OPTIMAL TAXATION IN
DYNAMIC STOCHASTIC
ECONOMIES:
THEORY AND EVIDENCE

by

Kenneth L. Judd

Hoover Institution, Stanford University

and

National Bureau of Economic Research

opttax.tex

ABSTRACT

This paper examines the nature of optimal taxation of labor and capital income in various models of dynamic, stochastic economies. We also compute Markov perfect equilibria (a.k.a. "consistent policies") for environments in which governments cannot make binding commitments about tax policy. We find that the random walk test for optimality, popularized by Barro, applies only to labor taxation, whereas optimal capital income tax rates are more volatile, sometimes being white noise. In contrast, we find that labor and capital income tax rates follow qualitatively similar processes (e.g., in one case both are AR(1) processes) in Markov perfect equilibrium outcomes. Previous analyses of tax rates tend to support the hypothesis that Markov perfect equilibrium characterizes observed tax rates. However, these results are due to the fact that it is inappropriate to use conventionally defined *ex post* capital income tax rates for these purposes. We find that when appropriate tax rates are used, the optimality hypothesis is favored by the evidence from the postwar era.

This model has implications for both fiscal and monetary policy analyses. First, several seemingly irrational features of U.S. tax policy (e.g., nominal depreciation allowances and income taxation instead of consumption taxation) may contribute to the efficiency of taxation in a stochastic environment. Second, examinations of the optimality of monetary policy should focus on the nominal aspects of the tax system since seigniorage is relatively insignificant. When we do so, the previous empirical support for optimality of monetary policy disappears. However, those tests were misspecified, thereby not contradicting the support for optimality found here. Third, observations that fiscal surpluses are positively related to changes in the unemployment rate are consistent with optimal taxation in a real business cycle model and, therefore, do not give support to Keynesian fiscal policy interpretations.

Optimal Taxation Policy in Dynamic Stochastic Economies

In the past decade there has been substantial interest in characterizing optimal tax policies in dynamic economies and constructing empirical tests of the optimality hypothesis. Barro (1979) argued that optimally chosen tax rates would display strong persistence, a proposition formalized by a random walk null hypothesis in empirical tests. Lucas and Stokey (1983) reexamined these problems in a fully articulated general equilibrium model, focussing on the potential role for state-contingent government-issued securities. These analyses assumed (implicitly or explicitly) that all income is generated by the use of labor only, and is proportional to labor supply. These assumptions imply that the direct effects of taxation on resource allocations are largely contemporaneous, a feature inconsistent with taxation of capital income where current supply is governed by past expectations about current tax policy. The importance of capital income taxation limits the applicability of these analyses. The primary purpose of this paper is to incorporate capital formation into the analysis of optimal taxation.

Empirical tests of the optimality hypothesis have generated a mixed collection of results. Univariate analyses in Barro, Sahasukul (1986) and Kingston (1984), show some support for the random walk hypothesis. Vector autoregression analysis in Skinner (1989) is supportive of the random walk hypothesis for both interest and wage taxation. Bizer and Durlauf (1988) discusses difficulties with the linear-quadratic specification used to derive the random walk test, and find that the information contained in the frequency domain indicates a richer process at work. While the strict random walk hypothesis may be rejected, all analyses show strong persistence in tax rates.

Since capital income comprises a nontrivial portion of national income and is the source of a nontrivial source of tax revenue, it is important to explicitly incorporate capital into both the theoretical and empirical analysis of optimal tax policy. Most of the empirical work does not distinguish between capital and labor income. However, the literature on optimal taxation in deterministic models gives strong indication that labor and capital income should not be treated equally (see Atkinson and Sandmo (1980), and Judd (1985)). Judd (1987) shows that the dynamic nature of the efficiency costs of capital and labor taxation differ

3

substantially. The many proponents of consumption taxation also argue for differentiated treatment. In short, the deterministic theory gives no support for the usual procedure of lumping capital and labor income when examining the optimality hypothesis. We shall see that similar considerations also argue that the random walk hypothesis is inappropriate for periods when tariffs comprised a major source of revenue.

These random walk arguments have also been adapted to study optimal monetary policy in Mankiw (1987). Mankiw assumed that social and private inflation costs depended only on the contemporaneous inflation rate, implicitly assuming that money demand depends only on contemporaneous inflation and ignoring anticipation effects and the stock nature of money. The theoretical analysis of optimal monetary policy in Turnovsky and Brock (1980) differs substantially from that in Mankiw because they treat money as being a stock held in private hands at all times instead of a good which is rented to individuals. While it may be difficult to choose among the alternative monetary theories on *a priori* grounds, we will show that the stock nature dominates when one considers all nominal liabilities, making the optimal money policy problem more similar to that of optimal capital income taxation than optimal wage taxation.

This paper first develops a simple model for optimal taxation of both labor and capital income, using a quadratic loss function approach. We model the important differences between labor and capital supply, focussing on the flow nature of labor supply and the stock nature of capital supply. We assume that labor supply is determined by the contemporaneous tax rate, the standard assumption. However, we assume that today's supply of capital is relatively insensitive to today's taxation, being determined largely by yesterday's market value of current tax liabilities.

Our results show that the optimal tax policy in uncertain environments treats capital and labor income in drastically different ways. We continue to find that the optimal labor income tax rates are highly persistent, as in the Barro analysis. In stark contrast, we find that the optimal tax rate for capital income displays a very different behavior, generally being much more volatile.

Since it may appear that the results for the linear-quadratic loss function depend on

4

special features of the linear-quadratic specification, we also examined some numerical calculations of finite-horizon economies with isoelastic utility functions over consumption and leisure, and linear technology. We found that the results were even more extreme than the linear-quadratic loss specification – after the initial period, the optimal labor tax rate is constant, all expenditure shocks were absorbed by innovations in the capital income tax rate, but the *ex ante* capital tax rate was zero.

This robustness is not too surprising when one takes a contingent security view of the basic problem, as in Lucas and Stokey. Their analysis differs from Barro's in that they allow the government to issue securities contingent on the outcome of expenditure shocks, and show that the shocks are absorbed by these securities not in labor income tax rates. Even though we assume safe debt, our results are similar since stochastic *ex post* income tax rates essentially create a state-contingent return for owners of capital. With stochastic income taxation, investors buy both a physical asset as well as a state-contingent bond. In general, any state-contingent taxation which they would implement via risky bonds could also be implemented by stochastic income taxation of capital.

We apply these findings to a number of tax policy issues. For example, these results show that income taxation dominates consumption taxation because of its flexibility in responding to expenditure shocks, that nominal features of the U.S. tax code may help to implement the state-contingent tax policy, and that optimal tax policy may lead to a positive correlation between unexpected increases in budget surpluses and unexpected increases in unemployent rates even though we have nothing in the model which justifies a Keynesian interpretation of this relation between fiscal policy and unemployment.

A well-understood problem (see Kydland and Prescott, and Turnovsky and Brock) with dynamic optimal tax policies is that they tend to be dynamically inconsistent; that is, if governments can renounce past promises and rewrite tax policy, it will have strong incentive to do so. A common feature of optimal tax policy in the presence of a stock is to heavily tax the income of that stock in the short run, but promise to tax it lightly in the future. This is optimal since the short-run supply elasticity of capital is smaller than the long-run elasticity. However, when the future arrives the government will want to rewrite the law,

5

again imposing high levels of taxation on the stock and promising relief later. It is often argued that governments will fall into a situation where no one believes promises about the future, leading the government to do what is only myopically best, and resulting in an inefficient sequence of policies.

In order to investigate the importance and nature of these problems, we develop a model where governments are not able to commit, either through explicit institutional arrangements or implicit reputation mechanisms. We specify a specific sequential decisionmaking framework for the government and compute the Markov perfect equilibrium tax policies. We show that Markov perfect policies differ substantially from optimal policies. In particular, we find that labor and capital income taxation follows qualitatively similar stochastic processes. Another interesting robust finding is that the Markov equilibrium has the government essential endow government spending – it builds up a stock of assets generating sufficient interest income to finance expected expenditures, and uses income taxation to absorb deviations. Therefore, debt follows a mean reverting process in the Markov equilibrium, whereas it follows a random walk with the optimal policy.

However, building on work by Barro and Gordon (1983) in monetary policy and Chari, Kehoe, and Prescott (1989) in fiscal policy, we know that optimal policies can be implemented even if a government lacks a precommitment technology. Therefore, there is a large multiplicity of subgame perfect equilibrium for our policymaking game. This multiplicity of equilibria for the sequential decisionmaking environment implies that theory gives no guidance as to which type of outcome we should expect. However, since the nature of the two types of equilibria differ substantially, we do have the basis for a test to see which kind of equilibrium best describes the data. Whereas optimality implies that labor taxation and capital taxation follow substantially different processes, the two tax rate series are stochastically similar in a no-commitment equilibrium sequence of decisions. Given that the two hypotheses, optimality versus no-commitment equilibrium, give such wildly different predictions concerning tax rate processes, an examination of the data is in order. A first examination would seem to support the no-commitment equilibrium hypothesis. For example, Skinner's (1989) analysis distinguishes between capital and labor income taxation, finding

6

that they follow structurally similar processes, with a strong correlation between innovations in both labor and capital income tax rates. While we have no quarrel with labor tax rates as conventionally computed, we will argue that conventionally computed capital income tax rates are inappropriate for use in tests of the optimality hypothesis. We argue that when appropriate capital income tax rates are used, they are not persistent. We find that the optimality hypothesis is to some degree a better description of the data, though a strong finding is limited by the relatively small sample.

Linear-Quadratic Approximation

We will first examine a simple linear-quadratic specification of the social cost of distortionary taxation. This will allow us to build on the Barro analysis by specifying the important distinctions between labor and capital income taxation, to explicitly compute both the optimal policy and the Markov perfect equilibrium of a sequential game representing governmental decisionmaking, and facilitate consideration of reputation equilibria.

We first specify the environment in which the planner operates. We assume that government expenditures per period, g_t, are exogenously determined, and follow an AR(1) process:

$$\tilde{g}_{t+1} = \rho\, \tilde{g}_t - \tilde{\varepsilon}_{t+1}, \qquad |\rho| < 1$$

where $\tilde{\varepsilon}_t$ is an iid innovation process.

As in Barro, we allow the government to issue only risk-free debt. This contrasts with the Lucas-Stokey analysis which demonstrated that state-contingent securities could play an important role in allocating risk between the public and private budget sets. We focus on the risk-free case since government-issued securities are nominally risk-free, implying that only inflation is being used for risk-sharing purposes. In discussing the existing tax system, it is easier to fist assume risk-free debt, and then discuss the extensions to risky debt.

We assume that the government can raise revenue only through taxation of capital and labor income. Let L be the tax revenue from labor income taxation and K the tax revenue from capital income taxation. We will assume these taxes distort the economy, but only over a one-period horizon. More specifically, we assume that the government's loss in period t

depends only on taxation in period t *and period $t-1$ expectations* of period t taxation. loss, from the point of view of period $t-1$, of period t taxation is assumed to be

$$\mathcal{L}_t = E_{t-1}\left\{\frac{1}{2}\,\bar{\eta}\tilde{L}_t^2\right\} + E_{t-1}\left\{\frac{1}{2}\,\bar{\mu}\tilde{K}_t^2\right\} + \frac{1}{2}\,\psi\left\{E_{t-1}\{\bar{\pi}\tilde{K}_t\}\right\}^2$$

where $\bar{\pi}$ is the state-contingent marginal rate of substitution between consumption at time t and $t-1$.

This loss function combines many aspects of factor supply, distortionary taxation, and distributional considerations. First note how labor and capital income taxation are treated differently in these loss function. The cost of labor taxation depends only on the state-contingent labor tax revenue, whereas the efficiency cost of capital income taxation also depends on a weighted expectation of capital income tax revenue. This difference represents the critical difference for our analysis and is justified by the nature of factor supply. assume here, as in the labor supply literature, that labor supply depends on the contemporaneous wage. This implies that the labor market distortion in state ω depends on the labor tax revenue in state ω and that the total value of those distortions is the expectation of the state-contingent distortions represented by the $\bar{\eta}\tilde{L}^2$ terms. The efficiency cost of labor taxation depends first on the elasticity of labor supply, but administrative and enforcement costs may also generate social costs. These are all implicitly represented by $\bar{\eta}\tilde{L}^2$; if the only cost were the labor supply reduction and the value of labor supply were normalized to be unity, then $\bar{\eta}$ would approximately be state-contingent compensated elasticity of labor supply.

In contrast, the primary social cost for capital taxation is associated with the expected tax revenue. Investment decisions depend on the expected net returns and the responsiveness of savings to anticipated tax liabilities, represented here by ψ. An expected tax revenue on future capital income equal to $E\{\bar{\pi}K\}$ will reduce today's level of investment below its efficient level, generating a social cost of $\psi\left\{E\{\bar{\pi}\tilde{K}\}\right\}^2$.

This distinction between capital and labor supply is standard and realistic. For example, if one were told that all investment income over the next year were to be taxed at a rate of 100%, it is surely not the case that one would liquidate and consume all, or even a substantial portion, of his assets, whereas such a temporary confiscatory tax on labor income would drastically reduce current labor supply. The critical aspect is that labor is a flow factor

whereas capital is a stock factor. Even when one considers human capital the distinction is not substantially affected: no matter how much education one has, labor supply will likely be very low if subjected to a 100% tax rate.

In our general formulation, we have also added $\tilde{\mu}\tilde{K}^2$ terms to the loss function. These represent other state-contingent costs of capital income taxation to the social planner. For example, there may be administrative costs to imposing the taxes (or subsidies) since firms may manipulate their accounting to reduce their tax burden. Also, the planner may have some preferred income distribution, experiencing a loss if the tax policy moves the income distribution away from his most preferred point. Since these losses are nonlinear and state-contingent, they are not included in $E\{\tilde{\pi}\tilde{K}\}$. Of course, there may be similar costs associated with labor taxation; if so, we can consider them to be part of the $\tilde{\eta}$ coefficients.

The loss function looks special since it does not allow for interaction effects between capital and labor taxation. These interactions may be substantial: a high labor tax will reduce labor supply and possibly reduce investment earnings. By not allowing for these interactions we are implicitly making a small country assumption. We are also assuming that interest rate effects on labor supply are small. We will deal with these issues below when we examine numerical solutions to a more general model. The important results below are robust to these considerations since the intuition for the basic results is.

In order to keep the exposition clean, we will assume that $\tilde{\pi}$, $\tilde{\mu}$, and $\tilde{\eta}$ are all i.i.d. random variables. Assuming a richer time series structure for these variables will only increase the size of the state variable and add little of immediate interest to our analysis.

This loss function represents many possible social choice mechanisms. It could be the case that there is a representative agent whose preferences are represented by this loss function, or we may be modelling a social planner not part of the society. When we discuss optimal policy we are not distinguishing between these possibilities. Optimality just refers to being on the intertemporal utility possibility frontier.

Given these assumptions, the government's dynamic optimization problem is

$$\text{Min }_{\tilde{K}, \tilde{L}} \ E \left\{ \sum_{t=0}^{\infty} \beta^t \, \mathcal{L}(\tilde{K}_t, \tilde{L}_t) \right\}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} (1+r)^{-t} \, (\tilde{K}_t + \tilde{L}_t - \tilde{g}_t) = 0, \quad \text{a.s.}$$

We will take a dynamic programming approach to this problem. The state of the economy at any time is given by

$$x = (1 \ D \ g)'$$

where D is the stock of debt and obeys the law of motion

$$D_{t+1} = (1+r)D_t + g_t - K_t - L_t$$

To keep things simple, we will assume that $\tilde{\epsilon}$, $\tilde{\pi}$, $\tilde{\mu}$, and $\tilde{\eta}$ can take on only two values:

$$\tilde{\epsilon}_t \in \{\epsilon_1, \epsilon_2\}, \quad \tilde{\pi}_t \in \{\pi_1, \pi_2\}, \quad \tilde{\mu}_t \in \{\mu_1, \mu_2\}, \quad \tilde{\eta}_t \in \{\eta_1, \eta_2\}$$

The economy at any time is described by x, the financial state, and $\omega \in \{1, 2\}$, the shock to the economy in each period which determines the value of ϵ, π, μ, and η.

The government's controls at time $t - 1$ are commitments to the state-contingent tax rates to be in effect at time t, and are represented by the vector u_{t-1}

$$u_{t-1} = (K_{1,t}, \ K_{2,t}, \ L_{1,t}, \ L_{2,t})'$$

where $K_{\omega,t}$ and $L_{\omega,t}$ are period t capital and labor income tax rates if shock $\omega \in \{1, 2\}$ occurs. The law of motion for the state is

$$\tilde{x}_{t+1} = \tilde{A} \, x_t + \tilde{B} \, u_t$$

where, for $\omega \in \{1, 2\}$

$$A_\omega = \begin{pmatrix} 1 & 0 & 0 \\ \epsilon_\omega & r & \rho \\ \epsilon_\omega & 0 & \rho \end{pmatrix}$$

and $\tilde{B} = 0$ except for $B_\omega (2, \omega) = B_\omega (2, \omega + 2) = -1$

Note that the laws of motion for debt have random coefficients. This makes it somewhat different from the usual linear-quadratic planning problem, but introduces only minor complications. The loss function above can be represented as $u'Ru$ where

$$R = \frac{1}{2}\begin{pmatrix} \eta_1 & 0 & 0 & 0 & 0 \\ 0 & \eta_2 & 0 & 0 & 0 \\ 0 & 0 & \mu_1 + \frac{1}{4}\pi_1^2\psi & \frac{1}{4}\pi_1\pi_2\psi & \\ 0 & 0 & \frac{1}{2}\pi_1\pi_2\psi & \mu_2 + \frac{1}{4}\pi_2^2\psi & \end{pmatrix}$$

As with any dynamic problem, we need a terminal condition to rule out Ponzi games by the government. We will assume that the horizon is infinite to keep the convenience of a time autonomous problem, but we will always select the policy which corresponds to the limit of finite-horizon problems as the horizon becomes infinite, and where the loss function in the last period is concave and increasing in D.

This problem is solved in the conventional way. Let $V(D,g)$ be the value function for the government at the end of each period, where D is the end-of-period debt and g is the most recent government expenditure level. Then Bellman's equation for V is

$$V(D,g) = \max_{\tilde{K},\tilde{L}} \beta\, E\left\{\mathcal{L}(\tilde{K}, \tilde{L}) + V(\tilde{D},\tilde{g})\right\}$$

$$\text{s.t.}\quad \tilde{D} = D(1+r) - \tilde{K} - \tilde{L} + \tilde{g}$$

In our notation, this becomes

$$V(x) = \max_u u'\,Ru + E\left\{V(x^+)\right\}$$

$$x^+ = \tilde{A}x + \tilde{B}u$$

We assume that the value function for the government is quadratic in the state, being equal to $x'Px$ for some negative definite P. We determine P by standard methods. If we define $H = \beta\,E\{\tilde{B}'P\tilde{B}\}$ then the first-order conditions imply that $u = Jx$, where

$$J = -(R + H)^{-1}\beta\,E\{\tilde{B}'P\tilde{A}\}$$

The Ricatti equation for P is

$$P = \beta\,E\{\tilde{A}'P\tilde{A}\} + \beta^2\,E\{\tilde{A}'P\tilde{B}\}(R+H)^{-1}E\{\tilde{B}'P\tilde{A}\}$$

As usual, explicit calculation of P is not tractable. However, iteration techniques do allow us to compute P for any collection of values for $\tilde{A}$, $\tilde{B}$, r, and ρ. While a general closed-form solution of this problem is possible (the Riccati equation is only a quartic) it is rather inelegant. To highlight the basic insights we will first examine a simple case, which also generates surprisingly robust results.

Extreme Case

The nature of the optimal policy is easy to determine in one particular case. Suppose that the *ex post* state-contingent costs of capital income taxation are zero, i.e., there are no state-contingent redistributive or administrative costs to capital income taxation. Furthermore, assume that the state-contingent marginal rate of substitution, $\tilde{\pi}$, is unity, and that $\tilde{\eta}$ is constant, $\tilde{\eta} = \eta$. The loss function reduces to

$$\mathcal{L}_t = E_{t-1}\left\{\frac{1}{2}\eta \tilde{L}_t^2\right\} + \frac{1}{2}\psi\, E_{t-1}\left\{\tilde{K}_t\right\}^2$$

We also assume that the social planner's discount rate, β, equals $(1 + r)^{-1}$, a common simplifying assumption. For example, it would hold near a steady state in a representative agent model. We also assume that labor and capital income are constant over time and states.

The first-order condition trading between increasing labor tax revenue in state i in period t and increasing it in all subsequent states in period $t + 1$ is

$$\eta\,\tilde{L}_t = E_t\left\{\eta\,\tilde{L}_{t+1}\right\}$$

implying that $\tilde{L}$ is a random walk. This is the standard result on which most empirical tests of the optimality hypothesis are based. The Barro model is just this special case plus the assumption that there is no capital income.

In period $t - 1$, the first-order condition between $\tilde{L}_t$ and $\tilde{K}_t$ implies that

$$\eta\tilde{L}_t = \psi\, E_{t-1}\{\tilde{K}_t\}$$

Note that the right-hand side is independent of the state at t. Therefore, $\tilde{L} = \tilde{L}_t$. However, $\tilde{L}$ is a random walk, implying $\tilde{L}_t = \bar{L}$.

Next, we find that trading off between $\tilde{L}_t$ and $\tilde{K}_{t+1}$ implies $\eta\,\tilde{L}_t = \psi\, E_t\{\tilde{K}_{t+1}\}$. Since $\tilde{L}_t = \bar{L}$, $E_t\{\tilde{K}_{t+1}\} = \bar{K}$, a constant. Combining these results, we find that

$$\frac{\bar{K}}{\bar{L}} = \frac{\eta}{\psi}$$

In fact, assuming that $\tilde{K}$ is stationary we can explicitly calculate $\tilde{K}_t$

$$\tilde{K}_t = \bar{K} + E_t\left\{PVG_t\right\} - E_{t-1}\left\{PVG_t\right\}$$

Hence, $\tilde{K}_t$ is *white noise*!

Next, if we assume that ψ is nearly infinite, which is implicitly the case in perfect foresight analyses of optimal tax analysis (Judd (1985)), then we see that $\bar{K}$ should be nearly zero. Note that this does not affect the variance of $\tilde{K}$, as expressed above. The assumption of a large savings elasticity does not eliminate capital income taxation, it just reduces the *expected* rate to zero.

The results for this case are particularly striking. While we still find that labor tax rates should follow a random walk, we find that they should actually be constant, and that capital income taxation will be highly variable. The key idea behind these results is that any tax structure must address two problems: raise adequate revenues and deal with shocks to revenue needs. We find that in an optimal income tax scheme these roles are divided between labor and capital income taxation, with labor taxation raising most of the revenues and capital income taxation acting as the shock absorber.

The reasons for this division of roles are clear. The efficiency cost of capital taxation depends on the market value of tax liabilities at the time that the capital is formed, after which it is supplied (almost) independently of the *ex post* net return. Taxation of capital in place is close to a lump-sum tax; therefore, if times are "bad," the efficiency cost of raising capital income tax rates to finance unanticipated expenditures is low. Furthermore, if there is new information indicating that expenditures will be unexpectedly higher for many periods, the efficiency cost of the unexpected tax increases will be zero even if current taxes are raised enough to cover the unanticipated increases in future needs as well as current needs. In fact, the government would not want investors to think that future tax rates will rise to cover the increase in future revenue needs since that would lead to distortions in investment decisions. *Ex ante*, the anticipation of this possibility would discourage investment. However, one could offset this *ex ante* distortion with favorable treatment of capital whenever the government observes unanticipated *decreases* in revenue needs. This possibility surely exists since, by the definition of unanticipated, for every possible future state with an unexpected increase in government revenue needs there is a possible future state with an unexpected decrease in revenue needs.

13

The optimal tax treatment implements the policy we have outlined, calling on capital income sources in times of unanticipated increases in revenue needs and making unanticipated reductions in the tax burden of existing capital when revenues exceed needs. Therefore, the optimal income tax treatment implicitly has an investor sell insurance to the government: when an investor increases the capital stock, he implicitly agrees to absorb unexpected differences between revenues and expenditures through capital income taxation.

This insurance idea is essentially the same as in Lucas and Stokey where they argued that the government should issue debt which redemption is contingent on government expenditure and is financed by labor taxation. We see that when such bonds are absent but capital is present, the optimal tax treatment of capital implicitly implements this state-contingent debt instrument. It is straightforward to show that the introduction of state-contingent debt will not affect any of our results since whatever can be accomplished with state-contingent can be implemented with our state-contingent capital income taxation in this case.

Even when our optimal capital income tax policy does no more than implement state-contingent debt, it is of practical interest. It may seem odd to seriously consider other forms of state-contingent debt since we do not often see such instruments. Some argue that nominal debt implements such risk-sharing, but that can be true only if monetary policy is concerned solely with these budget considerations. Generally, it is unlikely that nominal debt and monetary policy can costlessly fill the shock absorption role, as would true state-contingent securities If there are costs of inflation and price volatility (see Carlton (1982)) then one wants to have low and constant rates of inflation. Therefore, even when the government can issue nominal debt, it may still want to implement some insurance through capital income taxation

It may appear excessive to spend so much time on this extremely special example. Examination of more realistic parametizations of our linear-quadratic specification, however, shows that the general points are robust. Optimal capital income tax rates will generally be more volatile than the optimal tax rates of labor income. In particular, the impulse response patterns are more severe and short-lived for capital income taxation than for labor taxation.

A second, and more important reason for this focus is the finding that these extreme

results appear to continue to hold when we leave linear-quadratic approximations and take a utility function approach, to which we next turn.

General Utility Functions

As always, the linear-quadratic model is only an approximation with many obvious limitations. For example, it assumes that it is possible to raise revenues in excess of income. Our particular specification ignores any interactions between capital and labor supply. Before we can have confidence about our results, we must determine whether they are dependent on these degenerate characteristics.

To test the robustness of our results, we numerically examined a simple finite-horizon model of uncertainty in government expenditure and/or factor productivity. We assumed that factor productivity was exogenous to the agents' actions, essentially making a small-country assumption. Specifically, we assumed that utility over consumption and labor supply was additively separable, and that both the elasticity of labor supply and the elasticity of intertemporal substitution were constant. In this model, we allow fluctuations in the wage and interest rates, the marginal value of government consumption (also given a constant elasticity, additively separable specification), and the level of exogenous, necessary government expenditure. We permitted independent linear taxation of both labor and interest income. We computed the state-contingent factor tax rates which maximize the utility of the representative agent subject to the constraint that all revenues go to finance government consumption.

Timing assumptions were made to keep the problem from being trivial. We assumed that each agent began with some wealth, but that in the first period only labor income could be taxed, thereby avoiding a capital levy which would provide all necessary tax revenue. Since pure nondistorting capital levies can occur only in the first period, a time which has little if any economic content, ruling out capital levies in this way allows us to focus on the long-run character of tax policy. After the first period, both capital and labor income are taxed. All state-contingent tax rates are known at the beginning.

Since this is a representative agent model, we are implicitly assuming that the social and market rates of discount are the same and that there are no redistributive motives. Also, we

15

abstracted from state-contingent administrative costs. These assumptions are qualitatively the same as in our special case above where $(1 + r)\beta = 1$ and $\bar{\mu} = 0$.

The results were striking. Strong nonconvexities make numerical computation of the optimal tax policy difficult, but as long as the share of GNP going to government consumption is not unrealistically large, we were able to compute the optimal tax policies. In all cases, the wage tax was *constant* after an initial period. Also, in the second period there was some taxation of capital income (an attempt on the part of the solution to impose something close to the capital levy not allowed in the first period), but thereafter capital income tax rates fluctuated in perfect step with news about government consumption, generating revenues with zero *ex ante* market value. These are exactly the same results which we computed in our seemingly special extreme example.

The pattern of capital tax rates also fit the "good news, bad news" rule discussed above. Capital income tax rates were high whenever there was new information indicating that the share of GNP going to government consumption was going to rise. This included cases where a negative shock to the marginal product of capital lead to an increase in the capital income tax rate. This conforms to the general intuition: a shock to revenue needs relative to GNP must be absorbed somewhere, and the best absorber is one which has a low *ex post* welfare cost.

Given the similarities between the special case and these nonlinear cases, it is not too surprising that the results are the same. These exercises do show, however, that the unattractive features of the linear-quadratic approximation do not drive the main implications. The absence of saving-leisure interactions was apparently unimportant since our nonlinear examples allow for such interaction and still get the same result. The same can be said for the myopia implicit in assuming that the efficiency cost of capital income taxation depends only on the immediately succeeding period's tax rates. Therefore, these exercises lend strong support for the general qualitative conclusions of the linear-quadratic specification; in particular, optimal taxation will have capital taxation fill a shock absorber role with labor being the primary source for tax revenue on average.

Tax Policy Implications

At first blush, our results appear to have no chance of describing actual policy. For example, the corporate income tax rate has changed only a few times in the last 40 years, mostly "fluctuating" between .48 and .52. To demonstrate that it is plausible that much of this volatility is going on, we next relate our general results to specific provisions of the tax code.

The first source of fluctuations in capital income taxation is inflation. Inflation has a strong impact on the taxes businesses pay. For example, depreciation allowances are nominal; therefore, high rates of inflation reduce the value of those deductions and raise the *ex post* tax rate. Furthermore, inflationary shocks reallocate real income between bondholders and equityholders, also resulting in revenue changes since debt and equity are taxed differently. These effects are substantial, as shown in Feldstein and Summers (1979). The nonindexation of capital gains is also a major source of revenue, and is also criticized in Feldstein et al. ().

An examination of the stock of nominal tax shields shows that the potential magnitude of the nonindexation effects is certainly significant. Gross business fixed investment is about ten per cent of GNP, with thirty per cent of investment being structures and seventy per cent being durable equipment. Since structures are written off by about four per cent per year, and equipment around fourteen per cent per year, if we assume an inflation rate of six per cent then the stock of unused tax shields for equipment equals 35% of GNP and for structures equals 30% of GNP. When the corporate tax rate equalled .5, this implied that the real value of the nominal liabilities implicit in depreciation tax shields was a third of GNP, equalling the current ratio between GNP and privately held debt. At a corporate tax rate of .34, these liabilities equal almost a quarter of GNP. While this calculation is valid for investment in corporations wholly owned by pension funds, the considerations of investment held in proprietorships and partnerships and the personal taxation of dividends adjust the calculations in contrary directions. In any case, we see that the nominal liabilities implicit in nominal depreciation roughly equals (and, in the recent past, probably exceeded) the stock of nominal privately-held debt.

The impact of inflation on corporate income taxation has been extensively discussed. Feldstein and Summers decompose the impact of inflation into depreciation and interest

components, and demonstrate that inflation substantially increases corporate income taxation. For the years 1954 through 1977 they calculate the cumulative effect of inflation on corporate income tax payments. For example, corporate income taxation was almost thirty billion dollars greater in 1974 due to inflation's impact on depreciation allowances and that almost all corporate income taxation in 1974 was due to inflationary effects. They did not distinguish between anticipated and unanticipated inflation. Since most inflation in the 1970's was unanticipated, much of the excess taxation was unanticipated. Their calculations show that the excess taxation was small in the 1950's and 1960's when inflationary shocks were much less. Therefore, the excess taxation which arises from inflation is substantial, but much of it was unanticipated. Their empirical findings are consistent with the optimality hypothesis, finding that inflation is a potent tool for raising tax revenue, but is used sparingly on average and heavily only during periods of poor macroeconomic performance.

In light of our results, however, it is unclear whether these nominal features of the tax code are undesirable. To the extent that inflation-induced shocks to capital income taxation are appropriately correlated with "bad" news, various "bad" features of tax policy and policymaking habits could be helping the tax system implement the "shock absorption" features of the optimal tax code. For example, if there is an adverse productivity shock but no reduction in government consumption, then some tax rate must be increased. However, (assuming the quantity theory of money) if the Fed did not reduce the money supply then there would be an increase in the price level. Since depreciation allowances are not indexed, their real value would decline, effectively increasing real government revenues. Furthermore, the nominal treatment of capital gains taxation would also lead to an increase in tax revenue. Therefore, with nominal depreciation allowances we have an automatic increase in taxation when the economy experiences an adverse productivity shock. Symmetrically, there is a reduction in capital income taxation when there is a positive productivity shock. These arguments show that various nominal aspects of the tax code may be consistent with optimality.

Recent years has seem much advocacy of replacing the current income tax with a consumption tax (e.g., Hall and Rabushka (1983), McLure (1989), Bradford (1986)). These

arguments are supported in the theoretical literature. Atkinson and Sandmo (1980) examined an overlapping generation model, showing that interest taxation should be zero under plausible separability conditions. Judd (1985) analyzed optimal tax policy in a model with multiple types of infinitely-lived agents with Uzawa-type utility functions, showing that any Pareto social welfare objective will impose a zero tax rate on capital income asymptotically if it imposes a convergent policy.

However, this model implies that income taxation strictly dominates consumption taxation if there is uncertainty about factor productivity or government expenditure. The superior performance of an income tax arises since fluctuations in the optimal capital income tax are engineered to provide insurance for the government, allowing it to keep labor tax distortions smooth. If (proportional) consumption taxation were the only tax instrument, than the optimal policy would adjust consumption tax rates in response to expenditure shocks, inducing variance in labor tax distortions and increasing the total cost of tax distortions. Since one of the options in our policy space, but not chosen, is to allow labor and capital income tax rates to move as they would under consumption taxation, we find that income taxation strictly dominates consumption taxation.

Another interesting aspect of optimal tax policy is how it reacts to "news." Any time there is a decision to "reform" the tax system, there is a debate as to whether income tax rates should be cut or investment incentives should be increased. The difference is that cuts in tax rates generates a lump-sum rebate to the owners of old capital, whereas investment incentives are targeted to encourage new investment. The differences may be quite dramatic; for example, Auerbach and Kotlikoff (), and Judd (1987) have shown in both overlapping generations and representative agent contexts that investment tax credits may indeed be self-financing, whereas there are no plausible theoretical examples of such for tax rate cuts. These results appear to argue that tax reforms should be focussed on investment incentives. However, in our model we find that tax reforms motivated by new 'information" about expenditure needs or factor productivity should lead to relief for old capital, and only moderate investment incentives for new capital. This nonintuitive result is due to the insurance nature of the optimal capital income tax – if the government has good

19

luck than it should pay the owners of old capital since they bore the risks associated with the chance that bad luck could have happened instead.

The impact of inflation (or, to be more precise, inflationary expectations) on the cost of capital has been recognized in both academic and policymaking circles. Congress has often addressed the adverse effect inflation has on investment incentives. However, there has been a tendency to counter inflationary expectations with investment incentives instead of indexing, a habit which is often criticized in the academic literature. Such a policy is also consistent with optimality since it reduces the *ex ante* tax rates without reducing the ability of the tax system to absorb shocks.

This also implies that the variability of *ex post* tax rates should be high. These results are consistent with the argument (see, e.g., Bulow and Summers (1984)) that the cost-of-capital formula should be adjusted for inflationary impacts on depreciation allowances and inflationary shocks to debt vs. equity allocation of corporate income, but only to the extent which that inflation is correlated to real shocks. Furthermore, those adjustments do not indicate inefficiencies in tax policy and do not argue for indexation.

Our model does not contradict recomendations for indexing and consumption taxation as much as supplement them. These earlier theoretical and policy discussions focussed on a deterministic world, analyzing the best way to raise revenue on average and ignoring the problem of how to absorb shocks to the government budget constraint. The *ex ante*, i.e. average, tax distortion imposed on capital income in our model is generally low, related to differences between the planner's discount rate and the interest rate, results which are consistent with the earlier deterministic analyses. The usefulness of capital income taxation lies with its ability to absorb shocks without generating much inefficiency. Indexation and consumption taxation would be fine if alternative shock absorbers were also introduced.

In discussing these issues, we must note that we are not saying that any use of indexing is bad nor that every twitch in monetary policy has been optimal. The main message is two-fold. First, the absence of indexing is not necessarily suboptimal, as argued forcefully by many, and a rush to introduce indexing into the U.S. tax code may reduce its ability to deal with various shocks. Second, focussing on the nominal aspects of the U.S. tax code and

20

the potential importance of monetary policy in implementing an optimal tax policy shows that any study of optimal monetary policy must take into account the tax interactions. To that end, we next discuss connections with related monetary policy analyses.

Monetary Policy Implications

The arguments above discussed the role of inflation in affecting real tax liabilities, in-dicating that inflation should also be considered a part of tax policy. More generally, this implies that any analyses of optimal monetary policy should consider *all* budgetary impli-cations of inflation, not just seigniorage. This is not just a debating point, but strongly supported by quantitative considerations. Recall that both Federal nominal debt and capi-tal consumption allowances each exceed ten per cent of GNP, which currently is about 4.5 trillion dollars. Since the monetary base is roughly $200,000,000,000, it is small relative to the net outstanding nominal debt of the government and the stock of unused depreciation allowances. When we properly include the latter components, we find that the monetary base is less than a fifth of the Federal governments total nominal liabilities. Below we shall see that corporate debt also must be included when considering the impact of inflation on the government's budget constraint as long as there is a substantial gap between the tax rates of corporations and debt-holders.

These observations have two implications for monetary analysis. First, monetary policy discussions which focus on seigniorage miss the bulk of its effects on the government budget constraint. Second, inflation should be explicitly treated as a tax on a *stock* of liabilities. This last observation is most important for our purposes since it indicates the appropriate modelling choice. Monetary analysis has taken two modelling approaches yielding dramat-ically different results. Mankiw assumed that the social and private costs of inflation were solely a function of the contemporaneous rate of inflation. This is akin to assuming that individuals rent money during a period, paying a higher implicit rental fee in periods of high inflation. This specification yielded the same random walk characterization for optimal inflation policy as in Barro. This result contrasted strongly with Turnovsky and Brock, who instead chose to model explicitly the money demand process (via a transaction cost specifica-tion), making the more realistic assumption that the private sector held the stock of money

at all times, and solved for the optimal monetary policy. They found that the optimal policy involved a high initial rate of inflation, followed by low inflation. Furthermore, they found that the optimal taxation of capital income involved the same kind of policy, the similarity being intuitive because both involved the taxation of a stock.

Since different monetary theories may yield different money demand specifications, it is difficult to choose which of these models should be used for an analysis of optimal monetary policy. However, we do not need to make such a choice since it is clear that the great bulk of the government's nominal liabilities are stocks, making the Turnovsky and Brock approach substantially more sensible. This turns Mankiw's empirical conclusions upside-down. It is obvious that once we add stochastic elements to a Turnovsky and Brock analysis, the optimal inflation rate process will be white noise, not a random walk. Therefore, Mankiw's acceptance of the random walk specification is a *rejection* of optimal monetary policy.

The use of inflation in implementing tax policy also implies that one should consider fiscal policy reactions to inflationary shocks. For example, while an inflation shock may be used to impose a tax on the holders of nominal assets, continued inflation could raise the cost of capital and reduce the incentive to invest. There are two ways to correct this problem. One would be to bring the inflation rate down to some desirable permanent level, as a white noise specification would do. If there were some considerations which would make it difficult to do this immediately, then an alternative would be, for example, to increase the invesment tax credit Both responses would bring the cost of capital down to the desired permanent level

These considerations show that, when we integrate the simple models typically used for optimal policy analysis, there are no predictions for the separate instruments of tax policy since they are redundant. In particular, we can not predict that inflation will be a random walk, nor can we predict that the income tax rate will be a random walk. The only predictions of theories which focus on basic factor supply and money demand considerations concern the overall net tax rate

No Commitment: Markov Perfect Equilibrium Policy

Our optimal taxation analysis used a dynamic programming approach which was equiv-

22

alent to assuming that the optimal state-contingent policy is formed at some initial time with no changes permitted later. Optimal taxation of capital income is a well-known example of dynamic inconsistency, i.e., if a government is permitted at some future time to make a permanent change in its policy, it will (see Chari,Kehoe, and Prescott, Kydland and Prescott, Turnovsky and Brock, Sargent). For example, under our optimal policy, when the government is confronted with bad "news" it taxes capital heavily, essentially making the following claim:

> "Today, we need extra revenue; therefore we will heavily tax current
> capital income accruing to investments made in the past; but this should
> not deter you from making investments today because we promise that the
> expected taxes on future income arising from today's investment will be low.
> In fact, if we get lucky and receive more revenues than we need we will give
> the excess to you.

Any such speech would be met with skepticism. If a good state of the world would occur in the next period, the government be tempted to use the extra revenue to permanently reduce labor income taxation instead of giving it to the owners of old capital. Reducing labor income tax rates permanently will reduce the present value tax distortions, whereas transfers to capital will have no such salutary effects on economic efficiency. Therefore, if the government were not tied by its previous speeches, it would renege on its promise and not make the rebates to capital. In fact, if the government could rewrite the tax law, it would impose a heavy tax burden on the owners of capital since capital cannot go anywhere in the short-run.

The inconsistency here is due to a difference in short- and long-run elasticities of capital supply. *Ex ante*, the elasticity of capital supply is moderate, so policymakers want to keep the expected capital income tax rate low. *Ex post*, the elasticity of supply is essentially zero, making capital income a low-distortion source of revenue.

Casual examination of our political institutions indicates that it is unrealistic to assume that a government can commit to future tax policies. By affecting the level of debt (which we assume cannot be totally renounced at the margin by future governments) a current

23

government can affect future decisions. But future Congresses and Presidents have free reign to change tax rates and depreciation rules. Furthermore, precommitment to a tax policy is equivalent to precommitment to a monetary policy since the tax code has many nominal features. We should therefore examine the range of possible outcomes when governments lack an explicit precommitment technology.

We next how governments will act when they can make neither implicit nor explicit credible commitments. Governments will interact since current policy will affect future policies through the debt it hands to the future governments; we are assuming that the obligations implicit in debt are credible, secured, for example, by some constitutional provision. Also, current policymakers will care about how future governments act. Therefore, both the current government and private agents will make predictions about future policy, and how it reacts to current choices. This is important in our model since investors will make predictions about how tomorrow's government will treat income from capital.

Given these predictions, today's government sets only *current* tax rates on labor and capital income, having current information about government expenditure needs and business cycle shocks. Today's tax decisions depend only on the current level of debt and "real" information. We call the resulting sequence of decisions a **Markov perfect equilibrium**.

Some discussion of the terminology is in order here. We have chosen not to use the term "consistent equilibrium" since it is incoherent. The predicate "dynamic consistency" applies to solutions of optimal control problems, not to Nash equilibria of games. Some have argued that the concept of "consistency" is equivalent to the notion of subgame perfection (see, e.g., footnote 40, page 621 in Blanchard and Fischer.) That is clearly false since there are many examples where subgame perfect equilibria support the dynamically inconsistent optimal precommitment policy; in fact, the policy reputation literature is devoted to establishing this. Therefore, we will use the concept "Markov perfect equlibrium" as defined in Bernheim and Ray. The key idea is that current policy depends only on the current "real" conditions, independent of "unreal" aspects of the past, such as commitments.

We can compute the equilibrium sequence by adapting dynamic programming. Let $V(D, g, \omega)$ is the current value of future tax distortion losses if the beginning-of-period in-

debtedness is D, the previous period's government expenditure was g, and ω is the current shock. If the current shock is ω, and the debt handed to the next government is D, the loss function *perceived* by today's government as it chooses K and L is

$$\frac{1}{2} \, \eta_\omega \, L^2 + \frac{1}{2} \, \mu_\omega \, K^2 + \beta E_\omega \left\{ V(D, g, \omega') \right\}$$

Note the absence of the *ex ante* market value distortion term – that is a sunk cost.

Our basic result is that optimality for the current government implies that a Markov perfect equilibrium choices for K and L must satisfy

$$\eta_\omega \, L = \mu_\omega \, K = E_\omega \left\{ V_D(D, g, \omega') \right\}$$

This implies that if $\tilde{\eta} \approx \eta$, $\tilde{\mu} \approx \mu$, then $\tilde{L}$ and $\tilde{K}$ follow the "same" stochastic process. We shall establish this formally by computing the Markov perfect equilibrium.

We assume that the government has observes ω at the beginning of each period, thereby knowing the current level of government consumption and the current parameters for the loss function when it sets current taxes. Furthermore, the current government believes that tomorrow's government will follow the policy $u = Jx$, where $x \equiv (1, D, g)$. Specifically, if tomorrow's government finds itself in financial state x and shock ω, it will set u equal to $J_\omega x$; therefore, $J \equiv (J_1', \quad J_2')'$. These beliefs are important to the current government since expectations about the next government's treatment of capital income affects current investment decisions and the current loss. However, the next government will not consider this effect of their decisions since today's investment decisions are sunk when tomorrow's government chooses tomorrow's tax rates. Other than that, however, losses due to tax policy are viewed the same by the current and the next government. Therefore, the problem the current government faces if the current state is $\omega \epsilon \{1, 2\}$, is given by

$$\min_u \quad u' R_\omega \, u + E \left\{ (J \, (A_\omega x + Bu))' \Phi J \, (A_\omega x + Bu) + \beta V(A_\omega x + Bu, \omega') \right\}$$

where $A_\omega x + Bu$ is the law of motion contingent on the current state being state ω.

If we assume that $V(x, \omega) = x' \, W_\omega \, x$, then the government's problem is

$$\min_u \quad u' R_\omega \, u + E \left\{ (A_\omega x + Bu)' \, (J'\Phi J + \beta W_{\omega'}) \, (A_\omega x + Bu) \right\}$$

The first-order condition is

$$R_\omega u + B' \left(J'\Phi J + \beta \bar{W} \right) \left(A_\omega x + Bu \right) = 0$$

where $\bar{W} \equiv E\left\{W_\omega\right\}$. The optimal strategy is

$$u = -(R_\omega + B' \left(J'\Phi J + \beta \bar{W} \right) B)^{-1} \left(J'\Phi J + \beta \bar{W} \right) A_\omega x.$$

$$\equiv J_\omega x$$

Combining the definition of the value function and the computation of the optimal strategies for each state gives us a system of equations for the W_ω and the J_ω:

$$W_\omega = J'_\omega R_\omega J_\omega + E\left\{ (A_\omega + B J_\omega)' \left(J'\Phi J + \beta W_{\omega'} \right) (A_\omega + B J_\omega) \right\}$$

$$J_\omega = -(R_\omega + B'(J'\Phi J + \beta \bar{W})B)^{-1} \left(J'\Phi J + \beta \bar{W} \right) A_\omega$$

Again we can compute the solution by iteration. This demonstrates the existence of a Markov perfect equilibrium sequence of tax decisions.

The main important result is evident from the first-order condition where we see that each government is myopic about tax decisions, considering only the immediate effects of its tax decisions. In particular, labor and capital tax rates are treated differently only in so far as they have different short-run inefficiencies. In our model this implies that they are both affinely related to the shadow price of debt. In particular, their laws of motion are qualitatively the same; if one is a random walk then so is the other, and if one is AR(1), then so is the other with the same autoregressive parameter. This contrasts strongly with the joint law for taxation under the optimal program.

The other interesting result from examination of many examples is that the government essentially endows government expenditure: it raises assets to a level sufficient to finance mean expenditures, imposing taxes on labor and capital only to finance deviations from the mean. Therefore, average tax rates are low in the long run, but very high initially when the endowment is built up. This financing pattern is much different from the optimal one, which generally has debt following a random walk.

It may appear that we have given a strong case against the hypothesis that tax policy is optimal. However, we have only computed one subgame perfect equilibrium. We next consider another family of equilibria.

No Commitment: Reputation Equilibria

The fact that the Markov perfect equilibria do not implement the optimal policy does not imply that we should conclude that it is not possible to observe the optimal policy. Perhaps there is among policymakers a strong ethic for pursuing the dynamically optimal policy. One solution to the problem of inconsistency (discussed in Rogoff (1985)) is to select policymakers that have a strong personal preference for the optimal policy. There are many political, noneconomic arguments which could be incorporated into our model which would lead to implementation of the optimal policy. Therefore, we cannot rule out the optimal policy as a possible outcome.

Even if we don't add extra political considerations, the absence of a formal precommitment technology for tax policy does not imply that the optimal tax policy cannot be an outcome of the game studied above. We shall next show that optimal tax policies can result as a subgame-perfect outcome if we use standard "reputation" constructions. This will then provide us with a much fuller range of equilibrium possibilities and allow us to consider again the optimality hypothesis.

Construction of a reputation equilibrium supporting the dynamic optimum is standard Suppose that all governments know the optimal state-contingent tax policy. Suppose that each government obeys the following rule: if all previous governments have followed the optimal policy, then it will also; but if any previous government has deviated from the optimal rule, then it will engage in the no-memory, no-commitment, subgame-perfect tax policy. Can this be a subgame-perfect Nash equilibrium strategy profile? It is straightforward to show that if the discount factor is sufficiently close to unity and if μ is not zero, then that strategy profile is subgame-perfect and Nash. The reader is referred to Chari, Kehoe, and Prescott for a formal development of these ideas in very similar models.

Theorem: If each government cares sufficiently about the actions of later governments, does

not have access to a capital levy, and cannot renounce its debt, then the first-best optimal policy is the outcome of some subgame-perfect equilibrium.

The main importance of these arguments is that optimal policy is a possible equilibrium outcome even if there were no explicit institutions whereby an initial government can determine future policy. It is not necessarily the case that policy will fall into the no-commitment, no-memory, Markov perfect equilibrium. We do not intend here to describe the full range of Markov perfect equilibria, only to argue that reputation considerations indicate that the optimal policy cannot be ruled out as a possible political outcome. Therefore, we next move to the empirical implications of our model.

Empirical Tests for Optimality

The multiplicity of possible outcomes implies that theory alone does not make any strong predictions about the nature of tax policy, especially in the absence of formal precommitments. Empirical analysis is therefore necessary to determine which among the collection of possible equilibrium outcomes most closely fits the data.

Since we can compute both the optimal tax policy and the Markov perfect equilibrium for the linear-quadratic loss function, and both are possible outcomes of subgame perfect equilibria, it is natural to ask if there are differences which can serve as a basis for an empirical test. Earlier models focussed on labor taxation where optimal policy is dynamically consistent, or, as in Lucas and Stokey, where consistency problems could be dealt with through debt management. In either case, existing institutions were able to implement optimal policy.

Our analysis of optimal and Markov perfect tax policies indicates empirically detectable differences between optimal and alternative outcomes. However, we must be careful in deciding on which differences to focus. Initially it would appear that there are many aspects which could be used as a basis for an empirical test. For example, the prediction of a low *ex ante* tax rate on capital from our optimality theory would be something which we should be able to test. However, some of the quantitative properties of our solutions are not empirically useful because of obvious nonrobustness of the model and the lack of precise information

concerning structural parameters of the economy. For example, the prediction of a zero *ex ante* capital income tax rate in our numerical simulations is probably due to the separable demand system, as in the deterministic analysis by Atkinson and Sandmo. Because of its dependence on the separability specification, it is not possible to base an inference about optimality on tax rate levels without also making unappealing joint hypotheses concerning the structure of tastes. The same considerations also imply that we cannot use *ex ante* tax rate data to infer anything about whether tax rates fit the predictions of the Markov perfect equilibrium

Even we if allowed a flexible utility specification, it is unlikely we could construct an informative test based on tax rate levels. Such tests would proceed by comparing the empirical characteristics of factor supply with our formulas for optimal and Markov perfect tax policies, attempting to determine whether the empirically observed combinations of tax rate levels and factor supply characteristics were most consistent with optimality or myopia. This is unlikely to be fruitful since the estimates of the crucial factor supply characteristics have insufficient precision for these tests. Because of the dynamics of our problem, we would also have to be careful about timing considerations when picking among the several possible factor supply elasticities

Fortunately there are qualitative features of the variance-covariance structures of the alternative outcomes which are arguably robust to most important considerations ignored in our simple model. This key difference is that capital and labor tax rates follow qualitatively different processes if tax rates are set optimally, whereas they follow similar processes, with capital income taxation being more volatile if tax rates are set myopically. This difference would likely continue in models with, for example, more general technology and taste specifications. Therefore, by focussing on the qualitative time series structures we can examine the optimality hypothesis without making unappealing auxiliary assumptions.

More specifically, under the usual maintained hypothesis that $\bar{\eta}$ and $\bar{\mu}$ are deterministic, the wage and capital income tax processes should both be similar in the Markov perfect equilibrium since they both follow the shadow price of debt. In sharp contrast, optimal outcomes have the labor and capital income tax rates follow substantially different processes,

with labor tax rates being substantially more persistent. These differences lead to a three-way breakdown of the possibilities

Optimality: "τ_K shocks are less persistent, more volatile than τ_L shocks"

Markov perfect: "$\tilde{\tau}_K$, $\tilde{\tau}_L$ processes are the same"

Model failure: "τ_L shocks are less persistent, more volatile than τ_K shocks"

Before proceeding with our tests, we should discuss the old evidence on optimality. Under the random walk hypothesis, the evidence in favor of optimality has been fairly supportive. While the random walk hypothesis has not passed all statistical tests, all studies have found strong persistence in tax rates, implying an important smoothing role for debt. Bizer and Durlauf have pointed out that the random walk hypothesis is, when taken literally, ludicrous since it would imply that either the tax rate is a constant or that the asymptotic variance is infinite. If the tax rate were constant then revenues could not match uncertain expenditures and if the asymptotic variance were infinite then tax rates will surely exceed 100% at some time. Therefore, rejection of the pure random walk hypothesis is not surprising. However, it is also clear that any reasonable adjustment of the theory (in particular, assuming a loss function which is infinite at 100% tax rates) will imply substantial smoothing and strong persistence in tax rates.

Little attention has been paid to heterogeneity in the tax code. The tax rate is generally taken to be the ratio between government revenue and national income, ignoring distinctions among commodity taxation, tariffs (both of which were important sources prior to World War II and the dominant sources prior to World War I), capital income and labor income taxation. The earlier arguments showing that the optimal treatment of labor and income taxation differ substantially also can be used to show that optimal tariff and commodity tax rates are not random walks. Suppose that all revenues are raised through tariffs on imported goods and that there are adjustment costs associated with shifts of factors across sectors. In the short run, the only distortions from an unexpected increase in tariff rates is the reduction in demand by consumers for importables. However, in the long-run there will be a movement of factors from the export sector to the import-competing sector. The

short-run distortion is therefore less than the long-run distortion and the optimal reaction to an expenditure shock would impose a large immediate increase in the tax rate followed by a smaller persistent increase. (See Sargent for a formal analysis of a similar problem.) If the collection of commodities which were subject to taxation does not include all goods then similar arguments apply for optimal dynamic commodity taxation.

Skinner did distinguish between wage and capital income tax rates, using the Joines and Seater tax rates for capital income. He estimated a vector autoregression for expenditures, debt, earnings, the real interest rate, wage tax rates, interest tax rates, finding several important relations. First, the interest tax rate is highly persistent, with its own coefficient being .987. Second, the wage tax rate was also persistent, but with only an own coefficient of .613. Third, the correlations among the residuals also revealed interesting information. Innovations in expenditures were strongly related to both the wage tax (.533) and the interest tax (.416). One of the odd results was that unexpected falls in the real interest rate was associated with unexpected increases in the interest tax rate. This is predicted by our theory if the falls in the real interest rate are due to negative shocks to the marginal product of capital.

The similarity between the wage and interest tax rate processes as estimated in previous work appears to indicate acceptance of the no-memory Markov perfect equilibrium hypothesis: both processes have statistically significant autoregression. In fact, innovations in the interest tax rate appear to be more persistent than innovations in the wage tax. Such a finding would strongly indicate that the institutional failures arising from the absence of precommitment technologies are substantial and that institutional innovations making precommitments possible would be of substantial value.

However, this conclusion is not warranted since accounting problems with the standard approaches to computing *ex post* capital income tax rates render the tax rates used in previous tests inappropriate for our purposes. The critical fact is that conventional computation of capital tax rates inappropriately smooths the *ex post* tax rate. This is seen clearly when we examine how these definitions deal with a simple example. Suppose that an investment project requires a $1,000,000 initial investment, generates $200,000 in annual cash flow for

31

ten years, is depreciated on a straight-line basis for ten years, and that the safe interest rate is zero; these assumptions are chosen to yield simple calculations, not to be precisely realistic. Suppose that the corporation is taxed at 50%. If there is no inflation than the tax payment is $50,000 per year, which is 50% of the $100,000 in true income ($200,000 gross income minus $100,000 economic depreciation). In this case the nominal tax rate equals the effective tax rate in each year of the investment.

Now suppose that immediately after the investment is put in place that there is a ten per cent increase in the price level, but that there is no further inflation after that. Since the depreciation deductions are nominal, their real value falls to $90,000 in each year. However, since the price level rises uniformly, the real gross cash flow remains $200,000. Therefore, the real tax liability increases to $55,000 each year, which is 55% of true economic income. It appears that the effective tax rate faced by the firm is not the nominal 50% rate, but rather the 55% rate which equals the ratio between real tax payments and real income.

This is clearly the wrong way to view this example if one wants to calculate tax rates relevant for investment incentives. Since the inflation is over by year two, if the firm were to invest in another project then the true effective tax rate on that investment equals 50%. The fact that the corporation is paying taxes at an average rate of 55% on an old investment is irrelevant for new investment decisions.

This example shows that the correct way to compute the tax rate faced by the firm is to take into account any unexpected changes in the market value of the firms assets, particularly the unused depreciation allowances. The true effect of the ten per cent inflation shock is to reduce the value of all future depreciation allowances by $100,000, from $1,000,000 to $900,000 and the increase in future tax payments is $50,000. Even if the extra taxes are paid in the future, the impact on the value of the firm is experienced immediately in rational markets, resulting in an immediate fall in the investors' budget constraints. Rational markets would also immediately recognize the favorable change in the government's financial condition. Therefore, the true impact of taxation on both the firm and the government in the first year is the $55,000 paid plus the $45,000 fall in the value of future tax shields, totalling $100,000. The fact that the government does not collect on all of this until later

years is irrelevant; it is as if the government sent the firm a tax bill of $100,000 but then agreed that $45,000 of the bill could be paid later. When viewed in this way, the true *ex post* tax rate in year one is 100%! In later years, there are no further revaluation of the tax shields and the true effective tax rate is only 50%.

We next construct an effective *ex post* tax rate which incorporates these capitalization effects. In constructing this rate, we will make many special assumptions, making choices which allow us to use standard data and proceed in ways similar to earlier studies. There are many alternative definitions which would also be useful. However, the critical element for our analysis is the variance-covariance structure of the time series of tax rates, and the critical distinguising feature is the attention we pay to capitalization effects of inflation on the market value of unused tax shields. It is doubtful that the special assumptions made here generate any particular bias relative to the issues we are studying.

Let θ be the current investment tax credit, θ' next year's ITC, τ tomorrow's tax rate, d the rate of depreciation allowed on today's investment, δ the rate of economic depreciation π_c the rate of inflation on GNP, π_I the rate of inflation of investment goods, and r the marginal product of capital. Let P be the present value of depreciation allowances on $1 of new investment. Suppose one buys $1 of new capital today, resulting in a cash outflow of $1 - \theta$ dollars. After one year, the real gross income will be r. In inflated dollars and after paying corporate income tax, $(1 + \pi_c)r(1 - \tau) + \tau d$ is net cash flow including depreciation credits of τd. After economic depreciation, $1 - \delta$ units of capital remains. We need to value it. Assuming that it is a perfect substitute for new capital, then we can value it by comparisons with new capital. The net cost of a new unit of capital is purchase price, $1 + \pi_i$, minus the value of investment tax credits, $\theta'\pi_I$, and the present value of future depreciation allowance θ, $P(1 + \pi_I)$.

Suppose the old asset is sold *but with no change in tax status*. The $1 - \delta$ units of capital would bring with it $P\tau(1 - d)$ dollars of unused tax shields. Therefore, its market value would be

$$(1 - \delta)\,(1 + \pi_i) + P\tau(1 - d) - P\tau(1 + \pi_I)\,(1 - \delta)$$

Therefore, an expenditure of $1 - \theta$ dollars today will yield a total real return of

$$(1 + \pi_c)^{-1} \left(r(1 + \pi_c)(1 - \tau') + d\tau' + (1 - \delta)(1 + \pi_I)(1 \quad \tau P) + P\tau(1 - d) - \theta'(1 \quad \delta)(1 + \pi_I) \right)$$

We define the effective tax rate tomorrow, $\hat{\tau}$, to be that rate such that the after tax net return is $(r - \delta)(1 \quad \hat{\tau})$. Hence, $\hat{\tau}_t$ is defined by

$$V_{t+1} = (1 - \delta)(1 + \pi_{I,t})(1 - \tau_{t+1}P_{t+1}) + P_{t+1}\tau_{t+1}(1 - d_t) - \theta_{t+1}(1 - \delta)(1 + \pi_I))$$

$$X_{t+1} = \frac{r(1 + \pi_{c,t})(1 - \tau_{t+1}) + d_t\tau_{t+1} + V_{t+1}}{(1 + \pi_{c,t})(1 - \theta_t)}$$

$$\hat{\tau}_{t+1} = 1 - (r - \delta)^{-1}(X_{t+1} \quad 1)$$

The construction of our effective tax rate includes some assumptions concerning asset transactions. Our valuation of the asset is really the valuation of the associated financial assets, and is implicitly associated with trading in the financial asset, not the physical asset. The difference lies in the tax treatment of asset transactions. If a used machine is sold, the new owner receives an investment tax credit (under the Long Amendment) and begins depreciation of the asset at its new value, and the old owner pays taxes on any profit and pays some recapture of tax credits. On the other hand, trade in financial assets involves no such tax consequences. In the case of commerical real estate, there is substantial churning of assets, encouraged by tax considerations. In this exercise, we ignore such considerations, a view appropriate for equipment and industrial structures.

If *ex post* tax rates are calculated in this way, then the resulting tax rate series indicates the true *ex post* impact of taxation. The resulting tax rates are ones which belong in prediction formulas on the right-hand side of investment equations. Tax rates such as those computed by Seater, Joines, Feldstein et al. all introduce excess smoothing relative to the true *ex post* impact. They are also clearly not appropriate to use for the cost of capital in investment equations. It is not surprising, therefore, that Feldstein, and Feldstein and Jun, and Shapiro, all show that the impact of tax rates on investment is weak.

Using our *ex post* capital income tax rate formula we can investigate the post-WWII U.S. experience. Our data covers 1950-1980, using depreciation rules, term structure of gov't bonds, corporate income tax rates, CPI investment goods inflation rates, and investment tax credit rates in effect during that period.

In computing the *ex post* effective tax rates, we need to make an assumption concerning the pre-tax return on investment. One choice would be to focus on marginal projects, that is, those investments which have a zero net return after taxation. This would require assumptions about the *ex ante* effective tax rates faced by investors. This tax rate is easily computed in a deterministic world, but is much more difficult to compute in a stochastic world. To do so would require much information concerning investor beliefs concerning the variance and covariance of taxation, inflation, and real returns. These issues are discussed in Bulow and Summers. No systematic attempt has been made to incorporate these elements into calculation of *ex ante* tax rates; we will not do so here.

In this section we will focus solely on the corporate portion of capital income taxation. We do this because integrating the corporate and personal tax structures to derive a total tax rate on capital income is very difficult. These issues will be discussed in the next section where we argue that such an integration will not likely reverse the results.

Because of the difficulties in defining the marginal project, we will instead focus on the taxation of projects with a fixed pre-tax return. We examine projects which have a 20% gross return and a 10% exponential depreciation rate. This choice roughly corresponds with the average gross marginal product of capital and depreciation over the period. The first univariate autoregression examined the series on the *ex post* tax rates on the aggregate average project. The results were:

$$\tau_{K,t+1} \quad .2\, \tau_{K,t} + .45$$
$$(.98)$$

The t-statistic indicates that the first lag is not significantly different from 0. The fact that the serial autocorrelation was so weak is evidence for the optimality hypothesis. Furthermore, the small size of the t-statistic indicates that it is unlikely that the random walk model generated the data. Higher order autoregression finds an alternating sign structure with nearly significant coefficients. In all cases the impulse response function indicates that any shock persists for only a short time before decaying rapidly.

To further investigate the univariate character of the aggregate tax rates, we also used a

spectral technique to test whether the data is white noise, the prediction of the extreme case examined above. We used the Cramer-von Mises test statistic discussed in Durlauf. This test rejected the pure white noise hypothesis at 1%

We next examine evidence from a disaggregated point of view of tax rates. An aggregate perspective is possibly error-ridden since not all assets are treated equally. For example, the investment tax credit discriminates between equipment and structures. Different assets are allowed different levels of acceleration in their depreciation schedules. The asset mix in new investment has changed substantially (see Jorgenson and Sullivan). In a multiple capital stock world, the theory generalizes to say that the tax rate on each asset is close to white noise. Furthermore, it is intuitively clear that the efficiency costs of deviations from white noise tax rates differ substantially across assets with different lifetimes and adjustment costs; therefore, in environments where there will be deviations from white noise tax rates (for reasons discussed above) we would expect fewer deviations from whiteness for assets with greater flexibility in short-run supply. For several reasons, it would be more appropriate to examine our optimality hypothesis on individual asset classes

In Table 1 we display the depreciation lives of various classes of assets during the period 1950-1980. Hulten and Wykoff have estimated the true depreciation of these 35 asset classes. During this period there have been various depreciation systems in place; these are displayed in columns two through four. In rough terms, as inflationary expectations increased, allowed tax lives decreased. This helped to keep the *ex ante* tax rates from growing (see Jorgensen and Sullivan for one calculation of *ex ante* tax rates for 1950-1980)

Using our *ex post* tax formula developed above, we can calculate the realized tax rate for each asset in each year, where we again assume a pre-tax net return of 10%. We then examined the time series of tax rates for each asset. Table 2 displays the results. For each asset class, we estimated the AR(1) equation for the *ex post* capital income tax rates, and used the Cramer - von Mises statistic to test for whiteness of the tax rates. For each asset class, the first two columns give the intercept and slope estimates with their t-statistics below. The final column of Table 2 reports the Cramer - von Mises statistic with the probability (computed by Monte Carlo simulation to avoid small sample biases) that the statistic is

36

smaller under null hypothesis that the tax rate process is white noise. The results were striking. For all assets, the first-order autoregression coefficient was small and statistically insignificant. Furthermore, we easily accepted the hypothesis that the time series of tax rates for each asset was white noise. In fact, the white noise hypothesis statistic could never be rejected even at the 48% level.

Our next test of optimality focusses on the joint determination of labor and capital income taxation. Optimality in our model says that the labor tax rate should be nearly a univariate random walk and capital income taxation should be more volatile, possibly white noise, whereas the Markov perfect equilibrium predicts serial and contemporaneous correlation among the tax rates. Skinner estimated a vector autoregression for tax rates, but used the cash-flow tax rates for capital. We instead used the tax rates computed above which avoid the artificial smoothing in cash-flow measures. We estimate the vector autoregressions of the labor tax rate series (taken from Barro) together with the *ex post* tax rate series computed above for each asset class.

Regression results for this collection of 35 VAR's, displayed in Table 3, again support the optimality hypothesis. For example, the tax rate for income from category 1 capital, furniture and fixtures, had a constant of .2241, a coefficient on lagged capital income tax rate of .0474, and a coefficient on lagged labor tax rate of 1.3753, the the coefficients on the lagged tax rates were both insignificantly different from zero. The VAR for labor taxation showed only strong and significant autocorrelation. As we examine Table 3, we consistently find that current labor tax rate depends only on lagged labor tax rates and that capital income tax rates were serially uncorrelated with both tax rates. Therefore, we find patterns which are consistent with even the most extreme specifications of our model, and strongly inconsistent with the Markov equilibrium alternative.

We should be careful in interpreting the evidence for optimality. The variance inequality which forms the basis of our test is a weak test. Perhaps we just have a crazy government which randomizes capital income taxation. The theory says that the shocks to capital income taxation should be related to new information about government consumption relative to output. It is difficult to test this aspect of the theory without making assumptions re-

lating actual expenditures and expectations. However, there is no evidence that the debt process is unstable, something which would occur if capital tax revenues were more volatile than expenditures but not related in the correct fashion. Furthermore, Eisner and Pieper have argued that when capitalization effects are taken into account, the budget surplus and business cycle shocks are positively correlated, a comovement consistent with our optimality theory.

The major conclusion to which we are drawn is that the capital income tax rate does not show much if any persistence and that the labor tax rate is persistent, the factor tax rates appear to follow distinct patterns and, whatever your prior belief over the two alternatives, the evidence will push your beliefs in the direction of the optimality hypothesis. However, it is clear that this is only weak evidence of optimality, and certainly does not justify every feature of the U.S. tax code.

While there may be alternative explanations for the data, some obvious candidates are not plausible. One reason for getting so much variance in capital income taxation could be that there is a struggle for power between the capitalists and workers over who will pay the government's bills. This could be represented by assuming a conflict between capitalists and workers. However, if this were true then one would expect a negative correlation between labor and capital income tax rates, which is not true. That is ruled out by the VAR results for the joint process of labor and capital income tax rates.

The evidence indicates that existing policy is not described by the Markov perfect equilibrium. There appear to be institutions which allow us to implement policies closer to the precommitment optimum than initially seems possible. This is an important consideration to remember when considering possible "reforms." If one were concerned about the lack of a precommitment technology, there might be a temptation to implement constitutional changes which block the Markov perfect equilibrium. This might be counterproductive since if the fear of falling into a Markov perfect equilibrium is providing the incentive for following the optimal policy, improving the Markov perfect equilibrim will reduce the cost of deviation and may in fact reduce the performance of policy.

Our discussion of the empirical evidence assumed all-equity financing and considered

only the corporate portion of income taxation. We next examine the implications of financial structure and integration of the corporate and personal tax structures.

Individual Income Taxation

The total burden of the tax system on factor supply includes taxation of personal income. To address the issue of optimality we should integrate the personal and corporate tax systems. Unfortunately, that is much more difficult than it first appears since the impact of such integration depends on the kind of personal tax system we have, a question which public finance economists have yet to resolve.

This assertion may seem silly since the tax system looks like and is called an income tax. However, various tax-sheltered investment instruments, such as pension funds, SRA's, IRA's, and Keough plans, push it towards being a consumption tax. If we had a consumption tax at the personal level, then the contribution of personal income taxation to the total capital income tax rate would indeed be zero, as is implicitly assumed in the previous section. However, liquidity limitations, such as contribution limits, early liquidation penalties, and limitations on using these assets as collateral, keep it from becoming a true consumption tax, instead turning it into a "hybrid" tax system.

The properties of a hybrid system can be very different from an income tax. Instead of being a compromise, it can be the worst of all possible worlds. For example, Balcer and Judd showed that allowing individuals to have IRA's may be worse than just reducing income tax rates, even though it reduces the total taxation of savings. In general, standard tax analyses which do not distinguish between realization and accrual taxation are highly unreliable (see Balcer and Judd (1985)). The important unresolved issues include how savings react to tax rate changes, how portfolio decisions altered, and the impact on precautionary versus life-cycle savings. Since the nature of the personal tax system is ambiguous, it is difficult to bring a complete analysis of the interactions of the capital structure and the tax system.

In our analysis we have focussed on the corporate income tax, neglecting the taxes paid at the personal level and the existence of corporate debt. An important aspect of the cost of capital is the fact that interest payments by firms to bondholders are tax deductible at

the corporate level (see Brock and Turnovsky for a complete general equilibrium analysis of the cost of capital). It is well-understood that if anticipated inflation is increased and the tax-adjusted Fisher relation holds (of course, the empirical work is ambiguous) then an increase in expected inflation will reduce the cost of capital for debt-financed investment. The substantial increase in corporate indebtedness in the postwar era has substantially reduced the effective tax rate.

However, we must keep distinct the interaction of inflationary expectations and debt on the *ex ante* cost of capital, and the impact of unanticipated inflation on *ex post* tax rates. The key fact is that unexpected increases in the price level will reduce the real value of the interest deductions, increasing both the real taxes paid by corporations immediately and the taxes paid eventually by the equity holders when they take the increased dividends or realize a capital gain. Offsetting this increase in corporate taxation is a change in taxes paid by the holders of the debt. However, this effect of unexpected inflation is enhanced by the clientele structure of asset holding. Interest-bearing assets tend to be held by low- tax rate agents, either individuals in low tax brackets or tax-sheltered investment instruments such as pension funds. The fact that the interest payments are deductions from the income of high-tax-bracket corporations and income to low tax bracket bondholders reduces even further the cost of capital for debt-financed investment. However, unexpected increases in the price level will increase the taxation of capital since it will reduce the real value of the tax deductions for the high tax bracket corporations, yielding an increase in tax revenues since it is only partially offset by the reduction in the real value of tax payments by the debtholders.

Despite our poor understanding of the net effect of the tax system on investment, we are able to make a reasoned guess about the impact on the optimality hypothesis. When we include the effects of personal taxation, debt, and tax-sheltered instruments, we will have a lower estimate of the net effective *ex ante* tax rate. Furthermore, there will be a downward trend in net *ex ante* rate in the post-WWII era since debt–equity ratios and use of pension funds have risen, and personal tax rates have fallen. However, all of this is consistent with the general optimality hypothesis since the general version of our quadratic loss function

implied a declining *ex ante* tax rate on capital income after an expenditure shock.

However, this discussion of the *ex ante* tax rate is not related to our test for optimality. Our test focussed on the relative variances of labor and capital taxation. Reflection shows that integrating the personal and corporate tax systems will, if anything, *increase* the variance of the *ex post* tax rates. First, inflation shifts real taxable income from debt-holders (low τ) to corporations (high τ) and equity holders. (See Feldstein-Summers and Gordon-Slemrod). Second, inflation shocks reduce the basis for capital gains taxation purposes. Therefore, it appears that integrating corporate and personal income taxation will not affect the general result that *ex post* tax rates on capital income are highly volatile. This supports the optimality hypothesis over the the Markov perfect hypothesis.

General Closed-Economy Optimal Tax Policy

The primary assertion of the foregoing has been that the optimal capital and wage tax rates follow substantially different stochastic processes, with the former being substantially more volatile. Some readers may not be convinced of the robustness of this assertion by our combination of linear-quadratic analyses and simulations. For them, we will now display the dynamic programming problem which describes the optimal tax policy in a general equilibrium, closed economy model. While we will not attempt any substantive analysis here, we will see the crucial feature which indicates that our qualitative results will be robust

Kydland and Prescott studied this problem in a deterministic context. The generalization to uncertainty is straightforward. Suppose that agents maximize the intertemporal utility function where c is consumption and ℓ is labor supply per period, and β is the utility discount factor. Suppose that net output equals $f(k, \ell)$, a CRTS production function where k is the physical capital stock per capita. We assume that the government can issue bonds, the stock of which is b.

In studying this problem it is best to approach it in a dual fashion. Suppose $\bar{\lambda}$ is tomorrow's promised marginal utility of consumption, $\bar{w}$ the promised after-tax wage rate, and $\bar{R}_i$ tomorrow's promised after-tax return on asset i, $i = k, b$. Let $C(\lambda, w)$ represent consumption as a function of λ and w, and $L(\lambda, w)$ and $u(\lambda, w)$ similarly represent labor

supply and utility. Let $\tilde{g}$ be government consumption.

The optimal tax policy is represented by the following dynamic programming problem:

$$V(k, b, \lambda, g) = \max_{\tilde{\lambda}, \tilde{R}_k, \tilde{R}_b, \tilde{w}} \beta\, E\left\{u(\tilde{\lambda}, \tilde{w}) + V(\tilde{k}, \tilde{b}, \tilde{\lambda}, \tilde{g}) \mid g\right\}$$

$$\text{s.t.} \quad \lambda = E\left\{\tilde{\lambda}\tilde{R}_k\right\}$$

$$\lambda = E\left\{\tilde{\lambda}\tilde{R}_b\right\}$$

$$\tilde{k} = k + f(k, L(\tilde{\lambda}, \tilde{w})) - C(\tilde{\lambda}, \tilde{w}) - \tilde{g}$$

$$\tilde{b} = b\tilde{R}_b - f(k, L(\tilde{\lambda}, \tilde{w})) + \tilde{w}L(\tilde{\lambda}, \tilde{w}) + k\tilde{R}_k + \tilde{g}$$

This problem says that the government makes promises about future tax rates, but that they must be consistent with the private incentives of the agents as represented by their intertemporal Euler equations.

The critical aspect for our purposes is that the problem is nearly linear in $\tilde{R}_k$ and $\tilde{R}_b$ but not in $\tilde{w}$. Standard intuition from bang-bang problems indicates high volatility in the optimally chosen $\tilde{R}_k$ and $\tilde{R}_b$. A full examination of this problem will not be attempted here, but this formalization makes clear that the qualitative results we found for the simple cases we examined will be robust to a fully structural analysis.

Conclusion

This paper has shown that optimal taxation of income would treat labor and labor income in substantially different ways. While the random walk prediction still holds for labor taxation, optimal taxation of capital income is closer to white noise. These differences arise directly from the fact that labor supply is a flow, with anticipation effects playing only a minor role, whereas capital supply is a stock with anticipated taxation being more important for current supply decisions than contemporaneous taxation. We also showed that "dynamically consistent" taxation will be very different, with capital and labor tax rates following the same process because the lack of commitment eliminates the intertemporal differences.

Since both outcomes can be supported in equilibrium by plausible descriptions of political institutions, there is no strong prediction as to what we should expect to observe.

However, these arguments do show that the optimal policy differs substantially in its statistical properties from "consistent" policy, giving us an empirical test as to which description is most consistent with the data. Previous empirical work on taxation appears to argue against the optimality hypothesis However, that is primarily due to accounting errors in defining the *ex post* tax rate on capital income. When the tax rate process on capital income is appropriately computed, we find that the U.S. experience is better described by the optimality hypothesis than by no-commitment Markov perfect equilibria.

Introspective consideration of various tax code provisions also supports this conclusion Several provisions which appear pointless from a deterministic perspective, such as the nominal character of depreciation allowances, promote appropriate risk-sharing under plausible assumptions concerning innovations in inflation and revenue needs. Also, we find that income taxation strictly dominates consumption taxation.

In conclusion, we have shown that explicit consideration of risk substantially alters the nature of optimal tax policy, and that the evidence is that to a nontrivial extent the U.S tax code incorporates these differences. While much more work is needed to incorporate more aspects of the tax system into our analysis, we have also shown that this exercise is tractable

References

Atkinson, Anthony B., and Agnar Sandmo. "Welfare Implications of the Taxation of Savings," *The Economic Journal* 90 (September, 1980).

Balcer, Yves, and Kenneth L. Judd. "The Efficiency of Capital Taxation: Capital Gains vs. Corporate Income," mimeo (1987).

Barro, Robert. "On the Determination of Public Debt," *Journal of Political Economy*, 87, (1979), 940–71.

Barro, Robert J., and David B. Gordon, "Rules, Discretion and Reputation in a Model of Monetary Policy," *Journal of Political Economy*, (1983).

Barro, Robert J. and Sahasukul, Chaipat "Measuring the Average Marginal Tax Rate from the Individual Income Tax," *Journal of Business*, 56, (1983), 419–452.

Bernheim, B. Douglas, and Debraj Ray, "Marker Perfect Equilibria in Altruistic Growth Economies with Production Uncertainty" *JET* 47 (1989): 195–202.

―――, "Altruistic Growth Economies. I, Existence of Bequest Eqluilibria" IMSS Technical Report No. 419, Stanford University, 1983.

―――, "On the Existence of Markov-consistent Plans Under Uncertainty." *RES* 54 (1987): 227–242.

Bizer, David S., and Steven N. Durlauf, "Are Tax Rates Martingales?" mimeo, Stanford University, 1989.

Bradford, David. *Untangling the Income Tax*, Harvard University Press, 1986.

Bulow, Jeremy I., and Summers, Lawrence, H. "The Taxation of Risky Assets," *Journal of Political Economy*, 92:1, (1984), 20–39.

Carlton, Dennis W. "The Disruptive Effect of Inflation on the Organization of Markets." In R.E. Hall, ed., *Inflation: Causes and Effects*, (University of Chicago Press, Chicago, IL).

Chari, V. V., Patrick J. Kehoe, and Edward C. Prescott. "Time Consistency and Policy," in *Modern Business Cycle Theory*, Robert J. Barro, ed., Harvard University Press (1989).

Diamond, Peter A. and Mirrlees, James A. "Optimal Taxation and Public Production," I-II, *American Economic Review*, 61, 8-27, (1971), 261-278.

Durlauf, Steven N., "Spectral Based Testing of the Martingale Hyptothesis," mimeo, Stanford University, 1989.

Feldstein, Martin C. "Adjusting Depreciation in an Inflationary Economy: Indexing versus Accelerated Depreciation," *National Tax Journal*, 34, No.1, (March, 1981), 29-44.

"Inflation and the Stock Market," *American Economic Review*, 70, 5, (1980a), 839-47.

"Inflation, Tax Rules and the Stock Market," *Journal of Monetary Economics*, 14, 3, 309-17.

Feldstein, Martin S., Dicks-Mireaux, Louis, Poterba, James. "The Effective Tax Rate and the Pretax Rate of Return," *Journal of Public Economics*, 21, (1983), 129-58.

Feldstein, Martin, and Summers, Lawrence. "Inflation and the Taxation of Capital Income in the Corporate Sector," *National Tax Journal*, 32, (December 1979), 445-470.

Fullerton, Don. "The Indexation of Interest, Depreciation, and Capital Gains and Tax Reform in the United States," *Journal of Public Economics*, 32(1), (1987), 25-51.

Gordon, Roger H. "Inflation, Taxation and Corporate Behavior." *Quarterly Journal of Economics*, 99, 2, (1982), 313-27.

Gordon, Roger H., and Joel Slemrod, "Do We Collect any Revenue From Taxing Capital Income?"

Hall, Robert E., and Jorgenson, Dale W. "Tax Policy and Investment Behavior," *American Economic Review*, 57, (1967), 391-414.

Hall, Robert E. and Rabushka, Alvin *Low Tax, Simple Tax, Flat Tax*, (McGraw-Hill, New York), 1983.

Hamilton, Jonathan, "Taxation, Savings and Portfolio Choice in a Continuous Time Model," Murphy Institute Discussion Paper 81/9, Tulane University, 1981.

Hulten, Charles R., and Wykoff, Frank C. "The Measurement of Economic Depreciation." In C.R. Hulten, ed., *Depreciation, Inflation, and the Taxation of Income From Capital*. Washington, D.C.: Urban Institute Press, 1981.

Joines, D. H. "Estimates of Effective Marginal Tax Rates on Factor Incomes," *Journal of Business*, 54, (April, 1981), 191–226

Jorgenson, Dale W., and Sullivan, Martin A. "Inflation and Corporate Capital Recovery." In C. R. Hulten, ed., *Depreciation, Inflation, and the Taxation of Income From Capital.* Washington, D.C Urban Institute Press, 1981

Judd, Kenneth L. "Debt and Distortionary Taxation in a Simple Perfect Foresight Model", Journal of Monetary Economics, (July, 1987a).

"The Welfare Cost of Factor Taxation in a Perfect Foresight Model," Journal of Political Economy (August 1987b)

"Redistributive Taxation in a Perfect Foresight Model", Journal of Public Economics 28 (October 1985), 59-84.

"Capital Gains Taxation by Realization in a Dynamic Model", *Journal of Public Economics* (forthcoming).

King, Mervyn A., and Fullerton Don, *The Taxation of Income From Capital: A Comparative Study of the U.S., U.K., Sweden, and West Germany.* Chicago: University of Chicago Press, 1984

Kingston, G. H. "Efficient Timing of Taxes," *Journal of Public Economics*, 24, (1984), 271–80.

Kydland, Finn, "Equilibrium Solution in Dynamic Dominant-Player Models," *JET* 14 (1977), 307–324

Kydland, Finn, "Noncooperative and Dominant Player Solutions in Discrete Dynamic Games,' *IER* 16 (1975), 321–335.

Kydland, Finn and Edward Prescott, "Dynamic Optimal Taxation, Rational Expectations and Optimal Control," *Journal of Economics Dynamics and Control* 2 (1980), 79–91

Kydland, Finn and Edward Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *JPE* 85 (1977), 473–491.

Lucas, Robert E. Jr. "Principles of Fiscal and Monetary Policy," *Journal of Monetary Economics*, 17, (1986), 117–134.

Lucas, Robert, and Stokey, Nancy "Optimal Fiscal and Monetary Policy in an Economy

Table 1
Asset Lifetimes for Tax Purposes

	Asset Category	Hulten-Wykoff Depreciation Rate	Bulletin F Lifetime	Guideline Lifetime	ADR Lifetime
1	Furniture and fixtures	11.00	17.6	10.0	7.8
2	Fabricated metal products	9.17	21.2	12.5	9.8
3	Engines and turbines	7.86	24.7	15.6	12.2
4	Tractors	16.33	9.4	4.3	5.0
5	Agricultural machinery	9.71	20.0	10.0	7.8
6	Construction machinery	17.22	10.6	9.9	7.8
7	Mining and oilfield machinery	16.50	11.8	9.6	7.5
8	Metalworking machinery	12.25	18.8	12.7	10.0
9	Special industry machinery	10.31	18.8	12.7	10.0
10	General industrial equipment	12.25	16.5	12.3	7.8
11	Office, computing, and accounting machinery	27.29	9.4	10.0	8.9
12	Service industry machinery	16.50	11.8	10.3	8.1
13	Electrical machinery	11.79	16.5	12.4	9.7
14	Trucks, buses, and truck trailers	25.37	10.6	5.6	4.4
15	Autos	33.33	11.8	3.0	3.0
16	Aircraft	18.33	10.6	6.3	6.0
17	Ships and boats	7.50	25.9	18.0	14.1
18	Railroad equipment	6.60	29.4	15.0	11.8
19	Instruments	15.00	12.9	10.6	8.3
20	Other equipment	15.00	12.9	10.2	8.0
21	Industrial buildings	3.61	31.8	28.8	25.3
22	Commercial buildings	2.47	42.3	47.6	41.8
23	Religious buildings	1.88	56.5	48.0	42.1
24	Educational buildings	1.88	56.5	48.0	42.1
25	Hospital buildings	2.33	56.5	48.0	42.1
26	Other nonfarm buildings	4.54	36.5	30.9	27.1
27	Railroads	1.76	60.0	30.0	26.3
28	Telephone and telegraph facilities	3.33	31.8	27.0	23.7
29	Electric light and power	3.00	35.3	27.0	23.7
30	Gas	3.00	35.3	24.0	21.1
31	Other public utilities	4.50	30.6	22.0	19.3
32	Farm	2.37	44.7	25.0	21.9
33	Mining, exploration, shafts, and wells	5.63	18.8	6.8	6.0
34	Other nonbuilding facilities	2.90	36.5	28.2	24.7
35	Residential	1.30	40.0	40.0	24.8

Table 2

1	0.5742 (4.6608)	0.0458 (0.2376)	0.1114 0.4770
2	0.5931 (4.8098)	0.0247 (0.1281)	0.1169 0.5080
3	0.6101 (4.9273)	0.0093 (0.0483)	0.0857 0.3420
4	0.5751 (4.7667)	0.0508 (0.2604)	0.0891 0.3620
5	0.5735 (4.6563)	0.0462 (0.2392)	0.1110 0.4730
6	0.5731 (4.6748)	0.0456 (0.2365)	0.1148 0.4970
7	0.5706 (4.6463)	0.0509 (0.2641)	0.1126 0.4850
8	0.5877 (4.6882)	0.0435 (0.2253)	0.1125 0.4850
9	0.5848 (4.7188)	0.0354 (0.1832)	0.1145 0.4960
10	0.5845 (4.7082)	0.0387 (0.2007)	0.1130 0.4880
11	0.5640 (4.5086)	0.0862 (0.4483)	0.1063 0.4540
12	0.5761 (4.6724)	0.0464 (0.2407)	0.1141 0.4950
13	0.5983 (4.8154)	0.0265 (0.1375)	0.1038 0.4340
14	0.5304 (4.3041)	0.1509 (0.7818)	0.0565 0.1840
15	0.4578 (3.6723)	0.3001 (1.5926)	0.0263 0.0300

16	0.5736	0.0466	0.1166
	(4.7449)	(0.2410)	0.5070
17	0.5926	0.0239	0.1165
	(4.7752)	(0.1240)	0.5060
18	0.5805	0.0342	0.1141
	(4.7138)	(0.1771)	0.4950
19	0.5783	0.0435	0.1142
	(4.6847)	(0.2254)	0.4950
20	0.5757	0.0458	0.1135
	(4.6716)	(0.2371)	0.4900
21	0.5833	-0.0037	0.0348
	(4.9870)	(-0.0182)	0.0670
22	0.5879	-0.0397	0.0473
	(5.2051)	(-0.1987)	0.1210
23	0.5695	-0.0246	0.0497
	(5.1417)	(-0.1235)	0.1320
24	0.5695	-0.0246	0.0497
	(5.1417)	(-0.1235)	0.1320
25	0.5779	-0.0220	0.0494
	(5.1273)	(-0.1105)	0.1310
26	0.5956	0.0044	0.0366
	4.9490	0.0219	0.0760
27	0.6000	-0.0442	0.0516
	(5.1846)	(-0.2300)	0.1490
28	0.6182	-0.0453	0.0493
	(5.2040)	(-0.2359)	0.1310
29	0.6146	-0.0447	0.0499
	(5.1975)	(-0.2324)	0.1330
30	0.6105	-0.0376	0.0492
	(5.1587)	(-0.1953)	0.1300

31	0.6230	-0.0324	0.0498
	(5.1456)	(-0.1684)	1.3320
32	0.5951	-0.0277	0.0830
	(5.1144)	(-0.1438)	0.3280
33	0.5776	0.0378	0.0475
	(4.8405)	(0.1934)	0.1230
34	0.6061	-0.0385	0.0833
	(5.1757)	(-0.2001)	0.3310
35	0.5565	-0.0146	0.0387
	(5.0637)	(-0.0730)	0.0860

Table 3

1	1	0.2241	0.0474	1.3753
		(0.2505)	(0.2330)	(0.3933)
	2	0.0340	0.0061	0.8598
		(0.7151)	(0.5635)	(4.6252)
2	1	0.2673	0.0255	1.2828
		(0.3198)	(0.1252)	(0.3928)
	2	0.0346	0.0053	0.8593
		(0.7253)	(0.4553)	(4.6115)
3	1	0.2131	0.0015	1.5874
		(0.2718)	(0.0073)	(0.5136)
	2	0.0357	0.0060	0.8532
		(0.7528)	(0.4835)	(4.5652)
4	1	0.1133	0.0400	1.8349
		(0.1846)	(0.1966)	(0.7515)
	2	0.0369	0.0028	0.8561
		(0.7771)	(0.1765)	(4.5309)
5	1	0.2190	0.0481	1.3914
		(0.2432)	(0.2364)	(0.3953)
	2	0.0339	0.0063	0.8596
		(0.7142)	(0.5856)	(4.6267)
6	1	0.2389	0.0456	1.3198
		(0.2750)	(0.2233)	(0.3892)
	2	0.0343	0.0053	0.8605
		(0.7191)	(0.4732)	(4.6202)
7	1	0.2357	0.0507	1.3230
		(0.2696)	(0.2485)	(0.3876)
	2	0.0344	0.0052	0.8604
		(0.7209)	(0.4667)	(4.6191)
8	1	0.2687	0.0448	1.2553
		(0.3022)	(0.2196)	(0.3618)
	2	0.0339	0.0057	0.8607
		(0.7119)	(0.5182)	(4.6260)

9	1	0.2577	0.0376	1.2835
		(0.2886)	(0.1847)	(0.3681)
	2	0.0337	0.0063	0.8605
		(0.7078)	(0.5785)	(4.6311)
10	1	0.2809	0.0397	1.1963
		(0.3169)	(0.1944)	(0.3460)
	2	0.0338	0.0058	0.8610
		(0.7094)	(0.5308)	(4.6284)
11	1	0.2129	0.0834	1.3958
		(0.2522)	(0.4081)	(0.4240)
	2	0.0356	0.0030	0.8606
		(0.7430)	(0.2583)	(4.6060)
12	1	0.2443	0.0465	1.3095
		(0.2802)	(0.2280)	(0.3847)
	2	0.0343	0.0053	0.8605
		(0.7186)	(0.4739)	(4.6204)
13	1	0.2457	0.0236	1.3992
		(0.3000)	(0.1158)	(0.4358)
	2	0.0348	0.0059	0.8569
		(0.7317)	(0.4990)	(4.5990)
14	1	0.0810	0.1392	1.7889
		(0.1302)	(0.6879)	(0.7215)
	2	0.0368	0.0041	0.8533
		(0.7754)	(0.2633)	(4.5119)
15	1	0.0234	0.2745	1.7585
		(0.0432)	(1.3862)	(0.7996)
	2	0.0375	0.0013	0.8572
		(0.7887)	(0.0776)	(4.4375)
16	1	0.2386	0.0449	1.3248
		(0.3243)	(0.2200)	(0.4603)
	2	0.0368	0.0012	0.8603
		(0.7692)	(0.0911)	(4.5950)
17	1	0.3073	0.0273	1.1161
		(0.3410)	(0.1341)	(0.3177)
	2	0.0331	0.0066	0.8619
		(0.6952)	(0.6176)	(4.6429)

18	1	0.2704	0.0375	1.2129
		(0.2974	(0.1840)	(0.3417)
	2	0.0333	0.0067	0.8611
		(0.7011	(0.6269)	(4.6399)
19	1	0.2451	0.0441	1.3135
		(0.2795	(0.2163)	(0.3837)
	2	0.0342	0.0056	0.8604
		(0.7162	(0.5009)	(4.6225)
20	1	0.2394	0.0462	1.3256
		(0.2727	(0.2268)	(0.3867)
	2	0.0342	0.0055	0.8603
		(0.7175	(0.4981)	(4.6217)
21	1	0.2219	-0.0252	1.4530
		(0.5498	(-0.1232)	(0.8796)
	2	0.0375	-0.0053	0.8725
		(0.7901	(-0.2191)	(4.4930)
22	1	0.3298	-0.0333	0.9882
		(0.8063	(-0.1635)	(0.6024)
	2	0.0386	-0.0082	0.8742
		(0.8130	(-0.3482)	(4.5893)
23	1	0.3308	-0.0148	0.9059
		(0.8010	(-0.0726)	(0.5483)
	2	0.0389	-0.0097	0.8758
		(0.8201	(-0.4136)	(4.6125)
24	1	0.3308	-0.0148	0.9059
		(0.8010	(-0.0726)	(0.5483)
	2	0.0389	-0.0097	0.8758
		(0.8201	(-0.4136)	(4.6125)
25	1	0.3321	-0.0134	0.9353
		(0.8064	(-0.0658)	(0.5671)
	2	0.0389	-0.0096	0.8761
		(0.8201	(-0.4111)	(4.6096)
26	1	0.2379	-0.0169	1.4387
		(0.5913	(-0.0823)	(0.8740)
	2	0.0376	-0.0061	0.8742
		(0.7931	(-0.2526)	(4.5041)

27	1	0.0970	-0.0649	2.0371
		(0.1257	(-0.3181)	(0.6620)
	2	0.0374	0.0043	0.8512
		(0.7892	(0.3438)	(4.5085)
28	1	0.1069	-0.0672	2.0730
		(0.1394	(-0.3291)	(0.6775)
	2	0.0374	0.0047	0.8499
		(0.7906	(0.3690)	(4.4972
29	1	0.1007	-0.0666	2.0834
		(0.1312	(-0.3263)	(0.6799)
	2	0.0375	0.0046	0.8500
		(0.7909	(0.3644)	(4.4978)
30	1	0.0773	-0.0604	2.1608
		(0.1004	(-0.2959)	(0.7021)
	2	0.0376	0.0046	0.8494
		(0.7935	(0.3700)	(4.4903)
31	1	0.0953	-0.0560	2.1424
		(0.1242	(-0.2739)	(0.6988)
	2	0.0375	0.0044	0.8498
		(0.7925	(0.3508)	(4.4886)
32	1	0.2021	-0.0353	1.5708
		(0.2679	(-0.1736)	(0.5276)
	2	0.0361	0.0052	0.8543
		(0.7609	(0.4034)	(4.5633)
33	1	0.0319	0.0122	2.1961
		(0.0526	(0.0598)	(0.8990)
	2	0.0376	0.0045	0.8496
		(0.7925	(0.2850)	(4.4483)
34	1	0.2371	-0.0458	1.4760
		(0.3164	(-0.2252)	(0.4995)
	2	0.0359	0.0051	0.8548
		(0.7571	(0.4001)	(4.5683)
35	1	0.2517	-0.0204	1.1965
		(0.6202	(-0.0996)	(0.7266)
	2	0.0376	-0.0052	0.8710
		(0.7933	(-0.2176)	(4.5233)