THE DYNAMIC EFFECTS OF TAXATION AND MONETARY POLICIES

by

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April 1985*

Revised July 1987

Abstract

This paper examines the impact of cost-of-capital-preserving changes in tax and monetary policy on the dynamic behavior of investment and output. Under the assumption of economic depreciation we find that an increase in the investment tax credit and an offsetting increase in the corporate income tax rate will increase the rate of convergence to the steady state. If shocks to the capital stock drive a business cycle, then such changes in the tax structure will stabilize output. Generally the structure of taxation and monetary policy will have a nonnegligible impact on the dynamic behavior of net investment and output, even when we hold fixed the effective tax rate. However, purely monetary effects associated with Tobin-Mundell portfolio balance effects appear to be trivial.

* The authors thank Roger Gordon for his comments. They also gratefully acknowledge the financial support of the National Science Foundation under contracts SES-8209247 to Professor Balcer and SES-8209544 and SES-840986 to Professor Judd. This paper was completed when Professor Judd was a National Fellow at the Hoover Institution.

1. Introduction

An important issue in the evaluation of tax policy is its impact on investment. Most work has concentrated on the effect tax rates has on the steady-state cost of capital and the associated steady-state level of capital supply (see, e.g., Brock and Turnovsky [1981], Feldstein, Green and Sheshinski [1978]). In this paper, we examine the impact of various tax rates on investment outside of the steady state and over a small "business cycle". We find that tax and monetary policies may have an important impact on the dynamic behavior of investment independent of the conventionally computed effective tax rate.

We examine a representative agent perfect foresight model of investment with inflation, taxation of nominal corporate profits, tax credits proportional to gross investment, and nominal depreciation deductions. The dependence of the steady-state cost of capital and capital stock on such a tax structure is well-known (see Brock and Turnovsky [1981]). This paper examines the impact of tax and monetary policies on the local dynamic behavior of an economy around its steady state. In particular, we determine the first-order approximation of the relation between investment and output if a business cycle is driven by shocks to the capital stock or productivity since the negative eigenvalue of the linearized deterministic system is the rate of convergence to the target level of capital stock when shocks are small.

We find that the structure of taxes is important in determining investment above and beyond the determination of the steady-state cost of capital. For example, if one increases both the investment tax credit and the corporate income tax so as to leave the cost of capital unchanged, the rate of convergence to the steady state level of capital is increased if depreciation allowances exist and utility is separable between consumption and money. The reasons for these effects are intuitive. Suppose that we increase the corporate tax rate and the investment tax credit so as to leave the steady state required rate of return and capital stock unchanged. In the steady state the term structures of the marginal product of capital and the interest are constant. However, if there is a shock which reduces the capital stock, the term structure of interest rates will become downward sloping, converging to the steady-state rate. Increasing the investment credit and the tax rate effectively gives the firm more money in the short-term and takes more in the long-run in response to a unit of investment.

If the present value of the tax burden is unchanged with a flat term structure then it is reduced with a declining structure converging to the original rate. This reduced tax burden makes the investment more attractive. Therefore, the investment response to this shock will be affected by the composition of the tax structure independent of the effective tax rate. This essay addresses the robustness and quantitative significance of this effect.

When the model is parameterized using existing empirical estimates of the key parameters we show these effects to be nontrivial. For example, if money has no real effects and true economic depreciation is used for tax purposes, then if the investment tax credit is increased by .05 and the reduction in the cost of capital is offset by an increase in the tax on corporate profits, then net investment is multiplied by about 1.1 around the steady state level of capital, that is, ten percent more net investment if capital stock is below the steady state and ten percent more disinvestment otherwise.

Manipulation of tax rates during the business cycle has often been suggested as a stabilization or stimulation instrument. The investment tax credit was initially instituted to stimulate fixed business investment and help bring the economy out of a recession. Individual income tax cuts have also been used to stimulate the economy, supposedly through income effects on the demand for consumption goods. We do not examine this kind of counter-cyclical tax policy. Rather, we examine how the cyclical behavior of the economy is affected by *level* and *composition* of capital taxation. It is known that the dynamic behavior of the economy around its steady state is affected by the level of the tax rate, essentially because of the income effects induced by taxation and because capital taxation effectively acts to reduce the curvature of the individual utility function (see Judd [1987]). In this paper, we hold fixed the effective tax rate, thereby inducing no steady-state income effect. However, there are many ways to combine the tax rate on corporate income, depreciation rules, expensing rules, inflation, and investment tax credit policy to accomplish any single cost of capital. We find that changing the mix of these various tax policies will affect the investment response of any firm to a shock which sends the economy away from its steady state. In particular, the variance of investment will increase and the variance of output will decrease if the capital stock is subjected to serially uncorrelated shocks.

Similar studies by Fischer [1979] and Asako [1983] have shown that monetary policy in a perfect foresight representative agent model may not be "superneutral", that is, monetary policy may leave the steady-state capital stock unchanged but affect the rate of convergence to the steady state because of a Tobin-Mundell portfolio effect causing agents to substitute out of money and into capital as inflation rises. Abel [1985] examined the same issue in a general equilibrium cash-in-advance model of money with capital, showing that the steady-state capital stock depended on monetary policy to the extent that the cash constraint applied to investment expenditures. He also examined the dependence of capital accumulation on monetary policy. None of those authors examined the quantitative significance of the effects. Our analysis subsumes the Tobin-Mundell issue and shows that in the absence of taxation this effect is negligible when plausibly parameterized.

However, changes in monetary policy are important *beyond the well-understood effect on the cost of capital* in the presence of nominal treatment of depreciable assets. One of the novel features of this general equilibrium analysis is the explicit incorporation of nominal depreciation. Comparing our analysis with that of Fischer and Asako indicates that if there were any empirical evidence of nonsuperneutrality it would more likely be due to interactions between monetary policy and nominal tax rules than due to any purely monetary effect.

Section 2 describes the general model we use. Section 3 determines the nature of equilibrium. Section 4 analytically examines the main point of this study in a particularly tractable reduced version of the general model. Section 5 discusses the case when utility is separable in consumption and money. If our dynamic analysis had only a deterministic interpretation then it would be of little interest since, in the long run, the economy is in the steady state. However, this analysis has broader implications and Section 6 discusses the connection between the eigenvalues we study and real business cycles. Section 7 investigates the more general model using a wide range of plausible parameterizations, showing that these effects remain important. Sections 8 and 9 present concluding observations.

2. Model

We adapt the general equilibrium model used previously by Brock and Turnovsky by including nominal depreciation to study corporate taxes assessed on nominal quantities, a better approximation of reality. This extension is particularly important for understanding the interaction of the tax system and monetary policy. For simplicity of exposition, all variables are expressed in real terms.

We assume an economy with one produced good, the numeraire, which serves both consumption and investment purposes. The agents in the economy are described by a representative consumer with an infinite life who maximizes the present discounted value of his utility flow:

$$\int_0^\infty e^{-\rho t}\, U(c(t),\, m(t))\, dt \tag{1}$$

subject to an instantaneous budget constraint

$$c(t) + \dot{k}(t) + \dot{m}(t) = r(t)k(t) + w(t)\ell - \pi(t)m(t) - \text{ Tax } (t) + \text{ Tr } (t)$$

where

$\rho =$ pure rate of time preference

$U =$ instantaneous utility

$c =$ rate of consumption of the numeraire good

$r =$ real rate of return on capital net of depreciation

$k =$ capital stock, in consumption units

$w =$ wage rate

$\ell =$ labor supply, set at one unit per period

$\pi =$ inflation rate

$m =$ real money balance

Tax $=$ taxes, to be specified below

Tr $=$ transfers

We choose to put money in the utility function since it is just a convenient way to generate a demand for money. It is equivalent to assuming a transactions cost function which depends on c and m (see Feenstra [1986]). Feenstra argues, the alternative Clower constraint approach to money demand is just an extreme transactions cost specification. We suspect that our analysis would generalize to that model of money demand. We did run simulations of the purely monetary model with Clower constraints, as formulated by Abel, finding that the monetary nonneutralities were equally insignificant. These considerations indicate that the transaction cost approach implicit in our model is an appropriate formulation of money demand for our purposes.

The production sector of the economy is described by a constant returns to scale production function in capital and labor. For simplicity, the labor supply is assumed to be exogenously determined, however we must emphasize that none of the results of this paper hinge on that fact. Since labor is inelastically supplied, it is convenient to choose the units of labor so that the capital stock equals the capital-labor ratio. Thus, output is equal to

$$F(k) = \delta k + f(k) = \delta k + kf'(k) + (f(k) - kf'(k)) = (r + \delta)k + w \tag{3}$$

where f' denotes the net marginal product of capital and δ the depreciation rate of capital, corresponding to exponential decay.

The government raises revenues by printing money and by taxing the gross return on capital. Furthermore, it spends part of that revenue on tax expenditures such as the investment tax credit, depreciation allowances and expensing. Some of the net revenue raised by government is removed from the economy, being expended in a fashion which affects utility in a separable way. Remaining government revenues are rebated to the consumer in a lump sum fashion, constituting the transfer component, Tr, of the agent's budget constraint. Because of the competitive assumptions, the consumer takes as given all government actions and the return from capital and labor. The analogy of these two approaches with static models is that rebating in a lump sum fashion is akin to constant yield incidence while discarding the revenues is more akin to equal yield incidence.[1]

The government has six instruments under its control to raise revenues:

[1] For the latter to be exactly true, welfare exercises should compare the tax system with a lump sum one at the margin. Another way of handling the excess revenue is through bond repurchase. Although bonds are not present in this model, they could be added with no difficulty (see Brock and Turnovsky (1981)).

$\mu =$ the growth rate of nominal money stock

$\tau =$ the tax rate on gross corporate income (net of wages)

$\theta =$ the tax credit on new investment

$\beta =$ allowed rate of depreciation of nominal undepreciated capital

$\alpha =$ the fraction of new investment expenditures that can be immediately expensed

$1 - \zeta =$ the level of indexing of the historical capital base.

The tax on gross income has full loss offset, although for our conclusions this feature is irrelevant. Although current legislation does not allow full loss offset, as long as profits are always positive, as is the case around a steady state in a deterministic model, no losses will arise.[2] Furthermore, mergers will accomplish the same. Both the tax credit and the partial expensing, α, are granted on all investment, replacement and new. Standard depreciation reduces taxes at a rate $\beta\tau$ of the real value of capital that was not expensed, has not yet been depreciated, and was not inflated away. The budget constraint of the consumer can now be rewritten to take into account both the production function and the specific taxes he must face:

$$\begin{aligned} c + \dot{k}(t) + \dot{m}(t) = w\ell + rk - (r+\delta)k\tau + \theta(\dot{k} + \delta k) \\ + \alpha\tau(\dot{k} + \delta k) + \beta\tau k = \pi m + \mathrm{Tr} \end{aligned} \tag{4}$$

$$\dot{K} = -\beta K + (1-\alpha)(\dot{k} + \delta k) - \zeta\pi K$$

where K is "book" capital, that is, the real value of undepreciated capital carried on the firm's tax accounts. In order to express all the variables in real terms, the historical (nominal) value of undepreciated capital must be devalued by the inflation rate with $1 - \zeta$ of that devaluation added back by indexing. If $\dot{k} + \delta k$ becomes negative, i.e., capital is reduced at a faster rate than it depreciates, we are implicitly assuming that the investment tax credit and the fractional expensing are fully recaptured at the corresponding fraction of the real value

[2] Although this provision is not in the current tax code, there were provisions in the past, such as tax lease-back agreements, that allowed unprofitable firms to sell their deductions to profitable firms.

of capital sales.[3] When $\dot{k} + \delta k < 0$, equation (5) is not an appropriate description of reality for the historical value of undepreciated capital, K, as $(1 - \alpha)$ $(\dot{k} + \delta k)$ indicates a reduction in the historical value of capital equal to the current value of capital taken out, while the capital taken out is carried on the books at a much lower value, lowered by inflation since purchase and by the fractional depreciation at rate β. However, since $\dot{k} + \delta k$ is positive around the steady state and our focus is on local behavior around the steady state, our assumptions are appropriate.

Even when the system of differential equations (4) and (5) truly represent the consumer's budget, i.e., when $\dot{k} + \delta k > 0$, we are implicitly making assumptions to guarantee that profitable trades cannot be made by various consumers to reduce their tax burden on capital. In the present context, we must be concerned about sales of capital in place which may generate tax benefits but leaves the aggregate level of capital unchanged.[4] To address this issue, it is assumed that all gains and losses on capital sales are taxed uniformly at a rate τ. We assume that when a unit of capital is exchanged, the buyer gets an expensing deduction of α and the seller is credited with α in taxable income, eliminating the possibility of gain from capital exchange under the expensing rule if all face the same tax rate. We assume likewise for investment tax credit, θ.[5] In the case of depreciation, we must be more careful. If the undepreciated base is smaller than the value of the capital being sold, then recapture, that is, augmenting taxable income by the excess of the sale price over the undepreciated value, generates a loss greater than the future depreciation value of the asset at the new base, and there is no tax gain from an exchange of assets. This is the case when $\beta > \delta - \pi$, the depreciation rate on the nominal base is greater than rate of decline in the nominal value of capital (increased by inflation and decreased by physical deterioration). This condition was

[3] This is not quite an accurate description of recent tax practices. Under 1986 Law, if an asset used ACRS, excess depreciation is recaptured as ordinary income, but if linear depreciation is used, the capital gains rate is used. We would argue that our assumption is an appropriate simplification.

[4] The obvious easy answer to this problem, although not very enforceable from a practical point of view, is to give no tax credit or expensing (or recapture) on sales of old capital and to pass along the undepreciated part of the capital base at the time of sale.

[5] Investment tax credits were forgiven at two percentage points per year; for example, if an asset received a ten percent credit and was held for two years, only sixty percent of the original credit is recaptured. This, in conjunction with the depreciation rule to be discussed below, is sufficient to guarantee the absence of net gain. The loss of recaptured depreciation more than offsets the gain from getting a new investment tax credit. Under the Tax Reform Act of 1986, there is no longer any investment tax credit.

certainly a good approximation of the recent past. Even the recent accelerated depreciation rules probably did not produce tax gains from exchanging assets sufficient to cover associated transactions costs. In summary, we assume rules sufficiently strict and enforceable that pure churning of assets is not profitable.

3. Dynamic Equilibrium

We are now ready to derive the equations describing the optimal paths by real capital, k, book capital, K, and their respective private shadow values, λ_k and λ_K. The Hamiltonian of the representative agent's problem is:

$$
\begin{aligned}
H = e^{-\rho t}\, U(c,m) + \lambda_k \Big\{ & w\ell + rk(1-\tau) - (\tau - \alpha\tau - \theta)\ \delta k \\
& + \beta\tau K - x - c + \text{ Tr } \Big\} (1-\theta-\alpha\tau) \\
+ \lambda_m \{ & x - \pi m \} \\
+ \lambda_K \Big\{ & -(\beta - \zeta\pi)K + (1-\alpha)\ \delta k \\
& + (1-\alpha)\Big[w\ell + rk(1-\tau) - (\tau - \alpha\tau - \theta)\ \delta k \\
& + \beta\tau K - x - c + \text{ Tr } \Big] (1-\theta-\alpha\tau)^{-1} \Big\}
\end{aligned}
\tag{6}
$$

where λ_m is the private shadow price of money balances and the identity $x = \dot{m} + \pi m$ has been used to eliminate $\dot{m}$ from the budget constraint (4). When we impose the equilibrium condition $r = f'(k)$, the equilibrium costate equations become:

$$
\begin{aligned}
\dot{\lambda}_k = \lambda_k \Big\{ & \rho + \delta - (f'(k) + \delta)\ (1-\tau)\ (1-\theta-\alpha\tau)^{-1} \Big\} \\
& - \lambda_K (1-\alpha)\, (f'(k) + \delta)\ (1-\tau)\ (1-\theta-\alpha\tau)^{-1}
\end{aligned}
\tag{7}
$$

$$
\dot{\lambda}_m = \lambda_m \{\rho + \pi\} - U_m \tag{8}
$$

$$
\begin{aligned}
\dot{\lambda}_K = \lambda_K \Big\{ & \rho + \beta + \zeta\pi - (1-\alpha)\, \beta\tau (1-\theta-\alpha\tau)^{-1} \Big\} \\
& - \lambda_k \beta\tau (1-\theta-\alpha\tau)^{-1}
\end{aligned}
\tag{9}
$$

The Maximum Principle implies

$$
0 = U_c - (\lambda_k + (1-\alpha)\lambda_K)\ (1-\theta-\alpha\tau)^{-1} \tag{10}
$$

$$0 = \lambda_k(1-\theta-\alpha\tau)^{-1} - \lambda_m + (1-\alpha)\lambda_K(1-\theta-\alpha\tau)^{-1} \tag{11}$$

From (10), we obtain an expression for consumption

$$c = c(m,\ \lambda_k,\ \lambda_K;\ \alpha,\ \tau,\ \theta) \tag{12}$$

for some function $c(\cdot)$, of $m,\ \lambda_k,\ \lambda_k,\ \alpha,\ \tau$, and θ. Using the time derivative of (11),

$$\begin{gathered}(\dot{\lambda}_K(1+\alpha) - \alpha\lambda_K + \dot{\lambda}_k)\ (1-\theta-\alpha\tau)^{-1} + (\lambda_k + (1-\alpha)\lambda_K)\ \cdot \\ (\dot{\theta} + \dot{\alpha}\tau + \alpha\dot{\tau})\ (1-\theta-\alpha\tau)^{-2} = \dot{\lambda}_m\end{gathered}$$

along with (7)–(9), (11), and (12), we obtain

$$\pi = \pi(m, k, \lambda_k, \lambda_K;\ \alpha, \tau, \theta, \dot{\alpha}, \dot{\tau}, \dot{\theta}) \tag{13}$$

for some function $\pi(\cdot)$. Thus, the inflation rate depends not only directly on current money and capital stocks and indirectly on future ones through the shadow values of real and base capital, but also directly on current levels and changes in tax parameters.

The dynamics of the system is completely described by three of the above differential equations, (5), (7), (9), and two additional differential equations (14) and (15). Since tax revenues are lump sum rebated, the material balance equation is

$$\dot{k} = f(k) - c - g \tag{14}$$

where g is the amount of input consumed by the government. The real money equation is[6]

$$\dot{m} = m(\mu - \pi)\ . \tag{15}$$

Note that this system of five differential equations, (5), (7), (9), (14), and (15), uses the two implicit functions c and π, defined by (12) and (13).

4. Steady State and Local Analysis

[6] This does not imply that m is a continuous function of time. If α, τ, or θ have discontinuities, then their time derivatives are Dirac delta functions, as π will be, causing a jump in m in accordance with (15).

In steady state, all the time derivatives in (5), (7), (9), (14), and (15) are zero. From (15), it follows that $\pi = \mu$ and from (14), $f(k) = c + g$. Also by (5), it follows that in the steady state

$$K = (1-\alpha)\,\delta\,(\beta + \zeta\mu)^{-1} k \ . \tag{16}$$

From (7) and (9), we have

$$\begin{aligned} \rho^* &= (\rho+\delta)\,(1-\tau)^{-1}[(1-\theta-\alpha\tau) - (1-\alpha)\,\beta\tau(\rho+\beta+\zeta\mu)^{-1}] - \delta \\ &= (\rho+\delta)\ (1-\tau)^{-1}\,[T - BR^{-1}] - \delta \ , \end{aligned} \tag{17}$$

where ρ^* is the steady state cost of capital and marginal product of capital, $T = 1 - \theta - \alpha\tau$, $B \equiv (1-\alpha)\beta\tau$, and $R = \rho + \beta + \zeta\mu$. T is the consumption cost of one unit of investment, B is the steady-state stream of depreciation deductions per unit of gross investment, and R is the discount rate for determining the real value of the depreciation stream.

From (9), we know that

$$\lambda_K = \beta\tau(RT - B)^{-1}\,\lambda_k \ . \tag{18}$$

in the steady state. Also, (8), (10), and (11) implies that relationship

$$U_m = (\rho + \pi)U_c \tag{19}$$

at all times, which can be used to solve m as a function c and π. Finally, from (10) and (18), we find that the marginal utility of consumption is proportional to the private shadow value of physical capital,

$$\lambda_k = [T - BR^{-1}]U_c \tag{20}$$

The final step in the analysis is to linearize the five differential equations (5), (7), (9), (14), and (15) around a steady state. The values at the steady state value of a variable x is denoted by x^*. By inspection, we observe that K does not appear in any equation but the one describing $\dot{K}$. This implies that the linearized dynamics can be analyzed separately for the other four equations and that one of the eigenvalues is $R - BT^{-1} - \rho$ which is always positive if $(1-\tau) > \theta$. The linearized system around k^*, λ_k^* and m^* is

$$\begin{pmatrix} \dot{k} \\ \dot{\lambda}_k \\ \dot{\lambda}_K \\ \dot{m} \end{pmatrix} = J \begin{pmatrix} k - k^* \\ \lambda_k - \lambda_k^* \\ \lambda_K - \lambda_K^* \\ m - m^* \end{pmatrix}$$

where J is the matrix

$$\begin{bmatrix} f'(k^*) & -c_{\lambda_k} & -c_{\lambda_k} & -c_m \\ f''(k^*)(1-\tau)(\lambda_k + (1-\alpha)\lambda_K)T^{-1} & (\rho+\delta)B(RT)^{-1} & (1-\alpha)(\rho+\delta)[B(TR)^{-1}-1] & 0 \\ \zeta\lambda_K\pi_k & -\beta\tau T^{-1} + \zeta\lambda_K\pi_{\lambda_k} & R - BT^{-1} + \zeta\lambda_K\pi_{\lambda_K} & \zeta\lambda_K\pi_m \\ -m\pi_k & -m\pi_{\lambda_k} & -m\pi_{\lambda_K} & -m\pi_m \end{bmatrix}$$

where c_x and π_x are the derivatives with respect to x in (12) and (13).

In the remaining sections we examine tractable special cases which demonstrate the essential features of how the composition of tax instruments affect dynamic behavior and discuss numerical examinations of the full system to validate the robustness of these properties.

5. Adjustment Speed in a Purely Real Model

In this section, we will analyze the importance of the composition of the taxes for the speed adjustment to a new steady state in a simple case. We do this to give an intuitive explanation of the effects we study. These effects are most clearly illustrated in the two-dimensional reduced version examined in this section.

We linearize the model around its steady state and note that the stable eigenvalue is the rate of convergence to the steady state. We find that this rate of investment depends crucially on the tax composition, even when we hold fixed the effective tax rate and the target level of capital. For this exercise, we will initially examine a reduced version of the original model to highlight the main points.

In this section, we assume that there is no money in the model and that the taxes are paid in real quantities. The absence of money is modelled by assuming $U_m = 0$ everywhere. If there are to be no effects of inflation then there are only two possible levels of depreciation for tax purposes, $\beta = 0$ or δ. In either case, there is no difference between an investment tax credit θ or an immediate expensing α such that $\alpha\tau = \theta$. Therefore, let $\alpha = 0$ in the remainder of this section. The cost of capital equation (17) becomes

$$\rho^* = (\rho + \delta)\ (1-\theta)/(1-\tau) - \delta - \beta\tau/(1-\tau) \tag{21}$$

When $\beta = 0$, the taxes affect ρ^* through the ratio $(1-\theta)/(1-\tau)$, but not when $\beta = \delta$. Since $U_m \equiv 0$, if $\beta = \delta$ the equilibrium system reduces to (14) and the sum of (7) and (9), yielding a differential equation in k and $\lambda_k + \lambda_K \equiv \lambda$. Linearizing the dynamic system corresponding to (14) and the sum of (7) and (8), we obtain

$$\begin{pmatrix} \dot{k} \\ \dot{\lambda} \end{pmatrix} = \begin{pmatrix} f'(k^*) & -1/((1-\theta)U_{cc}) \\ -f''(k^*)(1-\tau)U_c & 0 \end{pmatrix} \begin{pmatrix} k-k^* \\ \lambda-\lambda^* \end{pmatrix} \tag{22}$$

where $f'(k^*) = \rho^*$ defines k^*, the steady-state capital stock. This system has the following negative eigenvalue:

$$\nu = \frac{f'(k^*)}{2} \left[1 - \left(1 - \frac{4(1-\tau)}{(1-\theta)} \frac{\theta_c \theta_L}{\theta_K \sigma} \frac{U_c}{cU_{cc}}\right)^{1/2}\right] \tag{23}$$

where θ_L, θ_K, and θ_c are labor income, capital income, and consumption, respectively, expressed as fractions of net output in the steady state.

For a given steady-state cost of capital, the speed of adjustment to the steady-state for small deviations from the steady state depends on taxes solely through $(1-\tau)/(1-\theta)$. When $\beta = 0$, for a given cost of capital, ρ^*, the composition of taxes does not affect ν except through ρ^* since both ρ^* and ν depend only on $(1-\tau)/(1-\theta)$. However, in the alternative case of true economic depreciation, $\beta = \delta$, ν depends not only on ρ^* but also on the mixture of τ and θ which given ρ^*. [7] Theorem 1 summarizes.

Theorem 1: If $U_M \equiv 0$ and $\beta = \delta$, then if θ and τ are increased so as to leave ρ^* unchanged, then ν is increased in magnitude. If $U_m \equiv 0$ and $\beta = 0$, ν is unaffected by changes in θ and τ which leave ρ^* unchanged.

Proof: Straightforward differentiation

The intuitive explanation for Theorem 1 rests on term structure considerations. The differing results for the $\beta = 0$ and $\beta = \delta$ cases are due to the different dependence on the term structure of interest rates. In the case of no depreciation deductions, a unit of investment is undertaken if and only if the cost does not exceed the present value of returns, i.e.,

$$1 \leq PV \equiv \theta + \int_0^\infty e^{-\int_0^t \gamma(s)ds} e^{-\delta t} F(1-\tau)dt$$

where $p \equiv U_c$ and $\gamma \equiv \rho - \dot{p}/p$, the required net rate of return. A unit of investment at $t = 0$ results in an extra $e^{-\delta t}$ units of capital at t, earning $F'(1-\tau)$ per unit after taxes. Suppose

[7] It is interesting to note that a joint increase in θ and τ leaving ρ^* unchanged leads to higher tax revenue. This result can be interpreted as a lump sum tax on capital in place due to rise in τ, with the rise in θ compensating for the distortion and providing the necessary incentive to maintain the capital stock.

θ is increased by $d\theta$ and τ by $d\tau$ so as to leave the steady-state cost of capital unchanged. Then $d\tau = (\rho+\delta)/(\rho^*+\delta)\, d\theta$ and

$$dPV = 1 - \int_0^\infty e^{-\int_0^t \gamma(s)ds}\, e^{-\delta t}\,(-F')\,\left(\frac{\rho+\delta}{\rho^*+\delta}\right) dt$$

In equilibrium, however, $F' = (1-\theta)\,(\gamma+\delta)/(1-\tau)$. Substituting this into the expression for dPV and using $\rho^* = (\rho+\delta)\,(1-\theta)/(1-\tau) - \delta$ shows that $dPV = 0$, independent of the path of γ. Therefore, even outside of the steady state, this change in taxes will not affect investment.

However, if $\beta = \delta$,

$$PV = \theta + \int_0^\infty e^{-\int_0^t \gamma(s)ds}\, e^{-\delta t}\,[(1-\tau)F' + \delta\tau)\, dt$$

Also, $\rho^* = (\rho(1-\tau) - \delta\theta)/(1-\tau)$ and $d\tau = (\rho+\delta)/\rho^*\ d\theta$ for ρ^*-preserving changes. In this case

$$dPV = 1 + \int_0^\infty e^{\int_0^t \gamma(s)ds}\,(\delta - F')\,(\rho+\delta)/\rho^*\, dt\ .$$

Along an equilibrium path, $F' - \delta = (\gamma(1-\theta) - \delta\theta)/(1-\tau)$, hence

$$dPV = 1 - \ \frac{\rho+\delta}{\rho^*+\delta}\ \frac{1-\theta}{1-\tau}\ +\ \frac{\delta(\rho+\delta)}{\rho^*(1-\tau)}\ \int_0^\infty e^{-\int_0^t (\gamma+s)ds}\, dt$$

which is zero if $\gamma \equiv \rho$, as in the steady state. Hence $dPV > 0$ if γ is falling to ρ and $dPV < 0$ if γ is rising to ρ, implying that investment is greater when capital is below the steady-state level, and less otherwise.

Another way of seeing the difference between the two cases is to examine the implicit partnership between the government and the firm implicit in the tax structure. The intuition is best illustrated when we assume that $\tau = 0$. We saw above that if $\beta = 0$, then ρ^* depends on taxes only through $(1-\tau)/(1-\theta)$. In this case the government receives a fraction τ of all receipts and pays the same fraction of all investment expenses; it is essentially a full partner owning τ of the firm and since $\rho^* = \rho$ there is no distortion to the capital stock. Any equal change in τ and θ will preserve this arrangement, alter the government's share, but will have no impact on the firm's incentive to invest. In particular, there will be no effect on convergence to the steady state.

On the contrary, if there are depreciation deductions, then the relationship is not so clean since the firm receives depreciation deductions for the entire capital stock, not just its "share." Since an increase in τ increases the present value of those future depreciation allowances, a cost-of-capital preserving increase in τ and θ cannot raise θ as much as in the absence of depreciation deductions. Therefore, in the presence of depreciations deductions, if we raise τ but not ρ^*, less of the offset to the τ increase can take the form of a θ increase, an immediate subsidy to investment, and more will take the form of an increase in future depreciation deductions, a delayed subsidy to investment. Hence, while term structure considerations cannot matter in the absence of depreciation allowances since the government is essentially a partner, term structure considerations will matter for cost-of-capital preserving tax changes in the presence of depreciations deductions. While this analysis assumed that $\tau = \theta$, the basic principles hold in general.

This section has shown how the structure of taxes and investment subsidies affect the dynamic behavior of the economy if money does not matter. We will next discuss why we care about the eigenvalues of the deterministic system and then discuss monetary examples.

6. Eigenvalues and the Productivity Shocks

Before continuing with analysis of more general models, we will not discuss why we examine the eigenvalues of our deterministic models. The focus on earlier models, such as Fischer, Asako, and Able, was on its relation to the economy's convergence to its steady state. However, it is also related to the impact of productivity shocks in stationary stochastic models. In this section we will outline the standard connection between the eigenvalue ν and stochastic fluctuations. For more detailed presentations of this approach see Kydland and Prescott (1984) or Judd (1985).*

The relation between the eigenvalue ν and some business cycle fluctuations is direct. Suppose the capital stock is subject to additive shocks, as in the stochastic differential

* Kydland and Prescott (1984) take a deterministic model, compute its steady state, compute a linear approximation for the dynamics around the steady state, and then add shocks to the linear approximation. This si logically equivalent to the discussion above. Judd (1985) discusses a more direct approach to analyzing a more general family of shocks.

equation

$$dK = Idt + \sigma K^* dz$$

where I is net investment, dz is white noise, K^* is the steady-state capital stock when $\sigma = 0$, and σ^2 is the variance of the relative shock. Since $I = \nu(K - K^*)$ for K near K^*, the linear approximation for the K process when σ is small is

$$dK = \nu(K - K^*)dt + \sigma K^* \, dz$$

This is the standard stock adjustment formula for capital and investment. If ν is large in magnitude, then the economy is being strongly pushed towards K^*, and therefore acts to stabilize capital stock and output. This follows directly from the solution for K,

$$K(t) - K^* = -\sigma K^* \int_{-\infty}^{t} e^{\nu(t-s)} dz(t)$$

It is important to note that the effect described in Theorem 1 is nontrivial when we parameterize the model using empirical estimates of the important structural parameters. Define $\epsilon_{\nu x}$ to be the elasticity of ν with respect to ρ^* as $x = \tau, \theta$ is changed. More precisely,

$$\epsilon_{\nu x} \equiv \frac{\partial \nu / \partial x}{\nu} \Big/ \frac{\partial \rho^* / \partial x}{\rho^*} \, , \ x = \tau, \theta$$

When σ is varied between .4 and 1.0 (a range suggested by Berndt [1976], Berndt and Christensen [1973], and Lucas [1969]), cU_{cc}/U_c is varied between $-.5$ and $-.10$ (a range suggested by Weber [1970, 1975], Hansen and Singleton [1983], and Hall [1981]), τ is varied between .2 and .8, and θ between 0.0 and 0.1 (ranges suggested by Feldstein, et al. [1983], and King and Fullerton [1984]), $\epsilon_{\nu\tau}$ and $\epsilon_{\nu\theta}$ vary over a moderate range. Most important for our point is the fact that $\epsilon_{\nu\theta} - \epsilon_{\nu\tau}$ was at least .4 and at most .6. This means that if θ is increased enough to reduce the cost of capital by ten percent (e.g., from .10 to .09) and τ is raised enough to return to the initial cost of capital, then net investment is increased by four to six percent. Changes of this size are not unusual. For example, $\theta = .05$ and is eliminated (as done recently) with $\tau = 0.5$, $\beta = \delta = \rho$, then ρ^* moves from 1.8ρ to 2ρ, an eleven percent change, implying for these parameters a five to seven percent change in net investment at each point in a business cycle driven by shocks to the capital stock.

If the real business cycle were driven by shocks to a productivity our analysis is still applicable. For example, if $\tilde{f}(k) = \tilde{\epsilon} f(k)$, where $\tilde{\epsilon}$ is a Markov process taking two values, $\epsilon_1,\ \epsilon_2 > 0$, then there are two target levels of capital stock, one for each state of productivity. ν still will be the local rate of convergence to the current target if ϵ_1 and ϵ_2 are close. An increase in the magnitude of ν would imply that the economy more rapidly accumulates in response to a favorable productivity shock and more rapidly decumulates in response to an adverse shock.

In this section, we have examined a tractable special case, allowing us to precisely determine the interaction between the tax mix, investment incentives, and the term structure of returns. The true tax structure is substantially more complex than this case. Unfortunately, the kind of precise computations performed above are no longer tractable. Therefore we next examine numerically two successively more general models, finding that the effects examined above are of equal or greater quantitative significance in more general models.

7. The Case of Separable Utility

In order to examine the importance of inflation and nominal depreciation in the most, direct fashion, we next assume $U(c,m) = u(c)+v(m)$, i.e., utility is separable in consumption and money. This assumption leads to dichotomy—real and nominal quantities are determined independently. This dichotomy follows from the observation that (10) implies that consumption is solely a function of $\lambda_k,\ \lambda_K$, and tax parameters, not m. Therefore, (5), (7), (9), and (14) alone determine $k,\ c,\ \lambda_k$ and λ_K. We examine this case since it is the case where we have globally valid quantity theory of money with no real effects except through a nominal tax structure.

In Table 1 we examine dynamic effects of the full range of policy parameters—α, β, τ, θ, and μ in a noninflationary state. Again we allow cu_{cc}/u_c to vary between $-.5$ and -5 and σ to vary between .4 and 1.0 as 1.0. In this table, $\tau = .46$, $\theta = .05$, $\alpha = .01$, $\beta = .03 = \delta$, and $\mu = 0$. We again find the same basic results if economic depreciation is used for tax depreciation. Cost-of-capital-preserving tax changes which increase either investment subsidy—α or θ increases—and also increase future tax liabilities—μ and τ increases, or β decreases—cause ν to increase. The impact on ν is of the same magnitude as we found in the previous section. For example, a simultaneous α increase and β decrease, each of which would in isolation change a cost of capital of .1 by .01, will change the magnitude of net investment by four to six percent at every point around the steady state. Most interesting is that inflation will have a substantial effect on ν. In fact, ν is most sensitive to small changes in μ when utility is more concave. When utility is less concave, changes in β have the greatest impact on ν per unit effect on ρ^*. In general, in Table 1, the largest dynamic effects come from changes in β, θ, and μ with the sensitivity of ν to α and τ being substantially less. The key feature to note is that the wide dispersion of these sensitivities imply that the composition of the tax system has important dynamic effects beyond the level of the steady-state capital stock.

In an inflationary environment, the direction of dependence of ν on the policy parameters may change. If $\mu = .10$ instead of 0.0 in Table 1, an increase in inflation will reduce the magnitude of ν as it raises ρ^* if $\gamma = -.1$ or $-.3$. The range of $\epsilon_{\nu x}$, $x = \beta, \tau, \theta, \mu, \alpha$, is even greater in this case, being at least .4 and possibly 1.0. The essential point remains—ν responds substantially to cost-of-capital-preserving changes in tax and monetary policy.

8. Speed of Adjustment in the General Model

We next examine the importance of tax composition on the speed of adjustment of the general system given by (5), (7), (9), (14), and (15).

First, we specialize the model to (7), (14) and (15) with inflation and no taxation as in Fischer [1979]. It should be noted that for this model the equilibrium level of capital is independent of the level of inflation. As had been remarked by Fischer and Asako [1983], the speed of adjustment increases (decreases) with inflation if $U_{12}(c, m)$ is positive (negative). The point that we wish to raise here is that even though this effect exists, it is quantitatively negligible for parameter values representative of the U.S. For the set of over 1000 combinations[8] that we tried, an increase in inflation from zero to 100 percent per unit of time resulted in at most a 2% change in the negative eigenvalue. In particular, we examined a variety of parameterizations of the Fischer model. For almost all cases, such an increase in inflation affected ν by $0-.2\%$. Therefore, the Tobin-Mundell effects in Fischer and Asako are

[8] See the Appendix for a discussion of the combination of parameters examined.

trivial and certainly unobservable with standard econometric methods when these models are appropriately parameterized. We also examined the cash-in-advance monetary analysis of Abel coming to the same conclusions. The impact on eigenvalues was sometimes greater, not surprising since inflation affects the steady-state level of capital. However, the maximum impact was with log utility where a four per cent change in ν resulted from increasing inflation from zero to 100%, still a trivial effect.

In the presence of taxes, the situation is very different. Any change in any single tax parameter, α, β, μ, τ, or θ, changes the steady state level of capital if τ and β are not zero. For instance, an increase in inflation will simultaneously decrease the optimal level of capital accumulation and affect the speed of adjustment for each of the parameter combinations examined. While some of the impact ν may be a pure monetary effect, some will be due to the fact that ν will be different at a different steady-state capital stock, a tax effect. While the direction of dependence of ν on the tax parameters is ambiguous over the range of money demand parameters examined, the sensitivity of ν to the composition of taxes remain generally unchanged.

To give an example roughly similar to the 1970's U.S. inflationary experience we present Table 2. Table 2 assumes a steady state where depreciation is on an historical cost basis and inflation is 10% per period where a period is that amount of time during which utility is discounted 4%. We let the intertemporal elasticity of consumption u_c/u_{cc}, to vary between -1.0 and $-.2$, and we set the nominal interest elasticity of money demand, γ, to be $-.1$ or $-.3$, as suggested by Goldfeld's estimates. m/c is set at .4 and σ at .7 only since the results are insensitive to these parameters. We find that $\epsilon_{\nu x}$ varies by .2 to .45 among the instruments $x = \beta,\ \tau,\ \theta,\ \mu$, and α. Other choices of depreciation and inflation parameters will yield more extreme results, indicating the robustness of ν being sensitive to the composition of the tax structure.

These results are likely to be sensitive to the nature of money demand. If money entered the production function or a cost of adjustment function, the nature of ν's dependence on monetary and tax policy is likely to be different. However, there is no reason to believe that the dependence of ν on the composition of taxes is any less. We chose to represent money demand by $u(c, m)$ since that is the most popular way to model money demand and it allowed direct comparison with the similar work of Asako and Fischer.

9. The Large Model Versus Smaller Models

The basic model proposed in this paper is characterized by five dynamic equations to represent taxation on nominal quantities in the presence of money demand and savings, is larger than most models used to theoretically analyze monetary and/or taxation phenomena. There are three reduced models commonly used. (i) *Nominal taxes with no money* (5), (7), and (14): essentially money demand is removed from the model and inflation is set exogenously at the rate of growth of money supply; inflation matters because the tax shield generated by depreciation is expressed in nominal terms. (ii) *Real taxes with money demand* (7), (14) and (15): taxes are based on real quantities with β being 0 or δ. (iii) *Real taxes with no money* (7) and (14): the system discussed in section 5.

In the last two models, there is no unique way to approximate the large model by one of its smaller counterparts. Reducing the full model to one with the same steady state can be achieved either by increasing θ in the reduced model from θ to $\theta + \alpha\tau + \frac{\beta\tau}{\mu+\rho+\beta}$, holding τ constant and setting $\beta = 0$, thereby imputing all investment credits and depreciation allowances to the investment tax credit. One also could increase θ to $\theta + \alpha\tau$ and decrease τ so as to leave $\rho^* = f'(k)$ unchanged, imputing the depreciation allowance to the tax rate. These are just two possible combinations of θ and τ changes which leave ρ^* unchanged.

While the steady state capital stock is the same in all these models, the dynamic behavior around the steady state may differ. In models (i) and (iii), the speed of adjustment is independent of the tax mix when $\beta = 0$, whereas the speed of adjustment in the larger model depends on the tax mix, holding ρ^* constant. Therefore, these models cannot exactly represent the dynamic behavior of the large models. When $\beta = \delta$, the speed of adjustment depends on the tax mix, so it may be possible to choose the mix of θ and τ such that the speed of adjustment is identical. But that can be accomplished only by first computing the speed of adjustment for the large model, not *ex ante*.

10. Conclusion

This paper has explored the dependence of the dynamic behavior of an economy on the composition of taxation. We have found that in a representative agent model with no money, raising direct investment incentives and raising income tax rates will increase the economy's speed of adjustment in the presence of depreciation allowances. If depreciation is based on historical cost, increasing the rate of inflation away from zero will increase the speed of adjustment even if another parameter is adjusted to keep the long-run cost of capital unchanged. However, in an already inflationary environment, this may be reversed. In models with money demand, results are ambiguous, but of quantitative significance.

This analysis shows that in examining various alternative tax policies, we should examine the composition of the tax structure in terms of various instruments as well as the cost of capital since the composition itself may substantially affect the dynamic performance of the economy. Further analysis should examine the normative implications of this positive analysis.

Appendix

In the numerical examples of the general model referred to in the text, we relied on a wide range of parameterizations suggested by the empirical literature. We allowed σ, the elasticity of substitution in the net production function, to be .4, .7, and 1.0 and 1.3, an range encompassing most estimates (see Berndt [1976]). Note that a Cobb-Douglas gross production function would have a net production function with σ about .8. This should be kept in mind when comparing our σ values with estimates of the gross production function. We permit cU_{cc}/U_c to be $-.5$, -1.0, -2.0, -5.0 and -10.0, a range suggested by Hansen-Singleton, and Weber and Hall.

In addition to the elasticity of factor substitution and the intertemporal elasticity of substitution in consumption, the introduction of money demand requires specification of money demand parameters. Section 6 examined the case of zero income elasticity of money demand. In Section 7 the computations sometimes assume a unitary income elasticity of money demand by assuming that the constant elasticity of substitution between c and m is constant. We allow the elasticity of money demand with respect to nominal interest rates to be $-.1$, $-.3$, $-.5$, and $-.8$, a collection consistent with existing estimates (see Goldfeld [1973] and Hadjimichalakis [1982] for discussions of money demand estimates). We allowed the ratio between annual consumption and money holdings, m/c, to be .2, .4, and .6, a range encompassing U.S. postwar experience with $M1$. Using a variable elasticity of substitution functional form, we also determined that the results for an income elasticity of .5 was intermediate between our results for zero and unitary income elasticity. Therefore, this collection covers a wide range of values for the crucial parameters.

For our discussion of the Fischer and Abel models of pure monetary policy we let inflation vary between 0 and 100% in steps of 1%, and we let the intertemporal elasticity of consumption demand vary between .1 and 2.0 in steps of .1. The capital share was assumed to be .25 or .33.

Table 1

σ	cU_{cc}/U_c	β	τ	θ	μ	α
.4	$-.5$	.90	.39	.67	.74	.41
	-1.0	.87	.35	.64	.98	.37
	-2.0	.82	.29	.61	1.14	.32
	-5.0	.75	.21	.57	1.30	.24
.7	$-.5$	.89	.37	.65	.75	.39
	-1.0	.85	.32	.63	1.00	.35
	-2.0	.80	.26	.60	1.17	.29
	-5.0	.73	.18	.55	1.34	.21
1.0	$-.5$	.87	.35	.64	.76	.38
	-1.0	.83	.30	.62	1.02	.33
	-2.0	.78	.24	.58	1.19	.27
	-5.0	.71	.15	.53	1.36	.18

Entries under column x, $x = \beta$, τ, θ, μ, α, are $\epsilon_{\nu x}$ when $U(c, m) = u(c) + v(m)$ and $\beta = \delta = .03$, $\tau = .46$, $\theta = 0.05$, $\alpha = 0.01$, and $\mu = 0$.

Table 2

γ	cU_{cc}/U_c	β	τ	θ	μ	α
$-.1$	-1.0	.29	.58	.54	.17	.60
	-2.0	.59	.49	.52	.67	.50
	-5.0	.62	.40	.45	.84	.41
$-.3$	-1.0	.50	.54	.54	.49	.55
	-2.0	.60	.47	.51	.69	.48
	-5.0	.61	.39	.44	.82	.39
$-.8$	-1.0	.53	.53	.54	.54	.54
	-2.0	.59	.47	.50	.67	.47

-5.0	.59	.39	.42	.88	.38

Table 2 assumes $\tau = .46$, $\theta = .05$, $\alpha = .01$, $\beta = \delta$ .03, $\sigma = .7$, $m/c = .4$, $\mu = .1$, and a unit income elasticity of money demand. γ is the long-run elasticity of money demand with respect to the long-run nominal interest rate.

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