STRATEGIC INCENTIVE MANIPULATION IN RIVALROUS AGENCY*

by

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ABSTRACT

The paper examines "rivalrous agency" situations in which two pairs of principal–agent strategically interact. In such a case contracts simultaneously mitigate incentive problems and serve as precommitment device that enable principals to gain competitive advantage. The paper examines the interplay between moral hazard problems and strategic incentive manipulation.

1. INTRODUCTION

An important problem for a firm's owners is the choice of their managers' incentives. The principal–agent literature has provided a valuable framework in which to examine this problem. This literature (see, for example, Holmstrom (1979), Harris and Raviv (1979), Grossman and Hart (1983) and a recent survey Hart and Holstrom (1987)) has generally assumed, however, that the principal and the agent together play a game against nature, unable to affect the conditional distribution of payoffs available to them. In market applications this implies that the principal–agent model is concerned primarily with the two extremes of market structure—competitive firms which face prices and costs as given (see Hart (1983)), and monopolies which face the demand and marginal cost curves as given. It is also common, however, for firms with a separation of ownership and control to find themselves in oligopolistic situations. In this paper we combine principal–agent theory and oligopoly theory, producing a structure we call "rivalrous agency," in an effort to examine the strategic uses of *intrafirm* contracting and the implications for oligopolistic conduct.

We examine a duopoly in which the owners hire managers to supply the effort necessary for output, such effort being unobservable to the owners. The resulting structure is one where two principal–agent teams compete against each other as well as nature. Since effort is unobservable and the relation between output and effort is stochastic, the owners face the standard moral hazard problem. Contracts are first written between each manager and his owner, specifying the state–contingent payments to the manager. Once the contracts are written, the managers make simultaneous, unobserved, and noncooperative choices of their effort levels. It is assumed that the internal incentives of each firm is common knowledge when competing managers make their effort choices.[1]

[1]This distinguishes the following from Myerson (1982) and Katz (1987) where agents only know their own contract and make Nash conjectures about other agent's contracts.

When the information structure of managers' incentives in a duopoly is so described and each firm's expected profits are affected by its rival's behavior, we find that the managerial contracts chosen in equilibrium are substantially affected by the presence of the duopolistic interaction. For example, we show in one case that each owner will give his manager so much incentive to provide effort that, after writing the contract, the owner hopes that his manager fails, a result which contrasts strikingly with the usual monotonicity results of principal–agent theory in nonrivalrous contexts. Hence, in rivalrous agency, it is possible that the owner will bear an excessive amount of risk relative to the nonrivalrous case because of his strategic use of managerial contracts.

Fershtman and Judd (1987), Sklivas (1987) and Vickers (1985) also examined the importance of rivalrous agency on market structure and conduct in standard oligopoly models in a framework pioneered by Brander and Spencer (1983). Those analyses, however, restricted owners to offer contracts linear in profits and sales. In that case, the owners substantially alter their managers' incentives in order to accomplish a strategic advantage. For dxample, if firms compete in quantities then managers would be given positive incentive to augment sales, with the sales incentive increasing as the correlation of costs across firms increased and as the variance in costs decreased, while firms that competed in price would penalize their managers for a marginal sale in equilibrium. While the analysis there was intuitive, examining the dependence of contracts on several important structural parameters in familiar oligopoly models, the owner–manager relationship was not explicitly affected by imperfect information parameters in familiar oligopoly models, the owner–manager relationship was not explicitly affected by imperfect information and the contracts were restricted in an ad hoc fashion. In this paper's model each firm faces the standard moral hazard problem in its principal–agent relationship – the owner does not observe how hard the manager works, just the outcome of that effort. also allow contracts to be fully contingent on all publicly observable information, including the output of rival firms. We find that many of the insights of the earlier analysis continues to hold in this explicit moral hazard structure. These studies demonstrate the strategic aspects of incentive contracts are potentially important determinants of market structure and conduct.

This analysis is also related to the study of "common agency" by Berheim and Whinston (1986). They considered a problem where principals hire a common agent to make decisions of importance for them. They find that delegation of decisionmaking to a common agent can be used to implement some degree of cooperation even when the principals' interests are not common. In rivalrous agency, principals hire separate agents who compete on behalf of the principals. Agents naturally arise in many situations since there are often individuals who have a comparative advantage in the management resources, particularly when the owners have better things to do. Therefore, the existence of principal–agent relations is often implied by efficiency. However, in oligopolistic contexts the use of a common agent is legally equivalent to a merger and unlawful collusion. Therefore a rivalrous agency structure will arise. In this paper we show how these *internal* relations may be manipulated by owners in an effort to gain an *external* advantage.

The results in this paper contrast strongly with Fershtman, Judd and Kalai (1987). They showed that if in rivalrous agency, the contract between principals and their agents are fully observed and can thus be conditional upon the agents' game, a folk theorem holds. Namely, any outcome which is Pareto superior to a noncooperative equilibrium in the game without the agents is achievable as an equilibrium with agents. The key difference between our model here and that of Fershtman, Judd and Kalai is not the incomplete information framework which we assume here, but the fact that in the agents' game, strategies are not conditional upon the compensation agents receive from their principals. In the agents' game each agent chooses his strategy as the best response to the other agents' strategy independently from the compensation scheme the other agents have. By not allowing to condition strategies on compensation, agents do not have the ability to "punish" principals for giving the "wrong" compensation. Clearly, the fact that we do not attain a folk theorem type outcome highlights the importance of the difference between the two models.

2. THE GENERAL MODEL

In this section we lay out the structure for the general model with nonriskloving owners and managers. We will then describe our equilibrium concept in general, and discuss the concept of manipulability of compensation schemes which is unique to rivalrous agency settings.

Consider two rivalrous pairs of principals and agents. Agents may take an unobservable action that determines the probability of "success." For example, success could be thought of as a relatively high output, or in the case of an innovation competition as success in research. Contract R&D work is also a good example of our model, where we interpret the principals as entrepreneurs with competing ideas, and the agents as research laboratories hired to develop those ideas into marketable products. In such a situation, the two–state assumption is also reasonable since the research outcome is often either success or failure.

We assume that the probability that agent i succeeds, denoted by p_i, is a function of his effort. To simplify notation we view p_i as the agents' choice variable and we let

$g(p_i)$ be the cost associated with this choice. We assume that $g'(p_i) > 0$ and $g''(p_i) > 0$. Furthermore, we let that $g'(1) = \infty$, thereby assuming that the marginal return of effort at certain success is zero.

We will assume that success is independent across agents. This assumption is made not because we consider it realistic, but rather to avoid the kinds of considerations studies in the tournaments literature. Green and Stokey (1983), Demski and Sappington (1984), and Nalebuff and Stiglitz (1983) have examined models where one can learn about the effort of one agent by comparing his output with that of other agents in a similar environment. These other agents may be his own or those of another principal. inferences cause a principal to condition an agent's compensation on other agents' output. Since such payment interactions will arise in our model due to strategic reasons alone, we abstract from the conditions in the tournaments literature which also generate them. by abstracting from the tournament effects we can ask whether the two models have observable differences.

There are four possible state which we shall denote by $S \equiv \{11, 01, 10, 00\}$ – in state ij, firm one succeeds if $i = 1$, and firm two succeeds if $j = 1$. To economize on space, we let p_α denote the probability of state α occurring for $\alpha \in S$. For example, $p_{10} \equiv p_1(1 - p_2)$.

We let π^i_α denote the monetary return to firm i in state $\alpha \in S$. We first assume that it is better for a firm to succeed:

$$\pi^1_{1j} > \pi^1_{0j}, \ \pi^2_{j1} > \pi^2_{j0}, \ j = 0,1.$$

We next assume that firms have competing interests:

$$\pi^1_{j0} \geq \pi^1_{j1},\ \pi^2_{0j} \geq \pi^2_{1j},\ j = 0,1$$

We assume the possibility of fully contingent contracts for the agents. Since there are four possible states the general compensation scheme that principal i offers is described by $E^i = (E^i_{10}, E^i_{11}, E^i_{01}, E^i_{00})$ where E^i_α is the compensation by principal i to his agent in state α. Notice that under this formulation the compensation for an agent may depend on the success of his opponent. We let u_i be the i^{th} principal's utility function over money with $u_i' > 0 \geq u_i''$. Therefore, the final utility of principal i in state α is $u_i(\pi^i_\alpha - E^i_\alpha)$. Thus, given a choice of action by both agents and given a compenation scheme E^i, the i^{th} principal's expected utility is:

$$J^i(E^i, p_1, p_2) = \sum_\alpha p_\alpha u_i(\pi^i_\alpha - E^i_\alpha).$$

Let v_i be the utility function over money of agent i with $v_i' > 0 \geq v_i''$. Agent i's expected utility is $\sum_{\alpha \in S} p_\alpha v_1(E^1_\alpha) - g(p_1)$. We further assume that agent i's alternative utility is $\bar{v}_i$ thus the individual rationality constraint is $\sum_\alpha p_\alpha v_i(E^i_\alpha) - g(p_i) \geq \bar{v}_i$.

We will study a two-stage game. In the first stage, each principal will simultaneously determine the incentive scheme for its agent, subject to the expectation that it will compensate the agent for his opportunity cost of agreement. In the second stage the managers run the two firms and by their choice of effort levels generate the outcome.

We assume that in the second stage each agent knows both compensation schemes and simultaneously choose the effort levels, p_1 and p_2. The crucial assumption in our analysis is that internal incentives faced by the agents are common knowledge when the agents play their subgame. Intuitively, this turns each principal into a Stackelberg leader with respect to his rival's manager and generates the strategic elements of contracts. Since this assumption is pivotal, it requires some justification. First, it is in the interest of each owner in our model to make such information public. If an owner can avail itself of an effective and credible way to communicate the true nature of internal incentives it will do so, independent of the other owner's decision to reveal internal incentives. Hence, if we append an early stage to our game where firms decide whether to reveal internal incentives, the choice to reveal is the dominant strategy. Second, corporations are required to make public some information regarding managerial incentives, such as bonus policies and employee stock option plans.

This discussion also shows that our analysis is one of precommitment since the manager is given preferences over the outcome which differs from the owner. It is not surprising that such precommitment affects oligopolistic outcomes and is desired by players. However, precommitting a manager to preferences which differ from those of the principal is natural in a principal–agent context given the separation of ownership and management and the asymmetry of information. The focus of this study is to examine how these precommitments are altered for strategic reasons by the presence of rivalry with another principal–agent team.

Given each agent's knowledge of all agents' incentives, the second stage game is a full information game. If manager one believes that his oppponent is choosing p_2 he solves

$$\max_{p_1 \geq 0} \sum_{\alpha \epsilon s} p_\alpha v_1(E_\alpha^1) - g(p_1)$$

The first–order condition for manager one is:[2]

$$g'(p_1) = p_2(v_1(E_{11}^1) - v_1(E_{01}^1)) + (1 \quad p_2)(v_1(E_{10}^1) \quad v_1(E_{00}^1)).$$

If there is no solution to equation (2) due to a negative right–hand–side, implying that the agent is being paid to fail, then p_1 is chosen to be zero. We will refer to equation (2) as agent one incentive's compatibility constraint since it determines what principal i can successfully command agent i to do given the agent's incentives Since $g(p)$ is assumed convex there is a unique solution to this maximization problem and it is designated:

$$(3) \qquad p_1 = \phi_1(p_2, E^1)$$

ϕ_1 is therefore manager one's reaction function to p_2 given his own incentive structure, E^1. ϕ_1 is clearly continuous in p_2 and E^1 by the convexity of g and the continuity of ϕ_1 in its arguments whenever it is zero. Finally, it is bounded below by zero, and since $g'(1)$ is infinite, bounded above by unity

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[2]The second order condition is guaranteed by the convexity of $g(p)$.

Similarly, manager two has a reaction function: $p_2 = \phi_2(p_1, E^2)$ which is also continuous in its arguments and bounded above by one.

We define an equilibrium in the standard way for a two–stage game:

DEFINITION 1: An *incentive equilibrium* is (p_1, p_2, E^1, E^2) such that:

(i) (p_1, p_2) is an *agent equilibrium* in the game induced by (E^1, E^2), i.e., $p_i = \phi_i(p_j, E^i)$; $i,j = 1,2$; $i \neq j$.

(ii) (E^1, E^2) is a Nash equilibrium in the owners game, i.e., no owner can benefit by changing the compensation scheme he provides.

PROPOSITION 1: For any (E^1, E^2) there exists an agent equilibrium with respect to (E^1, E^2).

PROOF: The domain of each reaction function is $[0,1]$, the range is a subset of $[0,1]$, and both reaction functions are continuous functions. Therefore, by the intermediate value theorem there exists an equilibrium.

□

The possible multiplicity of agent equilibria is disturbing and makes the analysis clumsy at times. As for the continuation of the paper we would like to restrict attention to cases in which we have unique equilibrium in the agents' game. Proposition 2 provides, as an example, conditions under which uniqueness is assured.[3]

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[3]In proposition 2 we consider only the game between the agents given their compensation schemes (E^1,E^2). Thus the conditions provided in the proposition are on (E^1, E^2) and not on the primitives of the model. Clearly in order to claim that at the incentive equilibrium the agent equilibrium is unique one needs to show that the equilibrium

PROPOSITION 2: When E^1 and E^2 are such that both (i) and (ii) hold, where

(i) $g_1'(0) < \min\{v_1(E^1_{11}) - v_1(E^1_{01}), v_1(E^1_{10}) - v_1(E^1_{00})\}$,

$g_2'(0) < \min\{v_2(E^2_{11}) - v_2(E^2_{10}), v_2(E^2_{01}) - v_2(E^2_{00})\}$,

(ii) $g''' > 0$ everywhere or $g''' < 0$ everywhere,

then there is a unique agent equilibrium with respect to (E^1, E^2).

PROOF: See Appendix 1.

For every (E^1, E^2) we let the equilibrium of the managers' game be given by the functions $p_1^* = P^i(E^1, E^2)$, $i = 1,2$. P^i gives agent i's equilibrium choice of p_i when (E^1, E^2) are the incentives determined by the owners. $P^\alpha(E^1, E^2)$ will give the probability of state α when owners give the incentives (E^1, E^2) and is related to P^1 and P^2 in the obvious fashion; for example $P^{10}(E^1, E^2) = P^1(E^1, E^2)(1 - P^2(E^1, E^2))$.

Having dealt with the agents' game, we now discuss the game between the principals. In stage 1, the owners simultaneously choose their incentive schemes. In particular, if owner one believes that owner two will choose E^2, then he solves

$$\max_{E^1} \sum_\alpha u_1(\pi^1_\alpha - E^1_\alpha) P^\alpha(E^1, E^2)$$

s.t.

$$\sum_\alpha v_1(E^1_\alpha) P^\alpha(E^1, E^2) \geq \bar{v}_1$$

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incentives (E^1, E^2) are such that guarantee uniqueness. Such an analysis is beyond the scope of this paper.

Since the equilibrium selection mechanism is not necessarily continuous there may not be a solution to the owners' problem. Assuming one exists, it defines owner one's reaction correspondence:

$$(6) \qquad E^1 \in \Omega^1(E^2).$$

Note that Ω^1 may be multi–valued, or, if there is no solution to (4), null–valued. Similarly, owner two has a reaction correspondence $E^2 \in \Omega^2(E^1)$. An *owner's equilibrium* is any (E^1, E^2) pair such that $E^1 \in \Omega^1(E^2)$ and $E^2 \in \Omega^2(E^1)$.

DEFINITION 2: An incentive scheme E^1 is *manipulable* whenever agent one's optimal choice of p_1 depends besides on the compensation scheme he has, on agent two's choice of p_2. By symmetry, it is clear what "E^2 is manipulable" means.

The definition of manipulability is central to our analysis as it is a necessary condition for having the strategic element of the contract. The manipulability of agent i compensation scheme implies that principal j can affect his choice of p_i. When agent i has a nonmanipulable compensation scheme then principal j faces a standard principal–agent problem. He cannot affect p_i and thus faces a standard tradeoff between risk sharing and incentive provision. The strategic aspect arises when agent i is provided with manipulable incentives. In such cases the contract provided by principal j simultaneously mitigates the incentive problem and serves as a precommitment device that enables principal j to gain competitive advantage.

PROPOSITION 3: Agent 1 compensation scheme E^1 is non–manipulable only when the gain from success is independent of the success (or failure) of the second agent, i.e., E^1 is nonmanipulable if and only if:

$$v_1(E^1_{11}) - v_1(E^1_{01}) = v_1(E^1_{10}) - v_1(E^1_{00}) \tag{7}$$

PROOF: When condition (7) holds the first agent incentive compatible condition (2) becomes

$$v_1(E^1_{10}) - v_1(E^1_{00}) = g'(p_1). \tag{8}$$

Thus given the compensation scheme E^1 agent one's choice of p_1 is uniquely defined by (8) and it is not affected by agent two's choice of p_2.

When condition (7) does not hold the incentive compatible condition (2) depends on p_2 which implies that the optimal choice of p_1 is a function of agent two's choice of p_2.

□

One of the concerns of the following analysis is to identify the cases under which at equilibrium incentives are manipulable.

Consider now the first principal maximization problem:

$$\max_{E^1, p_1, p_2} \sum_\alpha p_\alpha u_1(\pi^1_\alpha - E^1_\alpha) \tag{9}$$

s.t.

$$\sum_\alpha p_\alpha v_1(E^1_\alpha) - g(p_1) \geq \bar{v}_1 \tag{10a}$$

(10b) $$p_2[v_1(E^1_{11}) - v_1(E^1_{01})] + (1-p_2)[v_1(E^1_{10}) - v_1(E^1_{00})] - g'(p_1) = 0$$

(10c) $$p_1[v_2(E^2_{11}) - v_2(E^2_{10})] + (1-p_1)[v_2(E^2_{01}) - v_2(E^2_{00})] - g'(p_2) = 0$$

$$p_1 \geq 0.$$

The first constraint (10a) is the agent's reservation utility and has a shadow value η. η is obviously positive for this problem since the owner will always choose an E^1 such that the incentive constraint is binding. Equation (10b) is agent one's incentive compatibility constraint and is given a shadow value of μ. Constraint (10c) is the incentive compatibility condition for agent two and is not degenerate if $v_2(E^2_{11}) - v_2(E^2_{10}) \neq v_2(E^2_{01}) - v_2(E^2_{00})$, i.e., if E^2 is manipulable. It is given a shadow price λ in the manipulable case; otherwise we will let λ be zero in the Kuhn–Tucker conditions below. This unifies the nonmanipulable and manipulable cases.

First, suppose that we are in equilibrium with incentives at (E^1, E^2) and effort choices of (p_1, p_2). Clearly, owner one may choose any incentive scheme E^1 which can result in that agent equilibrium. Therefore, the choice of E^1 for owner one must be optimal among all such schemes which are consistent with equations (10b, c) for the equilibrium (p_1, p_2). That is problem (9) with E^1 as the choice variable and p_1 and p_2 set equal to their values in the hypothesized equilibrium. The resulting first–order conditions for E^1 takes the form of:

$$(\eta\, p_1 + \mu)v_1'(E^1_{10}) - p_1 u_1'(\pi^1_{10} - E^1_{10}) = 0$$

$$(\eta p_1 + \mu)v_1'(E_{11}^1) - p_1 u_1'(\pi_{11}^1 - E_{11}^1) = 0$$

(13)
$$(\eta(1-p_1) - \mu)v_1'(E_{01}^1) - (1-p_1)u_1'(\pi_{01}^1 - E_{01}^1) = 0$$

$$(\eta(1-p_1) - \mu)v_1'(E_{00}^1) - (1-p_1)u_1'(\pi_{00}^1 - E_{00}^1) = 0.$$

From (11) and (12) we obtain that:

$$\frac{v_1'(E_{10}^1)}{u_1'(\pi_{10}^1 - E_{10}^1)} = \frac{v_1'(E_{11}^1)}{u_1'(\pi_{11}^1 - E_{11}^1)}$$

while (13) and (14) yield that:

$$\frac{v_1'(E_{01}^1)}{u_1'(\pi_{01}^1 - E_{01}^1)} = \frac{v_1'(E_{00}^1)}{u_1'(\pi_{00}^1 - E_{00}^1)}.$$

Equations (15) and (16) indicate the following:

COROLLARY 1: Given that agent i succeeds, the equilibrium incentives scheme provides optimal risk sharing with respect to the risk of agent j succeeding. Similar optimal risk sharing holds when it is known that agent i fails

These risk sharing rules (16) and (17) imply the following:

PROPOSITION 4: (i) When both the principal and the agent are risk averse then at equilibrium they both get higher payoffs when the rival agent fails, i.e.

(17a) $$E^1_{10} > E^1_{11} \; ; \; E^1_{00} > E^1_{01}$$

(17b) $$\pi^1_{10} - E^1_{10} > \pi^1_{11} - E^1_{11} \; ; \; \pi^1_{00} - E^1_{00} > \pi^1_{01} - E^1_{01}.$$

(ii) If the principal is risk neutral and the agent is risk averse then, $E^1_{01} = E^1_{00}$ and $E^1_{10} = E^1_{11}$ so the agent is fully insured with respect to its rival's success and the principal gets higher payoffs when the rival agent fails.

(iii)When the agent is risk neutral and the principal is risk averse then at the incentive equilibrium the principal is insured with respect to the rival agent's success while the agent receives higher payoffs when the rival agent fails.

PROOF: (i) Let us assume a–contrario that $E^1_{11} \geq E^1_{10}$. Equation (15) and the concavity of v_1 implies that $u_1'(\pi^1_{10} - E^1_{10}) \geq u_1'(\pi^1_{11} - E^1_{11})$ which by the concavity of u_1 implies that $\pi^1_{11} - E^1_{11} \geq \pi^1_{10} - E^1_{10}$. Since $\pi^1_{10} > \pi^1_{11}$ and together with our assumption that $E^1_{11} \geq E^1_{10}$ yields a contradiction. In a similar way we can prove that $E^1_{00} > E^1_{01}$.

Given that $E^1_{10} > E^1_{11}$ and using the concavity of u_1 and v_1, equation (15) implies that $\pi^1_{10} - E^1_{10} > \pi^1_{11} - E^1_{11}$. Equation (16), together with $E^1_{00} > E^1_{01}$ implies that $\pi^1_{00} - E^1_{00} > \pi^1_{01} - E^1_{01}$.

(ii) and (iii) are proven similarly by using the risk sharing rules (15) and (16).

□

PROPOSITION 5: (i) when the principal is risk neutral and the agent is risk averse the optimal compensation scheme is nonmanipulable.

(ii) when the principal is risk averse and the agent is risk neutral the optimal compensation scheme is manipulable.

PROOF: (i) when principals are risk neutral equation (15) implies that $v_1'(E^1_{10}) = v_1'(E^1_{11})$. Using the concavity of v_1, this condition implies that $E^1_{10} = E^1_{11}$. In a similar way we can use equation (16) to obtain that $E^1_{01} = E^1_{00}$. These two conditions imply that $v_1(E^1_{11}) - v_1(E^1_{10}) + v_1(E^1_{00}) - v_1(E^1_{01}) = 0$ and thus E^1 is nonmanipulable.

(ii) letting agents be risk neutral we can use equations (15) and (16) to obtain that $\pi^1_{10} - E^1_{10} = \pi^1_{11} - E^1_{11}$ and $\pi^1_{01} - E^1_{01} = \pi^1_{00} - E^1_{00}$. These conditions imply that $(\pi^1_{11} - \pi^1_{01}) + (E^1_{01} - E^1_{11}) = (\pi^1_{10} - \pi^1_{00}) + (E^1_{00} - E^1_{00})$. Since we assume that $\pi^1_{11} - \pi^1_{01} \neq \pi^1_{10} - \pi^1_{00}$ we obtain that any best reply by an owner is such that $E^1_{01} - E^1_{11} + E^1_{10} - E^1_{00} \neq 0$ which implies that E^1 is manipulable.

□

Proposition 5 indicates that in the commonly used setting of risk neutral principal and risk averse agent the strategic aspect of the incentives disappears and the equilibrium contracts take into account only the moral hazard element as they cannot affect the choice of action of the rival agent. Note, however, that in discussing rivalrous agency with risk neutral principal and risk averse agent the problem is still not identical to the standard principal–agent problem. The first principal in determining the optimal contract takes into account the contract given to the second agent. Specifically, although the incentives are non manipulable the optimal contract depends on the rival's choice of strategy. Thus in such an analysis we are still looking for incentive equilibrium, but the strategic precommitment aspect is missing as equilibrium incentives are nonmanipulable.

Going back to the principal's maximiation problem (9–10d) the first–order condition with respect to p_1 is

$$(18)\quad p_2u_1(\pi^1_{11} - E^1_{11}) + (1-p_2)u_1(\pi^1_{10}-E^1_{10}) - p_2u_1(\pi^1_{01} - E^1_{01})$$
$$- (1-p_2)u_1(\pi^1_{00} - E^1_{00}) - \mu g'(p_1) + \lambda[v_2(E^2_{11}) - v_2(E^2_{10}) - v_2(E^2_{01}) +$$
$$+ v_2(E^2_{00})] = 0.$$

When the second agent is given a manipulable incentive scheme and the first agent can affect the choice of p_2 the f.o.c. with respect to p_2 is

$$(19)\quad p_1u_1(\pi^1_{11}-E^1_{11}) + (1-p_1)u_1(\pi^1_{01}-E^1_{01}) - p_1u_1(\pi_{10}-E_{10})$$
$$- (1-p_1)u_1(\pi^1_{00}-E^1_{00}) + \eta[p_1v_1(E^1_{11}) + (1-p_1)v_1(E^1_{01})$$
$$- p_1v_1(E^1_{10}) - (1-p_1)v_1(E^1_{00})] + \mu[v_1(E^1_{11}) - v_1(E^1_{01})$$
$$- v_1(E^1_{10}) + v_1(E^1_{00})] - \lambda g''(p_2) = 0.$$

The strategic precommitment aspect of the optimal contract is better illustrated through (18)–(19). When E^2 is nonmanipulable $v_2(E^2_{11}) - v_2(E^2_{10}) - v_2(E^2_{01}) + v_2(E^2_{00}) = 0$ so that the last expression in (18) is zero and the f.o.c. with respect to p_1 is similar to the one in a regular P—A problem. Equation (19) on the other hand needs to hold only when E^2 is manipulable. Notice that eq.(10c) is the reaction function of agent 2 in the agents' game. Describing this reaction function in the $p_1 \times p_2$ plane yields that the slope of ϕ_2 is

$$\left.\frac{dp_1}{dp_2}\right|_{\phi_2} = -\frac{v_2(E^2_{11}) - v_2(E^2_{10}) - v_2(E^2_{01}) + v_2(E^2_{00})}{g''(p_2)}$$

The principals control the shape of the agents' reaction function through the compensation scheme they provide. When $v_2(E^2_{11}) - v_2(E^2_{10}) - v_2(E^2_{01}) + v_2(E^2_{00}) < 0$ then $\left.\frac{dp_1}{dp_2}\right|_{\phi_2} > 0$, i.e., the reaction function is positively sloped and in the language of Bulow Geanakoplos and Klemperer (1985) p_1 and p_2 are strategic complements. When $v_2(E^1_{01}) - v_2(E^2_{11}) + v_2(E^2_{10}) - v_2(E^2_{00}) < 0$ the reaction function ϕ_2 is negatively sloped and p_1 and p_2 are strategic substitutes. Indeed one can see that the sign of the last expression in (18) depends on whether the rival's reaction function, in the agent game, is upward or downward sloping.

3. INCENTIVE EQUILIBRIUM WITH RISK NEUTRAL AGENTS

In this section we study a special case in which both agents are risk neutral while the principals are risk averse. As Proposition 5 indicates this assumption guarantees that the equilibrium contracts are manipulable. While the above is admittedly a restrictive assumption there are situations where this assumption is quite reasonable. For example, consider the case of authors of similar books and their contracts with their publishers. The authors are principals and the publishers are their agents hired to market the books. In this case, assuming risk averse authors is reasonable, but publishers are likely to be risk neutral since the systematic risks are small. Similarly, in our earlier innovation example, the entrepreneurs are likely to be risk averse, but research laboratories handling many projects are likely to be substantially less risk averse.

The risk–neutral agent assumption has a strong implication in the usual principal–agent literature: the firm should be sold to the agent. When we add our strategic elements to the usual principal–agent model we will find systematic deviations from these simple results.

DEFINITION 3: An incentive scheme E^i is a *sell–out* of firm to its agent whenever π^i_α E^i_α is independent of α.

In a sell–out scheme, the owner of firm i receives a payment which is independent of the realized state of the market. Such contracts are a useful benchmark in comparing our results with the usual principal–agent problem where the optimal contract is a sell–out if the agent is risk–neutral. Therefore, any deviation from sell–out contracts in the risk–neutral manager case arises solely from the strategic elements. Propositions 4 and 5 indicate that given our assumptions the equilibrium incentive schemes are manipulable, the principals' equilibrium utilities are independent of their rival success or failure, i.e., $\pi^1_{j1} - E^1_{j1} = \pi^1_{j0} - E^1_{j0}$, j = 0,1 and that agents receive higher payoffs when their rival agent fails.

PROPOSITION 6: Assume that the agents' equilibrium is unique for any incentive scheme and that $g'(0) < \min\{(\pi^1_{1j} - \pi^1_{0j}),(\pi^2_{j1} - \pi^2_{j0})\}$, $j = 1,2$ and $\pi^i_{11} - \pi^i_{01} \neq \pi^i_{10} - \pi^i_{00}$ for $i = 1,2$, then in equilibrium.

$$((E^1_{1j} \quad E^1_{0j}) \quad (\pi^1_{1j} - \pi^1_{0j})) \cdot ((\pi^2_{01} \quad \pi^2_{00}) \quad (\pi^2_{11} \quad \pi^2_{10})) > 0, \ j = 0,1$$

i.e., agent one receives a marginal payment for success, $E^1_{1j} - E^1_{0j}$, which exceeds the marginal revenue to the firm, $(\pi^1_{1j} - \pi^1_{0j})$, if and only if the marginal return to effort for firm two is greater when firm one fails. In particular, the equilibrium incentive scheme will not be a sell–out to the agent and the owner of firm one is better off when his agent succeeds if and only if the opposing firm's marginal revenue is greater when firm one fails than when it succeeds.

PROOF: Given our assumption of risk neutral agents, and using our result that the principal payoffs are independent from the rival agent's success or failure the first–order conditions for E^1_{11}, E^1_{01}, and p_1 (12), (13) and (18) respectively are reduced to:

$$-p_1 u_1'(\pi^1_{11} \quad E^1_{11}) + \mu + \eta p_1 = 0$$

$$-(1 - p_1) u_1'(\pi^1_{01} \quad E^1_{01}) \quad \mu + \eta(1 - p_1) \quad 0$$

$$u_1(\pi^1_{11} \quad E^1_{11}) \quad u_1(\pi^1_{01} - E^1_{01}) \quad \mu g \quad (p_1) + \lambda((\pi^2_{11} - \pi^2_{10}) \quad (\pi^2_{01} \quad \pi^2_{00})) = 0.$$

Since Proposition 5 indicates that incentives will be manipulable, the first–order condition for p_2 (eq.(19)) reduces to:

$$(24) \quad \mu(\pi^1_{11}-\pi^1_{01}-\pi^1_{10}+\pi^1_{00})-\lambda g'(p_2)+\eta(p_1(\pi^1_{11}-\pi^1_{10})+(1-p_1)(\pi^1_{01}-\pi^1_{00}))=0.$$

At this point we can gain understanding about how this structure relates to the usual principal–agent case. Suppose that agent two is not manipulable. Then by (23), μ and $u_1(\pi^1_{11}-E^1_{11})-u_1(\pi^1_{01}-E^1_{01})$ have the same sign. By (21) and (22), μ also has the same sign as $u_1'(\pi^1_{11}-E^1_{11})-u_1'(\pi^1_{01}-E^1_{01})$. These implications for u_1, however, contradict concavity of u_1 unless $\mu=0$. Therefore, in the nonmanipulable case the owner's utility is independent of state implying that he sells the firm.

In the manipulable case, λ becomes important. Signing λ is accomplished by noting that the incentive compatibility constraint for agent one can be written as a binding nonpositivity constraint, in which case λ must be nonpositive. Such an inequality constraint loosens up the constraint relative to the initial equality constraint. However, if $\lambda=0$, in particular if the relaxed nonpositivity constraint were not binding, the previous paragraph implies that $\mu=0$ and ownr sells out. Such would cause (24) to degenerate to the equation $\pi^1_{11}-\pi^1_{10}=\pi^1_{01}-\pi^1_{00}$, which violates our assumption. Therefore, the inequality constraint would be binding, representing an inessential loosening of the incentive compatibility constraint but implying that $\lambda<0$.

To ascertain the nature of the owner's return for the manipulable case, we use the following chain of equivalences:

$$\mu > 0 \text{ iff } u_1'(\pi_{11}^1 - E_{11}^1) - u_1'(\pi_{01}^1 - E_{01}^1) > 0, \text{ by (21), (22), } \eta > 0$$

$$\text{iff } u_1(\pi_{11}^1 - E_{11}^1) - u_1(\pi_{01}^1 - E_{01}^1) < 0, \text{ by concavity of } u_1$$

$$\text{iff } \mu g \ \ (p_1) - \lambda(\pi_{11}^2 - \pi_{10}^2 - \pi_{01}^2 + \pi_{00}^2) < 0, \text{ by (23).}$$

Hence, by (25) and $\lambda < 0$,

$$\mu > 0 \text{ iff } (\pi_{11}^2 - \pi_{10}^2) - (\pi_{01}^2 - \pi_{00}^2) < 0.$$

$\mu > 0$ iff $u_1(\pi_{11}^1 - E_{11}^1) - u_1(\pi_{01}^1 - E_{01}^1) < 0$, we find

$$(u_1(\pi_{11}^1 - E_{11}^1) - u_1(\pi_{01}^1 - E_{01}^1))(\pi_{11}^2 - \pi_{10}^2 - \pi_{01}^2 + \pi_{00}^2) > 0$$

i.e., owner one's utility is greater when his agent succeeds only if firm two's *marginal* return to effort is also greater when agent one succeeds.

The last thing to prove is the nature of agent one's incentives relative to his marginal product. If, conditional on firm two succeeding, the owner is better off when his agent succeeds then $\pi_{11}^1 - E_{11}^1 > \pi_{01}^1 - E_{01}^1$, which implies that $\pi_{11}^1 - \pi_{01}^1 > E_{11}^1 - E_{01}^1$, that is, the marginal return to the firm of success exceeds agent one's marginal payment for success. Therefore, the marginal payment to the agent for success exceeds the firm's marginal revenue exactly when $(\pi_{11}^2 - \pi_{10}^2) - (\pi_{01}^2 - \pi_{00}^2) < 0$.

□

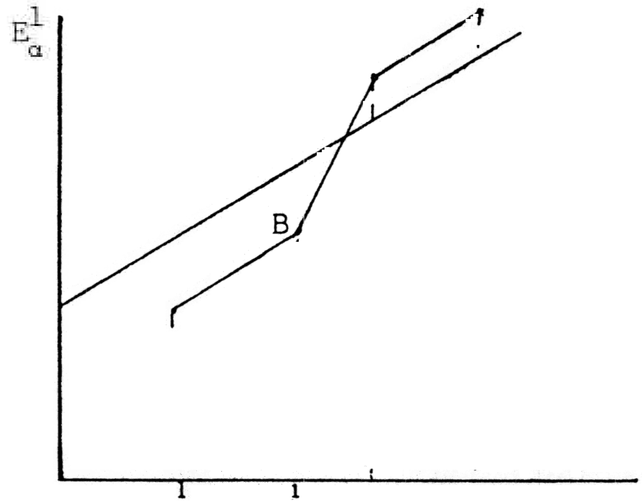
To understand the implications of this result, suppose that $(\pi^2_{11}-\pi^2_{10})-(\pi^2_{01}-\pi^2_{00}) < 0$. This would often be the case for producers of a homogeneous good since if my opponent is producing a substantial amount, i.e., he is successful, then that output so depresses price that I have relatively little to gain by high versus low production compared to the situation if my rival produced little. Proposition 6 says that an owner's return is greater when his manager *fails* since that is when his rival's marginal return to success is greater and also that the marginal incentive of a firm's manager to succeed *exceeds* the marginal return to the firm. Why would an owner do this? In order to reduce the rival's effort, one must reduce his incentives to succeed. By increasing one's own manager's incentives to succeed, a manager in this case shifts probability away from those states where my manager fails and my rival has the larger incentive to succeed. Such a shift reduces the rival firm's marginal revenue as long as that agent's incentive is greater when marginal revenue is greater, which is the case in equilibrium since $\pi^1_{j1} - E^1_{j1} = \pi^1_{j0} - E^1_{j0}$, $j = 0,1$, implies that $(E^1_{11} - E^1_{01}) - (E^1_{10} - E^1_{00}) = (\pi^1_{11} - \pi^1_{01} - \pi^1_{10} + \pi^1_{00})$.

While it may appear odd that the owner prays for his manager to fail, recall that in the nonrivalrous principal–agent model the owner would sell the firm and not care if the manager failed. The key fact is that the strategic elements of contracting investigated here shift risk from the manager to the owner. In our example it is efficient for the owner to bear no risk but he does so in order to strategically manipulate his agent's incentives.

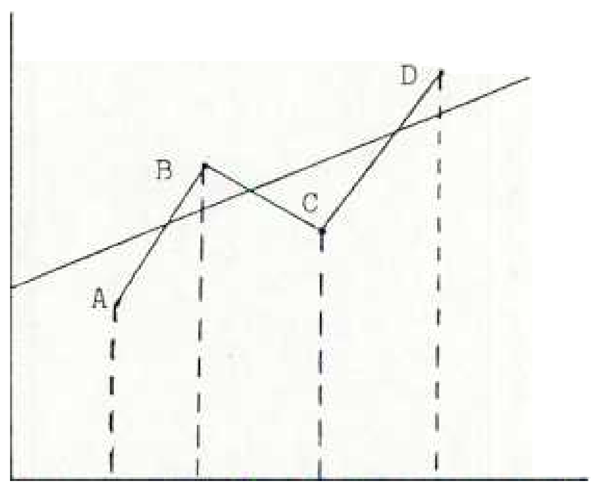
While the conditions of Proposition 6 are strong, it is apparent from its proof that the assumption of unique agent equilibrium in all possible agent games is excessive. Intuitively, the characterization of equilibrium in Proposition 6 holds for any equilibrium where all agent equilibria are unique given either owner's contract. For example, if the equilibrium contract of firm two is such that the agent equilibrium is unique given any contract choice by firm one, then firm one's choice will solve (9) and will be characterized by the resulting first–order conditions. This observation indicates that the content of Proposition 6 is not completely dependent on the strong uniqueness assumption.

Some may argue that it is more reasonable to assume that an agent's compensation is based on his firm's profits. That does not affect our analysis at all as long as π^1_α is different for different states α, and similarly for firm 2. However, it is an interesting way to view our compenation schemes. Suppose $\pi^2_{11}-\pi^2_{10} < \pi^2_{01}-\pi^2_{00}$, the case where each firm's incentives to succeed are greater if its opponent fails. If we graph compensation, E^1_α, against π^1_α, the sell–out compensation scheme is a straight line since $\pi^1_\alpha - E^1_\alpha$ is independent of α under a sell–out. When we graph managerial compensation against profits in our model we obtain two basic cases. If $\pi^1_{00} < \pi^1_{11}$, a natural case since it essentially says that a firm's own success is more important to its profits than the outcome at its opponent's firm, figure 1a obtains. Points A, B, C and D represent the possible $(\pi^1_\alpha, E^1_\alpha)$ pairs. The curve connecting these points is convexo–concave since the slope between A and B is one since $\pi^1_{01} - E^1_{01} = \pi^1_{00} - E^1_{00}$. Similarly, the slope between C and D is one. However, the slope between B and C exceeds one since $\pi^1_{11} - \pi^1_{01} < E^1_{11} - E^1_{01}$. The result is a compensation scheme which is convexo–concave in profits. In this case, the manager receives the greatest incentive to raise profits when profits are in the middle of the range of possible profits.

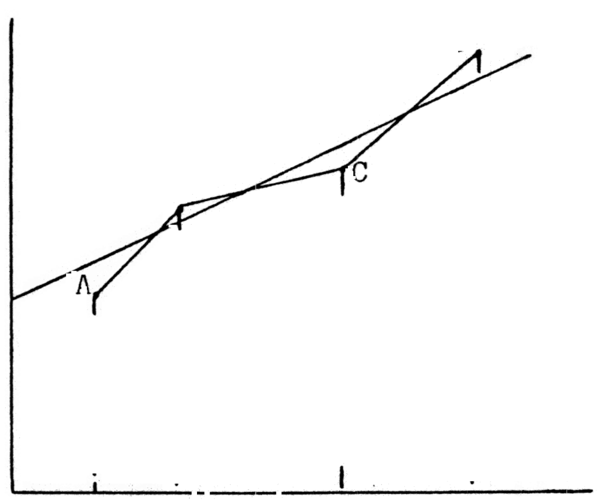
If $\pi^1_{00} > \pi^1_{11}$, figure 1b or 1c obtains. By similar reasoning, compensation is concavoconvex. In (1b) compensation is not monotonic in profits, but fails to be because the agent is paid more when he succeeds, a reasonable condition. In (1c), compensation is monotonic as well as concavo–convex.



E_{α}^{1}
B



D
B
C
A



C
A

If $\pi^2_{11}-\pi^2_{10} > \pi^2_{01}-\pi^2_{00}$, then the considerations are the same except concavo–convex compensation results when $\pi^1_{00} < \pi^1_{11}$. This is the case where outcomes are complementary, agent's reaction curves are upward sloping instead of downward sloping, and owners give insurance to managers against failure in order to get them to work less hard.

4. LOCAL EXISTENCE AND UNIQUENESS

Having established that equilibria have interesting properties, we should next prove that equilibria to our game do exist We will not prove a general existence theorem In the first place, there are cheap ways of specifying our model so as to assure existence. We could have assumed that contracts had to be confined to some large but finite collection of possible contracts (e.g., compensation must be expressed in whole dollars) and that g satisfies conditions sufficient to assume unique agent equilibria. In that case, an equilibrium always exists for the managers, and, given a selection mechanism for the managerial equilibria, an equilibrium exists for the owners' game. These existence results follow from the standard existence theorem for finite games. Of course, the equilibria at the owner stage may be mixed strategy equilibria.

Continuing with our continuous strategy approach we identify a collection of games for which equilibria with pure strategies exists and are unique. This will show that we are not talking about a class of situations of measure zero. In this demonstration we will appeal to the implicit function theorem to show that equilibrium will exist and be unique if the interactions between the firms are sufficiently small

If we assume that $\bar{\pi}$ represents our profit structure and that $\bar{\pi}^1_{i,j} = \bar{\pi}^1_{i,1-j}$, $\bar{\pi}^2_{i,j} = \bar{\pi}^2_{1-i,j}$, for i, j = 0,1, then each firm is unaffected by the others' success and the problems reduce to two separate principal–agent problems and as we have seen, this will cause the owners to sell their firms to their agents. In particular, $E^1_{10} = E^1_{11}$, etc.

The reaction curves of the owners in their game are degenerate in this case, with Ω^1 and Ω^2 being constant functions. This trivially implies that the equilibrium between owners is unique. Next suppose that

$$\pi^i_\alpha = \bar{\pi}^i_\alpha + \epsilon\,\Delta^i_\alpha$$

where ϵ is "small." By allowing Δ to be arbitrary but ϵ small, we assume that the interactions of the payoffs are arbitrarily complex, but of small magnitude. We will use the uniqueness of equilibrium for $\epsilon = 0$ to show uniqueness for small

First, note that ϵ has no impact on the game between the managers since their equilibrium depends only on the incentive schemes, E^i. Moreover, since the compensation schemes under a sell–out have no competitive elements, the reaction functions are "flat," i.e.

$$\frac{\partial\phi_1}{\partial p_2} = \frac{\partial\phi_2}{\partial p_1} = 0$$

at the equilibrium choices of E^i when $\epsilon = 0$. Therefore, there is no multiplicity problem in the managers' game and P^1 and P^2 are uniquely defined by the equilibrium conditions corresponding to (E^{1*}, E^{2*}), the equilibrium choices of (E^1, E^2) when $\epsilon = 0$. By the continuity of the reaction functions, ϕ_1 and ϕ_2, and the compactness of the strategy space for the agents, this uniqueness holds for $\vec{P}$ in a neighborhood of (E^{1*}, E^{2*})

To complete the argument, we must determine the nature of Ω^1 and Ω^2 around (E^{1*}, E^{2*}) as ϵ increases. Since ϵ affects profits, it is a parameter of Ω^i, $i = 1,2$. We are concerned with solutions to the equation

$$(30) \qquad \Psi(E^1, E^2, \epsilon) = \begin{pmatrix} \Omega^1 \\ \Omega^2 \end{pmatrix} - \begin{pmatrix} E^1 \\ E^2 \end{pmatrix} = 0.$$

At $\epsilon = 0$, we have a unique solution for (E^1, E^2). To show unique solutions when ϵ is small, we just need to show that Ψ has a nonsingular Jacobian in (E^1, E^2) and that $\partial\Psi/\partial\epsilon$ exists at $\epsilon = 0$. However, $\partial\Omega^i/\partial E^j = 0$ at $\epsilon = 0$. Hence

$$(31) \qquad \frac{\partial\Psi}{\partial\begin{pmatrix} E^1 \\ E^2 \end{pmatrix}} = \begin{pmatrix} I_4 & 0 \\ 0 & I_4 \end{pmatrix}$$

which is clearly nonsingular.

The existence of $\partial\Omega^i/\partial\epsilon$ is related to the maximation problem of owner i since its calculation holds fixed E^{3-i} at its nonrivalrous value. Smooth dependence of i's choice of E^i for ϵ near zero depends on the Jacobian of i's system of first–order conditions. That Jacobian must be negative semidefinite by the second–order condition of owner i. We would expect that the Jacobian will usually be negative definite since the sign of its determinant depends on high–order derivatives which do not affect the fact that the initial optimal contract satisfies the first–order conditions.

These arguments demonstrate the following proposition:

PROPOSITION 7: If the dependence of each firm's profits on its rival's output is sufficiently small, then equilibrium exists and is unique.

A more general analysis of existence for our model would be desirable, but is unattained as yet. However, Proposition 7 shows that our earlier analysis assuming existence is not vacuous and does apply to a nontrivial set of cases. Further development of the approach of Proposition 7 could also reveal more general insights about the interactions between the tastes of the principals and agents, payoffs, and the resulting contracts. For these reasons, Proposition 7 is interesting in spite of its local nature.

5. CONCLUDING REMARKS

The main focus of this paper is the study of rivalrous agency, the generalization of agency theory to situations where principal–agent teams compete and principals can effectively communicate the nature of the incentives they give to their agents. In so doing we adopt the standard framework of agency theory and assume that principals cannot fully observe the agents' strategy choices. Within such a structure we study the relationships between external conditions and determination of internal incentives as contracts simultaneously mitigate incentive problems and serve as precommitment devices that enable principals to gain competitive advantage.

APPENDIX 1: PROOF OF PROPOSITION 2

PROOF: By (i) and (2) $\phi_i(p_{3-i}) > 0$ for $i = 1,2$. Next recall that

$$\text{(A1.1)} \qquad g'(\phi_1(p_2)) = p_2[v_1(E^1_{11}) - v_1(E^1_{01})] + (1 - p_2)[v_1(E^1_{10}) - v^1(E^1_{00})].$$

$$\text{(A1.2)}\, \phi_1'(p_2) = \frac{[v_1(E^1_{11}) - v_1(E^1_{01})] - [v_1(^1_{10}) - v_1(E^1_{00}) - v_1(E^1_{00})]}{g''(\phi_1(p_2))} \equiv \frac{D}{g(\phi_1(p_2))}$$

$$\text{(A1.3)}\, \phi_1''(p_2) = -\frac{Dg'''(\phi_1(p_2))\phi_1'(p_2)}{(g''(\phi_1(p_2)))^2} = -\frac{D^2 g'''(\phi_1(p_2))}{(g''(\phi_1(p_2)))^3} \gtrless 0 \text{ iff } g'''(\phi_1(p_2)) \gtrless 0.$$

If g_1''' and g_2''' are both always positive then $\phi_1(p_2)$ and $\phi_1(p_1)$ are concave functions with domain $[0,1]$ and range $[a,b]$ for some $a > 0$ and $b < 1$. They are either as represented in Figure a or b when both reaction functions are decreasing functions. Similar pictures apply when both are increasing. If one is increasing and the other is decreasing, uniqueness is assured. This exhausts the cases since ϕ' is of one sign.

In either case, it is clear that there is a unique agent intersection. Similarly, if g_1 and g_2''' are both always negative there is a unique agent equilibrium under (i).

□

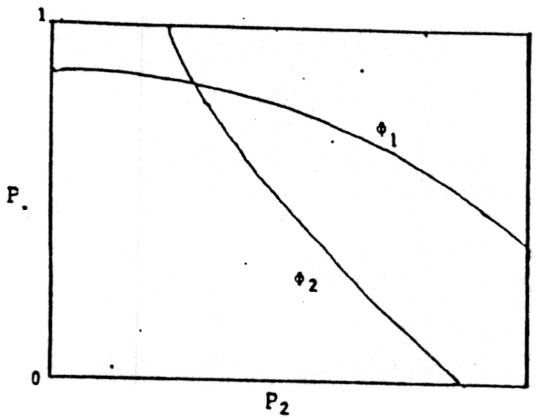


Figure a

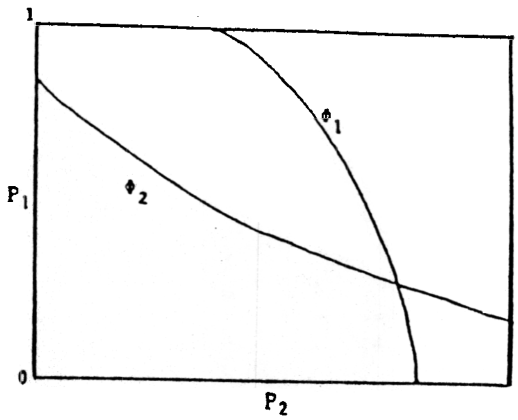


Figure b

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