Bertrand game. A nontrivial merger is often better for social surplus than no merger, but monopoly is not best except for very large cost savings.*

5. CONCLUSIONS

The analysis of mergers in oligopolistic markets is critical to the formation of rational antitrust policy. We use the Judd (1990) model of dynamic oligopoly to address merger questions. This model avoids the unrealistic strategic limitations of Cournot and Bertrand analysis, and allows for costly output adjustment in the short run. Static Cournot and Bertrand models differ in their predictions of firms' incentives to merge. We find that firms generally benefit from mergers. This agrees with the conclusions of the Bertrand analysis in Deneckere and Davidson (1985), but disagrees with the Cournot analysis of Salant et al.(1983). We show that the Cournot analysis of mergers is unreliable in predicting merger profitability in dynamic contexts even in cases where the static Cournot model correctly predicts the long-run equilibrium. The difference arises because we focus on the present value of merger activity whereas the Cournot model implicitly focuses on the long-run steady state of our model.

While this analysis is limited, it does show that dynamic analysis of mergers can be executed. It offers an alternative to using unrealistic static models which produce conflicting results. Most surprising, we show that much of the ambiguity and confusion of static analyses is avoided by examining a more realistic, explicitly dynamic model.

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6. Tables

Table 1: Cases Examined

Tastes:	$B \in \{.05, .10, .20, .30, .50, .75\}$, $C = 100$, $\beta = .99$
LRMC:	$m \in \{20, 30, 40\}$, $m_{-1} \in \{m, .9m, .8m, .7m\}$
Adjustment Cost:	$\gamma \in \{0, 5, 50, 500\}$
Inventory costs:	$g \in \{1, 10, 100\}$, $h = 0$
Products:	$N \in \{5, 7, 10\}$

Table 2: Profit Gains (%) and Elasticities

No Cost Savings: $N = 5$, $m = 30$

		Profit Gains for $M =$			Pre-merger		
					Demand Elas.		Elas. of
B	γ	2	3	4	Own-	Cross-	SRMC
0.1	5	2.0	13.2	39.5	5.5	1.2	1.0
0.1	50	-1.4	7.3	29.8	5.9	1.3	9.7
0.2	5	2.0	8.7	22.3	3.4	0.7	0.9
0.2	50	0.5	6.0	17.8	3.6	0.7	9.0
0.5	5	0.5	2.0	4.5	2.2	0.3	0.7
0.5	50	0.4	1.7	4.0	2.3	0.3	7.4

Table 3: Profit Gains (%)

No Cost Savings: $N = 10$, $m = 30$

B	γ	M =2	3	5	9
0.1	0	1.0	3.5	14.2	103.0
0.1	5	0.5	2.6	13.1	92.4
0.1	50	-1.4	-0.9	7.1	79.0
0.2	0	0.7	2.3	8.9	48.7
0.2	5	0.5	2.0	8.5	45.2
0.2	50	-0.1	0.9	6.5	40.2
0.5	0	0.2	0.5	2.0	8.23
0.5	5	0.1	0.5	1.9	8.1
0.5	50	0.1	0.4	1.8	7.6

Table 4: Post-Merger Surplus Changes (%)

No Cost Savings: $N = 5$, $m = 30$

		Consumer Surplus			Social Surplus		
B	γ	M=2	3	4	M=2	3	4
0.1	5	-4.9	-14.3	-30.9	-0.78	-2.85	-7.62
0.1	50	-6.6	-17.9	-35.3	-1.22	-4.07	-9.71
0.3	5	-4.6	-13.8	-28.7	-1.10	-3.72	-8.77
0.3	50	-5.5	-15.6	-30.1	-1.39	-4.42	-9.54

Table 5: Post-Merger Surplus Changes (%)

No Cost Savings: $N = 10$, $m = 30$

		Consumer Surplus			Social Surplus		
B	γ	M=2	3	4	M=2	3	4
0.1	0	-0.4	-1.3	-2.8	-0.04	-0.14	-0.3
0.1	5	-0.9	-2.6	-5.1	-0.10	-0.35	-0.8
0.1	50	-1.4	-3.9	-7.4	-0.19	-0.63	-1.4
0.5	0	-0.7	-2.2	-4.6	-0.19	-0.60	-1.3
0.5	5	-0.8	-2.4	-4.9	-0.21	-0.67	-1.4
0.5	50	-1.0	-2.8	-5.6	-0.26	-0.79	-1.6

Table 6: Profit Gains (%) and Elasticities: $N = 5$

				Profit Gains for M =			Average Pre-merger		
							Demand Elas.		Elas. of
m_1	m_{-1}	B	γ	2	3	4	Own-	Cross-	SRMC
18	20	0.1	5	9	22	48	3.6	0.8	1.9
18	20	0.1	50	5	15	37	3.9	0.8	18.3
18	20	0.3	5	5	10	18	2.0	0.3	1.6
18	20	0.3	50	5	9	16	2.1	0.3	15.4
36	40	0.1	5	19	34	60	7.0	1.5	0.7
36	40	0.1	50	14	26	48	7.5	1.6	7.3
36	40	0.3	5	12	18	27	3.5	0.6	0.6
36	40	0.3	50	11	17	24	3.6	0.6	6.0

Table 7: Profit Gains (%) for Mergers:
$N = 10$, $m_1 = 30$, $m_{-1} = 27$

B	γ	M=2	3	4	6	9
0.1	5	16.6	23.7	29.3	45.1	112.2
0.1	50	13.6	18.3	22.2	35.7	97.0
0.2	5	10.6	15.1	18.7	28.1	58.3
0.2	50	9.8	13.6	16.7	25.1	52.8
0.5	5	5.9	8.1	9.5	11.9	16.8
0.5	50	5.8	8.0	9.3	11.6	16.3

Table 8: Post-Merger (%) Changes in Surplus: $N = 5$

				Consumer Surplus		Social Surplus	
m_1	m_{-1}	B	γ	M =2	3	M =2	3
18	20	0.1	5	-4.34	-13.4	-0.01	-1.41
18	20	0.1	50	-6.11	-17.1	-0.52	-2.76
18	20	0.3	5	-3.89	-12.6	-0.21	-2.02
18	20	0.3	50	-4.87	-14.4	-0.53	-2.76
14	20	0.1	5	-3.29	-11.6	1.61	1.49
14	20	0.1	50	-5.26	-15.6	0.92	-0.12
14	20	0.3	5	-2.44	-10.1	1.61	1.43
14	20	0.3	50	-3.53	-12.0	1.24	0.59
28	40	0.1	5	-0.74	-7.3	5.95	8.76
28	40	0.1	50	-3.21	-12.0	4.69	6.35
28	40	0.3	5	1.19	-3.7	6.32	10.15
28	40	0.3	50	-0.18	-6.2	5.78	9.04

Table 9: Post-Merger (%) Changes in Surplus

$N = 10$, $m = 30$

			Consumer Surplus				Social Surplus			
m_{-1}	B	γ	M =2	3	4	5	M =2	3	4	5
27	0.1	0	0.4	0.1	-0.8	-2.6	0.8	1.6	2.1	2.5
27	0.1	50	-0.9	-3.0	-6.2	-10.5	0.6	0.8	0.6	0.0
27	0.2	0	0.1	-0.6	-2.2	-5.1	0.8	1.3	1.6	1.7
27	0.2	50	-0.8	-2.9	-6.2	-10.7	0.5	0.7	0.4	-0.2
27	0.5	0	0.1	-0.7	-2.4	-5.1	0.6	1.1	1.2	1.0
27	0.5	50	-0.2	-1.4	-3.6	-6.6	0.6	0.8	0.8	0.4
21	0.1	0	2.0	3.2	3.4	2.5	3.1	5.8	8.0	9.8
21	0.1	50	0.2	-1.2	-3.8	-7.7	2.4	4.0	5.0	5.5
21	0.2	0	1.7	2.4	1.9	0.02	2.7	5.0	7.0	8.5
21	0.2	50	0.5	-0.6	-3.1	-6.8	2.3	4.1	5.2	5.8
21	0.5	0	1.7	2.4	2.0	0.6	2.4	4.6	6.3	7.7
21	0.5	50	1.4	1.5	0.7	-1.3	2.3	4.3	5.8	7.0

7. Figures

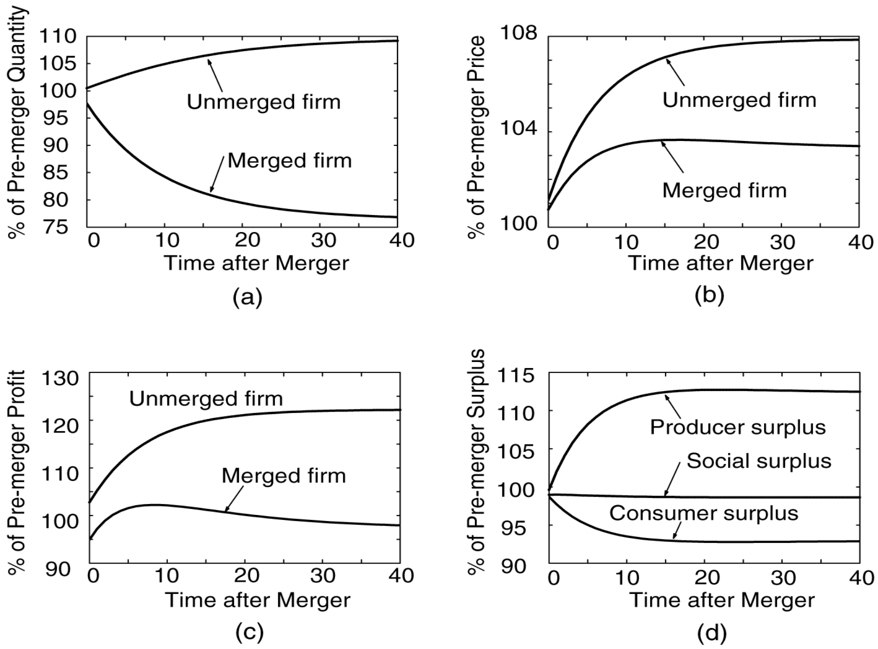

Figure 1: Impact of a merger